

# IMTECH

# 8

# Newsletter

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### Editorial

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Voilà a new issue of our annual **IMTech** Newsletter! This is a great achievement, mostly from all contributors, but also from **IMTech** mathematical community at UPC that make this possible, jointly with the Editorial Committee. In this issue you will find interviews to **IMTech** members that have been recently promoted to Full Professorship: [Imma Baldomà](#) <sup>↗</sup>, [Jezabel Curbelo](#) <sup>↗</sup>, [Francesc Pozo](#) <sup>↗</sup> and [Gemma Huguet](#) <sup>↗</sup>. We also bring forward diverse but some representative work in the **Research Focus** section: recent results in spiral wave solutions for reaction-diffusion systems, by [Imma Baldomà](#) <sup>↗</sup>, modelling of collective migration of cells and robots swarms, by [Gissell Estrada](#) <sup>↗</sup>, a review on geometric algebra, by [Sebastià Xambó](#) <sup>↗</sup>, and a summary of methods for the analysis of libration points orbits in celestial mechanics, by [Josep Masdemont](#) <sup>↗</sup>. The work of Prof. Xambó is dedicated to the memory of Professor Miguel C. Muñoz Lacanda, who passed away last year and with whom Prof. Xambó maintained a long collaboration. The summary of Prof. Masdemont corresponds to a keynote lecture he presented at the [International Astronautical Congress](#) <sup>↗</sup> that took place in Sidney during September-October last year. We are also delighted to include some summaries of recent **PhD theses** supervised by **IMTech** members, and authored by: [José Lamas](#) <sup>↗</sup> (sup. M. Guàrdia and T. M-Seara Alonso), [Pradeep Bal](#) <sup>↗</sup> (sup. M. Arroyo), [Rafel Perelló](#) <sup>↗</sup> (sup. A. Huerta and S. Zlotnik) and [Sergi Pérez](#) <sup>↗</sup> (sup. S. Fernández and I. Arias). Our **Outreach** includes contributions in the modelling of contaminant removal, by [Marc Calvo-Schwarzwalder](#) <sup>↗</sup>, and on surface effects in flexoelectricity, by [David Codony](#) <sup>↗</sup>. All these topics illustrate the high quality of the research in mathematics across different departments at **UPC** <sup>↗</sup>. No other field than mathematics can cover such a wide umbrella of areas at UPC: astrodynamics, neuroscience, aerodynamics, soft and living matter, and many more that we could not include. Please, do [contact us](#) <sup>↗</sup>, to avoid any omissions in our next issues.

We take this opportunity to thank the former director of **IMTech**, [Marc Noy](#) <sup>↗</sup>, for his monumental effort on building up the structure for the research institute and setting the strategic directions. The fact that we have received 36 new applications for **IMTech** memberships in our last call, with more than 30% coming from new young researchers reflects the interest that the institute is attracting, and the thrilling activity that mathematics is undergoing at UPC. We just mention some successful cases from **IMTech** members: [Eva Miranda](#) <sup>↗</sup> has been appointed **Gauss Professor** for the 2025–2026 academic year at the [Göttingen Academy of Sciences and Humanities](#) <sup>↗</sup> and in 2027 she will be the **Karen Uhlenbeck Lecturer** at the [Institute for Advanced Study in Princeton](#) <sup>↗</sup>, [Xavier Cabré](#) <sup>↗</sup> is an invited speaker at the International Congress of Mathematics (ICM 2026 <sup>↗</sup>), [Marino Arroyo](#) <sup>↗</sup> has received the *Programa Acadèmica Excel·lència* from AGAUR and is a partner of the [LaCaixa Research grant](#) <sup>↗</sup> in colon cancer research, [Francesc Pozo](#) and [Yolanda Vidal](#), jointly with student [Shun Wang](#) received the first prize in the [WeDoWind ASCE-EMI Structural Health Monitoring for Wind Energy Challenge](#) <sup>↗</sup>, [Gissell Estrada](#) has won a Leonardo grant from [Fundación BBVA](#), and [Jezabel Curbelo](#) has received the *Premio Nacional de Investigación "María Andresa Casamayor"* in the area of mathematics and information and communication technologies. We can just join in these recognitions that emphasize the high quality of their research.

This Newsletter 8 ends with a **Chronicles** section of the [2nd IMTech day](#) <sup>↗</sup> that took place last November, the [IMTech Colloquium](#) in March 2025 by [Anup Rao](#) <sup>↗</sup>, and the [International Conference on Mathematical Neuroscience \(ICMNS-2025\)](#) <sup>↗</sup>, organized by **IMTech** members. At this point, it is pertinent to announce our next [Colloquium](#) <sup>↗</sup> that will take place the [25th of February 2026](#), with a talk by [Gemma de les Coves](#) <sup>↗</sup>, ICREA Professor at Universitat Pompeu Fabra. Please, try not to miss!



Imma Baldomá<sup>✉</sup> has recently been appointed as a Full Professor in Applied Mathematics at the Universitat Politècnica de Catalunya (UPC). She is member of the [Department of Mathematics \(DMAT<sup>✉</sup>\)](#), the [IMTech<sup>✉</sup>](#), the [Centre de Recerca Matemàtica \(CRM\)](#),<sup>✉</sup> and the research group [Dynamical Systems](#).<sup>✉</sup>

She received her degree in Mathematics in 1984 and her PhD in Mathematics from Universitat de Barcelona in 2001 under the supervision of Prof. [Ernest Fontich<sup>✉</sup>](#). She joined UPC in 2007 with an Assistant Professor position (Professora Lectora). She has been Graduate Teaching Assistant and Assistant Professor at UB, Assistant Professor at Universitat Rovira i Virgili, Associate Professor at UPC, deputy director of research at DMAT during the years 2018-2024 and secretary of the [Societat Catalana de Matemàtiques<sup>✉</sup>](#) during the period 2019-2022.

Dr. Baldomá's research primarily focuses on the study of dynamical systems with a particular interest in how the invariant objects and the geometry of their invariant manifolds determine the qualitative behavior of the system. She studies mechanisms leading to instability and chaos through the transverse intersections between these manifolds yielding verifiable conditions suitable for computer-assisted proofs. Recently, she is exploring the question of how prevalent certain dynamical behaviors are, such as the transversality of invariant manifolds, analyzing the prevalence of the coexistence of chaotic dynamics with periodic orbits of any period (mixed dynamics) in different settings.

**NL.** *Congratulations on your recent appointment as Full Professor in Applied Mathematics! Could you share when your interest in mathematics first began, and how it developed throughout your undergraduate studies?*

My interest in mathematics began very early, and my family—especially my mother—encouraged it despite not having any formal education. By the age of 10, I was already saying that I wanted to study mathematics and work at NASA. In fact, together with history, mathematics was the only subject I truly enjoyed in primary school and later in high school. So when the time came to choose a degree, the decision was very clear to me.

I really enjoyed the degree—it was challenging and rewarding. During those years, my early interest became more structured and mature. I gradually discovered the depth and coherence of mathematics as a discipline, and I learned to appreciate both its internal beauty and its explanatory power. I remember a first-year Mathematical Analysis professor who often said, “now we are going to solve the problem in

an elegant way.” At first, it was hard to appreciate that elegance, but at some point it suddenly clicked, and you could really see what he meant.

**NL.** *At what point did you realize you wanted to pursue research in mathematics? Which areas initially captured your interest?*

During my undergraduate studies, I particularly enjoyed courses in mathematical analysis, differential equations, functional analysis, and dynamical systems. However, I did not make a deliberate decision to pursue a research career at a specific moment. Rather, my interest in research developed gradually. After completing my undergraduate studies, I accepted the first position that was offered to me, an assistant position in what was then the Department of Applied Mathematics and Analysis at the University of Barcelona. During the following two years, I combined teaching with doctoral courses and seminars, which exposed me more deeply to mathematical research on a daily basis. It was during this period that I began to see research as a natural continuation of my academic interests. In particular, my strong interest in dynamical systems—an area I had already enjoyed as a student—played an important role.

Looking back, my transition into research was not the result of a single decision, but rather a gradual process shaped by teaching, coursework, and a steadily developing interest in research.

**NL.** *Could you share your experiences during your doctoral studies at UB?*

My doctoral studies at the University of Barcelona were a very formative period, both in terms of teaching and research. Starting to teach at the university where I had studied was both exciting and challenging, as I had to (re)learn the subjects I was now responsible for explaining.

Regarding research, first the doctoral courses provided a demanding introduction to academic inquiry. I still keep the notes from many of these courses, and the assignments were tough—small contributions that allowed us to taste the rewards of research. I worked under the supervision of Professor Ernest Fontich, whose guidance had a strong influence on my mathematical training and my approach to research. Those years were crucial not only technically, but also in learning how to formulate meaningful questions, how to combine intuition with rigor, and how to persist when tackling difficult problems. Overall, my doctoral experience laid the foundations for my later work in dynamical systems.

**NL.** *In your view, what are the emerging research directions or open questions in dynamical systems? Where do you see the field heading in the coming years?*

This is a difficult question, as there are several classical conjectures that remain far from being resolved. One fundamental issue is understanding the size or measure (in an appropriate sense) of the chaotic set of a dynamical system. This question is still widely open, even for very simple systems such as planar dynamics. Other long-standing open problems include the question of the long-term stability of the Solar System, or Birkhoff's conjecture on billiards, which asks whether elliptical billiards are the only integrable ones.

At present, many researchers are combining different techniques in order to obtain more powerful results, both at a theoretical and a computational level. In particular, the development of computer-assisted proofs is playing an increasingly important role. If these methods continue to improve, they may allow for rigorous analysis

of high-dimensional dynamical systems, opening the door to mathematically solid results in areas such as biological models, N-body problems, neuroscience, or atmospheric dynamics.

**NL. As you step into your new role as a Full Professor, what is your vision for your research in the coming years? What key achievements are you aiming for?**

In the coming years, I aim to further develop a coherent research program in dynamical systems, with a particular focus on singular settings, invariant manifolds, and mechanisms of instability. A central theme of my work is the understanding of chaotic dynamics in dynamical systems with multiple time scales, where classical perturbation theory often fails to apply.

A paradigmatic example is the N-body problem, which exhibits strong multiscale features and for which fundamental questions—such as the long-term stability of the Solar System—remain open. In such systems, phenomena beyond all algebraic orders often play a crucial role, making explicit analytical computations particularly challenging.

Within this framework, I see two complementary research directions. On the one hand, one may ask whether a given class of dynamical systems exhibits chaotic behavior in a generic sense (for instance, in the sense of a countable intersection of open and dense sets). On the other hand, when dealing with a specific system of interest, the challenge is to establish instability or chaos through explicit and verifiable conditions. Typically, results can be obtained in both directions: either proving that chaos is prevalent within a broad class of systems, albeit without precise control on the hypotheses, or deriving concrete criteria for instability in more restricted settings. Bridging the gap between these approaches is one of my long-term research goals.

In parallel, I aim to further strengthen national and international collaborations and to consolidate several new research directions that my collaborators and I are currently developing.

Beyond specific results and publications, a key objective of my work as a Full Professor is to contribute to the creation of a strong and sustainable research environment. This includes supervising PhD students and postdoctoral researchers, fostering collaborative research, and ensuring the long-term continuity and impact of the research group.

**NL. With your strong track record in teaching, how do you currently view this aspect of your academic career?**

Teaching has always been an essential part of my academic life. I see it as closely connected to research: teaching helps clarify ideas, while research keeps teaching intellectually alive.

Over the years, I have taught a wide range of courses at different levels, I think that, for a professor, it can actually be very beneficial to change the courses you teach from time to time. Teaching the same subject repeatedly can make the material feel routine or even trivial, and it becomes difficult to explain it with enthusiasm or insight. By switching courses, you are forced to revisit the fundamentals, rethink your approach, and engage with the subject from a fresh perspective. This not only benefits the students, who receive a more dynamic and thoughtful teaching experience, but also keeps the teacher intellectually stimulated and deeply connected to the materia

I particularly value the opportunity to accompany students as they develop mathematical maturity. Helping students learn how to think rigorously and independently is, for me, one of the most rewarding aspects of the profession.

**NL. Drawing on your experience in university administration, and in particular your role as Deputy Director of Research at DMAT, what measures should UPC prioritize to promote and support research in mathematics?**

From my administrative experience, I believe universities need to foster environments that encourage collaboration, provide stable support for early-career researchers, reduce unnecessary bureaucratic hurdles that detract from scientific creativity and to lighten the teaching responsibilities.

At UPC, a technology-focused university, the Department of Mathematics (DMAT) is an exception. Unlike other departments, a big amount of the research done in DMAT is more fundamental and qualitative in nature. The current evaluation system for positions and other incentives is purely quantitative, assigning points for specific activities, which disadvantages mathematics. To better promote research in mathematics, it would be important to adapt the evaluation system to reflect the different discipline's characteristics. Emphasizing the depth, originality, and long-term impact of research, rather than simply counting publications or grants, could create a fairer environment and encourage high-quality mathematical work within the university.

**NL. Through your work with the SCM, you've been involved in initiatives that promote and make visible the presence of women in mathematics. How do you see the presence of women in mathematics evolving in recent years, and what responsibilities do you believe institutions and professional societies have in sustaining and accelerating this progress?**

Over the past 20 years, the number of young women choosing STEM careers has dropped significantly. When I studied mathematics, classes were fairly balanced, although this was not the case among faculty or those who later went into research. Today, women make up less than 30 percent of mathematics students. I believe that to strengthen the presence of women in mathematics, we first need to understand why fewer girls are choosing these paths. Identifying the root causes is essential.

During my time as secretary of the SCM board, under President D. Herbera, we launched the Catalan Mathematics Day in 2020. In 2021 and 2022, we produced two interview-style documentaries ([JMC2021](#)<sup>2</sup>, [JMC2022](#)<sup>2</sup>) to highlight the work of women mathematicians in our country. The goal was to give young women strong role models and to create a sense of community.

Progress has been made, but there is still a long way to go to achieve true equity. Institutions and professional societies have an important role to play by providing mentoring, increasing visibility, ensuring fair hiring and evaluation practices, and adopting family-friendly policies that reflect the realities of academic careers. However, much of this work still falls on the women who are already part of these institutions—and who are often overburdened. For example, some gender parity policies apply to committees or evaluation panels, but because women are underrepresented in the community, this often translates into more work for them without necessarily guaranteeing that outcomes, such as a university position, will go to a woman. It is therefore crucial that responsibility and support are shared broadly to make real progress.

**NL. As a member of IMTech, which strategies would you favor in order to maximize its potential?**

IMTech has the potential to be a central hub for mathematical research and collaboration. To maximize this potential, I would emphasize strategies that promote interdisciplinary engagement, international visibility, and support for early-career researchers. Organizing thematic programs, inviting leading researchers from around the world, and fostering connections between industry and academia can all strengthen IMTech's profile.

Encouraging collaborations across departments and institutions, and providing platforms for young researchers to present their work, are also key steps toward building a dynamic and influential research institute, even within the constraints of a tight budget.

NL. *We hope that you will enjoy your Full professorship and that it will turn out to be a most positive time for your academic*

*career. We also wish you good luck!*

Thank you very much!



Jezabel Curbelo<sup>ORCID</sup> is Full Professor at the Department of Mathematics (DMAT<sup>ORCID</sup>) of the UPC<sup>ORCID</sup> and member of IMTech<sup>ORCID</sup>. She has previously been a Ramón y Cajal researcher, from 2020 to 2025, and visiting researcher at University of California-Los Angeles (UCLA)<sup>ORCID</sup>. Her current research focuses on fluid dynamics for ocean sciences and geophysics, and complex systems in general. See [her website](#)<sup>ORCID</sup> for recent publications and full list of awards and outreach activities.

NL. *You have been awarded the Premio Nacional de Investigación from "Ministerio de Ciencia, Innovación y Universidades", and you have been also recently promoted to Full Professor. Congratulations for these deserved recognitions, jointly with other previous achievements. The award highlights among other topics, your work in geophysical fluid dynamics. When did you decided to work on this field, and what prompted you to do so?*

I have worked in geophysical fluid dynamics throughout my career. My interest began during my PhD, when I studied convective fluid flows with applications to Earth's mantle dynamics. Since then, I have explored new applications while remaining rooted in this fundamental area of applied mathematics and geophysical flows.

***Can you please briefly explain what are the Lagrangian Descriptors and their main applications?***

Lagrangian Descriptors are a dynamical systems tool designed to reveal geometrical structures in the phase space of systems with general time dependence. They have proven to be particularly powerful for characterizing transport and mixing processes in geophysical flows. Beyond geophysical applications, Lagrangian Descriptors have been successfully applied in a wide range of fields, including magnetohydrodynamics, transition state theory in chemistry, cardiovascular flow analysis, and billiard dynamics, among many others. <https://link.springer.com/article/10.1007/s40324-025-00382-y>

***Your work is also closely linked to the exploitation of experimental datasets. Do you find it difficult to adapt this data to your models, and to cross knowledge from other disciplines?***

Working with experimental datasets is certainly challenging, but I see it as a very enriching part of my research. Adapting data to mathematical models and integrating knowledge from other disciplines requires close collaboration, yet this cross-disciplinary effort often

leads to deeper insights and more realistic descriptions of complex phenomena.

***You have visited several universities and research centers abroad (École Normale Supérieure de Lyon and University of California Los Angeles), and you maintain numerous collaborations with national and international researchers. What are in your view the main differences with respect to the national research ecosystem, and which good practices do you see that we need to import?***

The main difference is the level of trust in scientists. In many international research systems, researchers are trusted both with funding their projects and with how they use those resources. In Spain, however, the amount of administrative paperwork is often excessive and sometimes even absurd, taking time away from actual research. Reducing bureaucracy and increasing trust would significantly improve the research environment.

***What have you enjoyed mostly from these collaborations?***

What I have enjoyed most is the exchange of ideas and perspectives. Collaborating with researchers from different backgrounds is intellectually stimulating and often leads to new ways of thinking and better science.

***What are the main current challenges in the modelling of fluids, and in your research?***

A main challenge in fluid modeling is capturing complex, multiscale dynamics accurately. In my research, this means building models that are both mathematically rigorous and able to integrate experimental data, while addressing important questions about turbulence, transport, and mixing.

***Among all your works and results, what are those you feel most proud of?***

I would say my first article. It is neither the most important nor the most cited, but it is where everything started. I came into this career almost by chance, so being able to complete that work and see that I could really pursue this way was incredibly meaningful for me.

***Which recommendations would you give to new PhD students starting research in mathematics?***

Read every day! It doesn't have to be in your specific field, and it doesn't have to be a research article, but keep learning and exposing yourself to new ideas constantly.

***You are member of the Board of IMTech (Junta). Which role do you see that IMTech should play in the next years, and which challenges does it face?***

I see IMTech as a hub for interdisciplinary research, fostering collaboration across fields, departments, and groups within the UPC, and applying mathematics and computation to real-world problems. The main challenges will be maintaining this collaborative spirit, attracting top talent, and securing stable funding for ambitious projects.

***Many thanks for your time, and best wishes for your next achievements!***



Francesc Pozo<sup>✉</sup> is Full Professor at the [Department of Mathematics \(DMAT<sup>✉</sup>\)](#) of the [UPC<sup>✉</sup>](#), member of [IMTech<sup>✉</sup>](#), and member of the research group [CoDaLab<sup>✉</sup>](#). He defended his PhD in 2004, co-directed by José Rodellar and Fayçal Ikhouane. Since then he has been developing and applying mathematical tools for control, applied to wind turbines and structural health monitoring. He has supervised

**NL.** *(You have been appointed Full Professor a couple of years ago. This is a well recognized position for your long and wide trajectory in research, teaching and service to the mathematics community at UPC. Congratulations. What made you decide to study mathematics?)*

Ever since I was a child, I've been fascinated by how things work. I used to take apart my toys just to see what was inside (springs, wires, batteries), anything that could reveal the hidden mechanisms. That curiosity for understanding systems and patterns eventually evolved into a love for mathematics.

At school, I discovered that I also enjoyed teaching. I often helped my classmates with math problems, and I found great satisfaction in seeing them suddenly understand something that had seemed difficult before. That experience made me realize that sharing knowledge could be just as rewarding as discovering it.

When it was time to choose a university degree, I considered computer science, architecture, and even journalism. But I kept coming back to mathematics. It was where I felt most at home, both intellectually and creatively. Studying math seemed the best way to combine my curiosity about how things work with my passion for explaining ideas clearly to others.

**You have a long experience in control theory, with a diverse set of applications, among them structural health monitoring. What do you think mathematics can furnish to this field?**

I have always believed that mathematics lies at the foundation of all knowledge, and that conviction has guided my entire research career. For me, mathematics is not just a toolbox of techniques, it is a way of thinking, of seeking clarity and structure in complex problems. Perhaps there is a certain obsession with understanding things deeply, but that curiosity still drives my work today.

In my research on structural health monitoring, mathematics plays a central role at every step, from data acquisition and preprocessing to modeling and interpretation. Beyond the technical aspects, I try to maintain a didactic dimension in my work: to explain not only what we do, but why we do it.

I often think of a scientific paper as a recipe in haute cuisine. The ingredients (the data, the algorithms, the models) may be simple on their own, but it is their combination and the way they are treated through mathematical reasoning that creates something original. That

creative synthesis is, to me, where mathematics shows its true power in engineering applications like control and monitoring.

**You have also been working on Machine Learning techniques for around ten years. What has changed during these years? What do you think mathematics can offer to the future of this field?**

Machine learning has evolved enormously over the past decade, especially with the rapid development of deep learning. The growth has been so impressive that, in some cases, we risk using these techniques indiscriminately and applying them without a full understanding of what we aim to achieve, what kind of data we are dealing with, or how to interpret the results.

This has led many scientific journals to become stricter: papers that simply apply existing techniques, without offering new insights or rationale, are often rejected. The key, in my view, is critical thinking, choosing the right method based on a deep understanding of the problem, the data, and the objectives.

That is why explainable artificial intelligence has gained so much attention in recent years. Beyond accuracy, we now ask why a model behaves in a certain way. Mathematics has a crucial role here, to help us understand the path from data to decision, to provide transparency, and to develop new theoretical frameworks that make AI both powerful and trustworthy.

**Shall we forget about classical statistics and deterministic models?**

Not at all. I firmly believe that we should always start by seeking the simplest possible solution. If a decision problem can be addressed through a straightforward hypothesis test, there is no reason to use a computationally complex machine learning model. Understanding the problem is essential before deciding on the sophistication of the approach.

In fact, early in my research on structural health monitoring, we were pioneers in using advanced hypothesis testing techniques for damage detection in structures. These classical methods, grounded in statistics and deterministic modeling, provided robustness and interpretability, qualities that remain invaluable today.

Mathematics reminds us that elegance often lies in simplicity. The goal is not to replace classical models but to integrate them intelligently with modern approaches, guided always by a deep understanding of the problem at hand.

**You have been heavily involved in teaching mathematics and generating contents for UPC students. What's your opinion on the appearance of Large Language Models — are they helping or impeding the learning process?**

The emergence of large language models will inevitably transform what and how we teach. A few decades ago, students were still taught how to compute square roots by hand. Today, that's no longer part of the curriculum. Yet, understanding what a square root is remains essential. The same applies to many mathematical concepts: it's not just about performing derivations, but about truly grasping what a derivative means.

In this new context, part of learning may involve developing the critical ability to recognize when a language model is wrong. These tools can accelerate access to knowledge, but they should not replace the cognitive process of thinking and understanding.

Human learning, especially at early stages, requires active reasoning and emotional engagement. So, while large language models can be powerful allies, our challenge as educators is to ensure they support thought rather than substitute it.

**How do you see the position of industry with respect to these new techniques? Are the new techniques going to remain and replace previous ones?**

I think the answer is clearly yes. Every technological revolution brings transformation: some industries evolve, some disappear, and

new ones emerge. It has been this way since the Industrial Revolution. The key to survival is adaptation.

Of course, the pace of change today is much faster, and that creates challenges, especially for universities, which naturally evolve more slowly. But change will come, and it will stay, at least until the next revolution appears.

These new techniques are not merely replacing the old ones; they are reshaping the way we think and solve problems. Those who adapt and integrate them thoughtfully will be the ones who thrive in this new technological landscape.

### **What are your next challenges?**

At the moment, one of my main challenges is to incorporate explainability techniques into models for fault detection, classification, and remaining useful life (RUL) estimation in wind turbines. Understanding why a model produces a certain prediction is essential if we want to build trust and make decisions based on its results.

In parallel, I am involved in several projects aimed at transferring the knowledge gained in the wind energy sector to other domains, such as the chemical industry and water management. I find this particularly exciting, as it shows how mathematical and data-driven approaches can have broad impact.

Finally, there is another challenge that I consider equally important: finding a healthy balance between work, body, and mind. Research and teaching are demanding, and maintaining that balance is key to sustaining creativity and long-term well-being.

### **Tell us about one of your results you feel most proud of.**

I would highlight two works that represent different but complementary sides of my research.

The first is “Nonlinear modeling of hysteretic systems with double hysteretic loops using position and acceleration information.” This paper introduces two new mathematical models that better describe how materials and structures deform and recover under repeated or dynamic loads, such as those caused by earthquakes. Building on the Bouc-Wen model, we proposed formulations using position or acceleration data to reproduce complex “double hysteresis loops” observed in experiments with materials like shape-memory alloys, reinforced concrete, wood, and steel. These models proved mathematically stable and were validated through simulations and prototype designs, offering improved tools for earthquake-resistant engineering.

The second is “Vibration-based detection and classification of structural changes using principal component analysis and t-distributed stochastic neighbor embedding.” This study proposes a data-driven approach for detecting and identifying structural damage through vibration analysis. Using PCA to simplify sensor data and t-SNE for visualization, the method creates distinct clusters representing healthy and damaged states. Tested on an aluminum plate with piezoelectric sensors, it achieved near-perfect accuracy, even in noisy conditions.

Together, these two papers reflect my dual motivation as a researcher: the pursuit of mathematical rigor and modeling precision on one hand, and the search for practical, interpretable solutions that improve the safety and reliability of engineering systems on the other.



**Gemma Huguet** is Full Professor at the **Department of Mathematics (DMAT)** and member of **IMTech**, of which she is the deputy director since 2021. She is also member of the **Centre de Recerca Matemàtica (CRM)** and the research group **Dynamical Systems**.

She received her degree in Mathematics in 2003 and her PhD in Mathematics in 2008 from **UPC** under the supervision of Prof. **Amadeu Delshams** and Prof. **Antoni Guillamon**. She did research stays at the Mathematical Biosciences Institute at Ohio State University and at the University of Texas at Austin. She held postdoctoral positions at the CRM (2008-09), the Center for Neural Science and the Courant Institute of Mathematical Sciences at NYU (2009-12), where she specialized in theoretical neuroscience. She was a Juan de la Cierva (2013-15) and Ramon y Cajal (2015-21) research fellow at UPC, contributing to the computational neuroscience research line within the Dynamical Systems group. Her work focuses on the development of analytical and numerical methods for the study of dynamical systems, with particular emphasis on models for neuroscience,

especially the analysis of oscillations in neuronal networks.

**NL. Congratulations for your recent promotion to Full Professor. This is a well deserved recognition after a strong research track in dynamical systems and computational neuroscience. This area has in fact attracted many mathematicians in recent years. Why do you think this is so, and when did you start being interested for this topic?**

Thank you very much! Neuroscience is a field that, earlier than other biological disciplines, developed a tradition of quantitative and mathematical approaches to describe neural mechanisms. In recent years, the explosion of experimental data has further increased the need for mathematical and computational tools to analyze and interpret complex datasets.

I first became interested in this topic during a dynamical systems course at UPC, where I came into contact with Prof. A. Guillamon. He had recently returned from a research stay at the Courant Institute with Prof. **John Rinzel**, a pioneer in computational neuroscience, and introduced several neuroscience models during the course, which motivated me to pursue a PhD. Then, my PhD thesis, under the supervision of A. Delshams and A. Guillamon, was more focused on Hamiltonian systems, but I maintained a strong interest in neuroscience. After completing my PhD, I decided to move more fully into computational neuroscience, which led me to pursue a postdoctoral position at the Courant Institute and the Center for Neuroscience under the supervision of Prof. Rinzel.

**NL. During your PhD and postdoctoral career, you have spent some periods in several national and international universities and centres: University of Austin Texas, Ohio State University, and New York University. What would you like to import from these institutions to UPC?**

I liked the close integration between theoretical work, computational methods, and experimental data. Although this has also become a strong trend here in recent years, these institutions have a longer tradition of integrating these approaches in a more natural and less

compartmentalized way. Another aspect I particularly valued is the strong institutional support for research activities, including greater administrative assistance and more balanced teaching loads, which allow faculty to devote more time to high-quality research.

**NL. *What would you change at UPC for creating an attractive environment for future mathematicians?***

Through the FME, our university has done an excellent job in training a high number of outstanding mathematicians and in creating an attractive environment for mathematics, particularly through the quality of its research and teaching. Building on this strong foundation, I would focus on further strengthening community and visibility. This could include creating more structured opportunities for interaction among students, postdocs, and faculty, supporting mentoring and career development, and increasing exposure to interdisciplinary applications of mathematics. Also, and in line with my previous comments, finding ways to reduce or better support administrative tasks could enhance the overall research environment.

**NL. *What do you like most of your job?***

Possibly, the opportunity to constantly learn new things and to witness how science, and mathematics in particular, pushes the boundaries of knowledge. I value being close to emerging trends and new developments, especially in such a rapidly evolving world. At the same time, I greatly enjoy working with young researchers, contributing to their training, and sharing their enthusiasm, expectations, and motivation as they begin their scientific careers.

**NL. *What do you think we are mostly missing, data or mathematical tools for better understanding our brain?***

Both. While data acquisition has progressed tremendously, several important types of data are still difficult to access. For instance, we cannot yet observe brain activity simultaneously across all relevant scales, from individual synapses to large networks, especially during natural behavior. In many cases, the key variables that determine how the brain processes information cannot be measured directly and must be inferred. At the same time, the volume and complexity of the available data often exceed our current ability to interpret them. For this reason, better mathematical tools are essential to bridge the gap between partial observations and mechanistic understanding.

**NL. *Which areas in computational neuroscience do you think that will undergo a higher progress in the incoming years?***

The field has grown tremendously, with many multidisciplinary groups working on several directions. Key areas include learning and plasticity, circuit dynamics, neural coding, and large-scale connectivity. Beyond these, the emerging field of NeuroAI is using foundation models to bridge biological and artificial intelligence, and Digital Twins promise to revolutionize personalized medicine by simulating treatments in virtual models of patients' brains.

**NL. *You are a good example of multidisciplinary and collaborations in mathematics. Is this something you specially enjoy?***

Yes, I particularly enjoy multidisciplinary work. One aspect I value most is the opportunity to see the applications of mathematics up close and to contribute to research with impact beyond mathematics itself. What I find especially rewarding is this balance: I value the rigor and elegance of mathematics, while neuroscience challenges me to work with models and hypotheses that are not always fully established.

**NL. *Your research demands some understanding of biological processes and biomedical applications. Do you find that there is also a better reception of mathematical tools from biomedical researchers and practitioners?***

Yes, biomedical researchers generally value mathematical tools when it directly helps answer concrete biological questions; purely abstract mathematics without immediate application is naturally less central for them. That said, I believe a strong mathematical foundation is essential for good applied work. It helps identify general principles, build robust methods, and avoid relying on superficial understanding. I was fortunate to receive rigorous training in dynamical systems, and that background has been extremely valuable in modelling.

**NL. *You have been deputy director of IMTech since nearly its first days. Where do you think that the Institute should prioritise its energies and funds?***

I think IMTech should prioritise making the mathematics done at UPC more visible across different departments, and actively promote interaction between research groups. Many groups are not sufficiently aware of each other's work, despite having strong common interests and clear potential for collaboration.

**NL. *What do you think are its strengths and weaknesses? What would you like to do within IMTech if you had unlimited funding (and time)?***

The main strength of IMTech is the broad diversity of mathematical research of high quality, ranging from very theoretical to strongly applied areas. A current weakness is that this diversity is still somewhat dispersed, and stronger cohesion across research areas could further enhance collaboration. If I had unlimited funding and time, I would focus on providing more institutional support for researchers, including resources for organizing scientific events, support for grant writing and strategic funding opportunities, and targeted hiring to strengthen key research directions. However, current funding constraints make this challenging.

***We hope that you will enjoy your Full professorship and that it will turn out to be a most positive time for your academic career. We also wish you good luck!***

Thank you!

▷ Editorial

## Research focus

### A rigorous derivation of the asymptotic wavenumber in spiral wave solutions of the complex Ginzburg-Landau equation,

by Inmaculada Baldomá Barraca<sup>✉</sup> (DMAT<sup>✉</sup>, CRM<sup>✉</sup>)

In a wide range of physical, chemical, and biological systems modelling the interaction among different species, the dynamics of each species is usually governed by a diffusion mechanism together with a reaction term, which takes into account the interactions with the other species. For example, such systems arise when modelling chemical reactions<sup>✉</sup> processes as pattern formation mechanisms as aggregation of amoebae *Dictyostelium discoideum*<sup>✉</sup>, in certain ecological models [Mur01] or even to describe transformations of cardiac muscle cells [ES22].

A reaction-diffusion equation is a system of differential equations, with a diffusion term:

$$\partial_\tau U = D\Delta U + F(U, a), \quad (1)$$

where  $U = U(\tau, \mathbf{x}) \in \mathbb{R}^N$ ,  $\mathbf{x} \in \mathbb{R}^2$ ,  $D$  is the diffusion matrix,  $F$  is the (nonlinear) reaction term,  $\Delta$  is the Laplacian, and  $a$  is a parameter (for instance a catalytic concentration in a chemical reaction) or a set of parameters.

We consider the so-called oscillatory systems, which are a particular class of reaction-diffusion equations. These systems produce oscillations when  $\Delta U = 0$  (the so-called homogeneous situation, meaning that the medium in which the reaction takes place does not influence the system). They correspond to systems of the form (1) for which the dynamical system

$$\frac{dU}{d\tau} = F(U, a)$$

has an asymptotically stable periodic orbit. More precisely, we focus on dynamical systems exhibiting a supercritical Hopf bifurcation at  $(U_0, a_0)$ . In this setting, we look for solutions of the form

$$U(\tau, \mathbf{x}, a) = U_0 + \varepsilon [A(t, \mathbf{x})e^{i\omega\tau} v + \bar{A}(t, \mathbf{x})e^{-i\omega\tau} \bar{v}] + \mathcal{O}(\varepsilon^2),$$

where  $\bar{z}$  denotes the complex conjugate,  $\varepsilon^2 = a - a_0$ ,  $\omega$  is the frequency of the oscillation, and  $A(t, \mathbf{x}) \in \mathbb{C}$  is the so-called amplitude. Under generic conditions and neglecting terms of order higher than two, it can be shown (see [Kur03]) that the amplitude  $A$  satisfies the celebrated complex Ginzburg-Landau equation (CGL):

$$\partial_t A = (1 + i\alpha)\Delta A + A - (1 + i\beta)A|A|^2, \quad (2)$$

with  $\alpha, \beta \in \mathbb{R}$  depending on  $F$  and  $D$ .

The CGL is an ubiquitous equation, widely studied in physics; for different regimes, it exhibits a plethora of patterns and it is far from being completely understood (see [AK02] and [SS23] for an extensive exposition about open problems).

Following the classical literature, [KH81], [Hag82], [Gre80], [YK76], we consider solutions of the following type:

**Definition 1.** Let  $n \in \mathbb{N}$ . In polar coordinates,  $A(t, r, \varphi)$  is a rigidly rotating Archimedean  $n$ -armed spiral wave if  $A$  is a solution of (2) of the form

$$A(t, r, \varphi) = \mathbf{f}(r) \exp(i(\Omega t + n\varphi + \Theta(r))) \quad (3)$$

where  $\mathbf{f}, \Theta$  are  $\mathcal{C}^2$  when  $r \geq 0$ . The boundary conditions are

$$\mathbf{f}(0) = \Theta'(0) = 0, \quad \lim_{r \rightarrow \infty} \mathbf{f}(r) = \sqrt{1 - k_*^2}, \quad \lim_{r \rightarrow \infty} \Theta'(r) = -k_*$$

with  $\Omega, k_*$  satisfying the dispersion relations:

$$C = \sqrt{1 - k_*^2}, \quad \Omega = \Omega(k_*) = -\beta + k_*^2(\beta - \alpha). \quad (4)$$

For any  $\Omega t + n\varphi$  fixed, the spiral wave goes to

$$A_*(\Omega t + n\varphi - k_* r + \theta(r)) := C e^{i(\Omega t + n\varphi - k_* r + \theta(r))},$$

when  $r \rightarrow \infty$ , with  $C, \Omega \in \mathbb{R}$  satisfying (4) and  $\theta'(r) \rightarrow 0$ . The contour lines of  $A_*$

$$\text{Re}(A_*(\Omega t + n\varphi - k_* r) e^{-i\Omega t}) = \cos(n\varphi - k_* r) = c$$

are Archimedean spirals (see Figure 1).

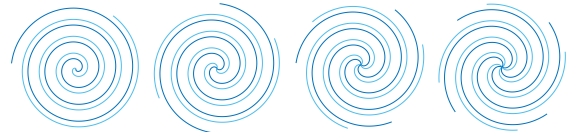


Figure 1: Representation of two Archimedean  $n$ -armed spiral waves for winding numbers  $n = 1, 2, 3, 4$ . These two spirals correspond to the contour lines  $\cos(-k_* r + n\varphi) = c \neq \pm 1$ .

We introduce the twist parameter  $q$  as:

$$q = q(\alpha, \beta) = \frac{\beta - \alpha}{1 + \alpha\beta} \quad (5)$$

which is well defined for  $\alpha, \beta$  satisfying  $|\alpha - \beta| \ll 1$ . This singular perturbation parameter plays an important role in the form of the spiral waves. Indeed, when  $q = 0$ , it is not difficult to check that the surface  $(r \cos \varphi, r \sin \varphi, \text{Re}(e^{-i\Omega t} A(t, r, \varphi)))$ , for  $r \gg 1$  looks like the one in the left hand side of Figure 2, that is there are not spirals waves when  $q = 0$ . However, when  $q \neq 0$  the level curves of  $\text{Re}(e^{-i\Omega t} A(t, r, \varphi))$  starts to bend and the spirals arise as  $r \rightarrow \infty$ .

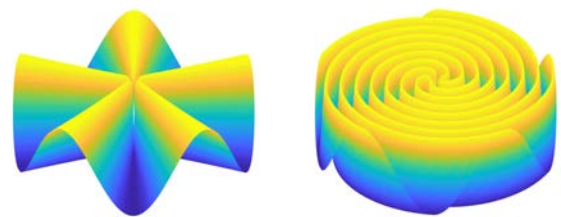


Figure 2: The cuore corresponds to  $r = r_0 \gg 1$ .

The result proven in [ABMS25] is the following:

**Theorem 1.** Let  $n \in \mathbb{N}$ . There exist  $C_n$  and a unique odd function of the form

$$\kappa_*(q) = \frac{2}{q} e^{-\frac{C_n}{n^2} - \gamma} e^{-\frac{\pi}{2n|q|}} (1 + \mathcal{O}(|\log |q||^{-1})), \quad q \rightarrow 0 \quad (6)$$

with  $\gamma$  the Euler-Mascheroni constant, such that the CGL equation has an Archimedean spiral wave as in Definition 1 if and only if the asymptotic wave number  $k_* = \kappa_*(q)$  with  $q = q(\alpha, \beta)$  as in (5) and the frequency  $\Omega$  satisfies (4).

The existence of the function  $\kappa_*(q)$  was already known since the 80's by the pioneer works by N. Kopell and L. Howard [KH81] and the formula (6) was conjectured by [Hag82] using formal matching techniques. This formula has been used widely in the literature, [CH93, AK02] but has not been proven until now. The novelty in our work, [AB11, ABMS16] and finally [ABMS25], is the use of a functional analysis approach that allow us, after 40 years, to design a beyond all orders selection mechanism of the asymptotic wavenumber.

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## Collective migration across scales: From PDEs to physical swarms,

by Gissell Estrada-Rodríguez <sup>✉</sup> (DMAT <sup>✉</sup>, CRM <sup>✉</sup>)

Mathematical modelling of biological, physical, and social phenomena using partial differential equations (PDEs) to describe the average behaviour of underlying microscopic (individual-level) dynamics has received significant attention [6, 11]. Since A. Turing's foundational work on reaction-diffusion systems, followed by the influential chemotaxis model by Patlak, Keller, and Segel in the 1950s [13, 16], PDE models have become essential tools in understanding self-organisation.

Despite their success, PDEs are often constructed directly at the continuum level; interactions are typically based on phenomenological considerations at the population scale. This top-down approach can limit their ability to capture micromechanical behaviours and complicates validation against experimental data. A concrete illustration appears with the classical Keller–Segel model for chemotaxis: while rigorous derivations from stochastic particle systems exist [18], recent work [10] suggests an alternative microscopic mechanism (“resident-explorer” interactions) producing analogous aggregation patterns. Establishing robust links between PDE models and their microscopic counterparts enhances predictive power and makes macroscopic models more valuable for experimental data analysis.

In collective cell migration, these limitations arise naturally. Cell–cell adhesion, a fundamental mechanism during tissue formation, has been studied using coupled nonlinear nonlocal PDE systems for two populations of cells [3, 9, 14], where

a mix of nonlocal adhesion and local saturation captures biologically relevant behaviours. Nonlinear diffusion (short-range repulsion) plus nonlocal aggregation imposes analytical and numerical challenges, especially under realistic conditions. Interaction parameters require careful calibration and are hard to measure experimentally. In addition, identifying biologically relevant interaction potentials remains a difficult task. Numerical results in [3] show such PDE systems qualitatively capturing experimental outcomes under different adhesion hypotheses, see Figure 3. Data-driven identification of interaction potentials [4] and efficient parameter estimation [2] can further refine these descriptions.

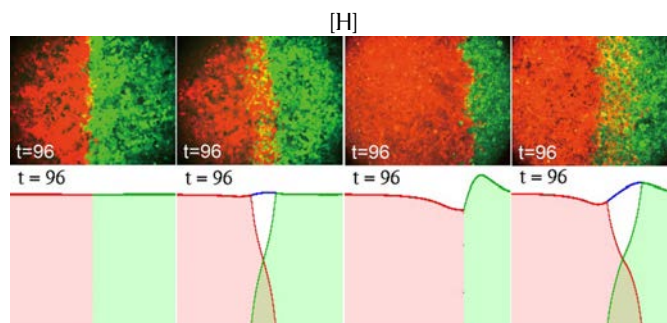


Figure 3: Top row: Cell-cell adhesion experiments from [12]. Bottom row: Numerical results from [3] using the system (7).

When mathematical models are improved using biological data, they accelerate discovery. Yet experimental validation of refined hypotheses is a bottleneck due to the time, cost, and complexity of biological experiments. To address this, we propose robotic swarms as controlled, measurable model systems for testing biological hypotheses. Compared to purely computational simulations, robot swarms introduce natural noise (e.g., sensing/actuation delays, limited communication), analogous to biological limitations, and may exhibit emergent behaviour (jamming, fragmentation) not predicted by simulations.

This offers a realistic and flexible platform to speed the cycle of prediction, testing, and refinement, bridging modelling and experimental validation.

In this context, the present work aims to develop a comprehensive analytical, numerical, and experimental framework for studying nonlocal PDEs with biologically relevant features such as collective cell migration and adhesion. Central to this approach is the use of programmable robotic swarms as controllable physical testbeds for validating and refining PDE-based models. By integrating mean-field limit theory, data-driven parameter estimation, and asymptotic-preserving numerical schemes with systematically designed robot experiments, this framework enables direct testing of movement hypotheses under realistic conditions that include noise and emergent effects. Two principal objectives guide this effort: (i) to perform a qualitative comparison between PDE solutions and observed robotic swarm behaviours, with particular emphasis on parameter inference, and (ii) to establish rigorous error bounds that quantify discrepancies between microscopic and macroscopic descriptions, thereby characterising finite-size effects inherent to experimental systems.

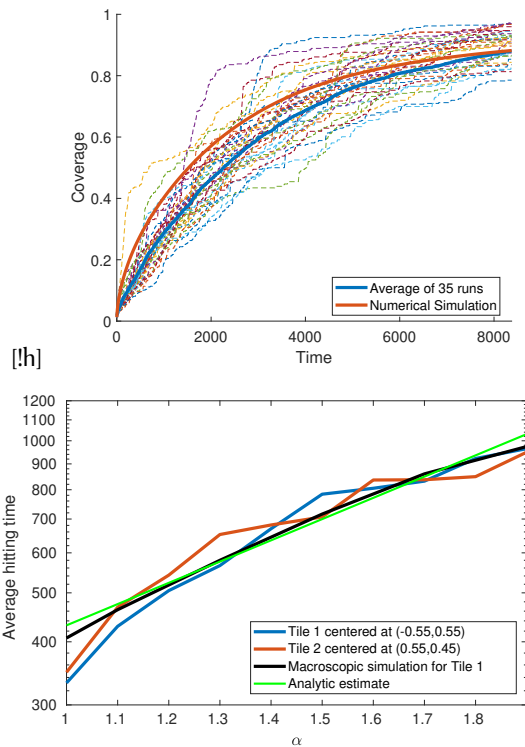


Figure 4: Top: Comparison of area coverage from a nonlocal PDE with averaged robotic simulations. Bottom: Comparison of hitting times. All plots correspond to 5 robots. Taken from [7].

Previous works [7, 8] derived nonlocal macroscopic PDEs for individual robot movement from detailed models of local communications and interactions, identifying limiting equations across temporal/spatial scales (see Figure 4). The combination of mathematical tools and robotic systems opens a zoo of questions between mathematical biology, analysis, and optimisation. For instance, spontaneous cell organisation leading to shape formation (morphogenesis) has been recently studied using a swarm of robots [5, 17]. In these works, robots exhibited collective behaviour by following classical reaction-diffusion PDEs *à la* Turing, relying entirely on local interactions with their neighbours. Differential adhesion hypothesis to program robot swarms was explored in [15], where no mathematical

model was used, only simple rules mimicking cell behaviour. The use of tailored PDE models for cell adhesion provides a rigorous framework to scale swarm behaviours across system sizes and it enables efficient inference of key parameters such as adhesion strength and sensing radius directly from data.

#### PDE solutions vs. robotic swarms for cell-cell adhesion.

Cell-cell adhesion involves binding through surface proteins. Although well studied theoretically, its quantitative calibration remains limited by scarce biological data. We therefore propose to calibrate the following two-population adhesion PDE system:

$$\begin{aligned} \partial_t \rho_1 &= \nabla \cdot \left[ \rho_1 \nabla \left( b_1 (\rho_1 + \rho_2) + W_{11} * \rho_1 + W_{12} * \rho_2 \right) \right. \\ &\quad \left. + \Sigma_1 \nabla \rho_1 \right], \\ \partial_t \rho_2 &= \nabla \cdot \left[ \rho_2 \nabla \left( b_2 (\rho_1 + \rho_2) + W_{22} * \rho_2 + W_{21} * \rho_1 \right) \right. \\ &\quad \left. + \Sigma_2 \nabla \rho_2 \right], \end{aligned} \quad (7)$$

posed in a 2-dimensional bounded domain with smooth boundary conditions. The calibration is performed using data collected from robotic experiments—designing two robot populations with different neighbour-to-neighbour communication strengths and sensing distances, which naturally map to adhesion and interaction radii. Here  $\rho_i(x, t)$ ,  $i = 1, 2$ , are the population densities of each cell-type,  $b_i, \Sigma_i > 0$  are the nonlinear and linear diffusion coefficients, respectively, and  $W_{ij}$  are the interaction potentials. The parameters we seek to estimate include the nonlinear diffusion coefficient and the coefficients governing the attractive and repulsive forces within  $W_{ij}$ .

As a first step, we will perform a qualitative comparison between the PDE numerics and the observed robotic configurations, extending area-coverage metrics previously used for single-population settings [7]. Parameter refinement will follow a hybrid approach combining a finite number of trajectories with PDE descriptions [2]. The advantage of our approach in [2] lies in combining two key elements: a finite number of individual trajectories, for which we have accurate biological/robotic data, and the use of PDE descriptions, for which we have efficient parameter estimation schemes. This combination makes parameter estimation more efficient and computationally feasible compared to individual-based models, which require all trajectories for calibration. Beyond fixed-form (e.g., Morse) potentials, we will test multiple interaction kernels and nonlinear diffusions at both experimental and PDE levels.

**Finite-size effects in PDE descriptions.** While studying collective behaviour, a major difficulty arises when trying to quantitatively measure how well the robots swarm achieved a prescribed spatial configuration, since often the target density is given as a continuous function and the robots swarm is actually composed of a finite number of individuals at discrete locations. These finite size effects are inherent in continuum PDE models, which constitute a problem when comparing PDE descriptions with individual particle simulations, as corroborated in [20]. Robotic systems are often small, compared to their biological counterparts, and finite size effects are measurable.

The validity of the continuum limit as the number of particles tend to infinity for weak interactions, an unrealistic scenario in robotics and cell-cell interacting systems, was established in [18]. Preliminary work towards the derivation of rigorous error bounds on the discrepancy between microscopic and macroscopic models, which depend on the swarm population size and other physical characteristics such as sensing radius,

was derived and validated in a series of papers by A. Bertozzi et al. [1, 19, 21] for a swarm of robots in the absence of interactions. Our aim is to extend the previous analytic framework to include (i) specular-type interactions between individuals of the same population, and ultimately, between individuals of different populations, (ii) different sensing radii depending on the different species. As a first step, a robot population will perform a simple task of area coverage where robot-to-robot interactions are considered. A second step will include two populations adhesion models.

Let us start by considering a bounded region  $\Omega \in \mathbb{R}^d$ , the desired robots distribution  $\rho(z)$  in  $\Omega$  and  $N$  the number of robots with positions  $x_1, \dots, x_N$  in  $\Omega$ . To describe the discrete positions of the robots A. Bertozzi et al. introduced a *blob function*  $B_\varepsilon(z) = \varepsilon^{-d}K(z/\varepsilon)$ , which is normalised to 1 and  $\varepsilon > 0$  describes the sensing radius. This is analogous to the purely repulsive potential introduced in [2] to describe nonlinear diffusion for the case of cell-cell interactions. The function  $K(\cdot)$  is generally assumed to be a Gaussian, namely, the robot is more efficient at its task locally than at larger distances from its current position. Then, if  $\varepsilon$  is small, we can derive a continuous distribution of the discrete positions of the robots given by  $\rho_\varepsilon^N(z) \simeq \frac{1}{N} \sum_{i=1}^N B_\varepsilon(z - x_i)$ , which is analogous to the so-called empirical measure ( $\rho^N(x) = \frac{1}{N} \sum_{i=1}^N \delta_{x_i}(x)$ ) when  $\varepsilon \rightarrow 0$ . The error metric is naturally given now by

$$e_N^\varepsilon(x_1, \dots, x_N) = \int_\Omega |\rho_\varepsilon^N(z) - \rho(z)| dz,$$

which converges to zero as  $N \rightarrow \infty$ .

Building on these ideas, a quantitative measure is introduced to characterise finite-size effects in the presence of strong interactions, which can provide a formal mathematical validation of the swarm control strategies, allowing us to determine the required number of robots and optimal robot sensing radius to achieve a final prescribed configuration. Gradient based optimisation algorithms will be used to minimize the error between the target distribution  $\rho(z)$ , and the positions of the robots given by  $\rho_\varepsilon^N(z)$ , given  $\rho$ ,  $N$  and  $\varepsilon$ .

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**Geometry and physics with geometric algebra**, *Geometric Mechanics*, Vol. 2, No. 3 (2025), 337–383. [↗](#) by [Sebastià Xambó](#) [↗](#) (DMAT<sup>↗</sup>.)

This work is dedicated to the memory of [Miguel Carlos Muñoz Lecanda](#) (MCML) (1946-2023), who was a Full Professor at [UPC](#). Broadly speaking, his scientific interests were focused on the interactions between mathematics (mainly differential geometry) and physics (geometric mechanics, classical field theory), but also on engineering applications (control theory, for instance). For a biographical sketch of [MCML](#) (by [María Barbero-Liñán](#), [Manuel de León](#), [Eduardo Martínez](#), and [Nar-](#)

[ciso Román-Roy](#)), see pages 221-223 of the journal cited above.

The paper under review surveys several topics about which the author and [MCML](#) exchanged ideas for over five decades, but in this occasion phrased in the [geometric algebra](#) formalism. The paper pretends to be the fruit of a pending conversation reminiscent of the fascination experienced when they studied texts such as [George Mackey](#) [↗](#)'s celebrated book [Mathematical Foundations of Quantum Mechanics](#) [↗](#) (AMS review by [Jacob Feldman](#)).

For any real vector space  $E = E_q$  (its elements, denoted  $v, v', u, \dots$ , are called *vectors*) endowed with a non-

degenerate quadratic form  $q$  (usually called the *metric*), the corresponding geometric algebra  $\mathcal{G}_q = \mathcal{G}(E_q)$  is defined in §1 to be Grassmann's exterior algebra  $\mathcal{G} = \mathcal{G}(E)$  (its elements  $x, x', \dots$  are called *multivectors*; its product,  $x \wedge x'$ , is the *wedge product*) enriched with the *geometric product*  $xx'$  (no infix symbol is used for this product: factors  $x, x' \in \mathcal{G}$  are just juxtaposed). If the metric  $q$  has signature  $(r, s)$ , so that  $n = r + s = \dim \mathcal{G}$ , the geometric algebra is also denoted by  $\mathcal{G}_{r,s} = \mathcal{G}(E_{r,s})$ . The power of the geometric product stems from the fact that *it is the only associative, bilinear and unital product satisfying two conditions*: (1) *Contraction*,  $v^2 = q(v)$  for any vector  $v$ ; (2) *Transference*,  $v(v_1 \wedge \dots \wedge v_k) = v \wedge v_1 \wedge \dots \wedge v_k$  whenever  $v$  is orthogonal to  $v_j$  for  $j = 1, \dots, k$ . In particular we see that a vector  $v$  is invertible provided  $q(v) \neq 0$ , which confers a flexibility to calculations in  $\mathcal{G}$  akin to numerical calculations, although with the precaution that the geometric product is not commutative. For example (see Eq. (11)),  $vv' = v \cdot v' + v \wedge v'$ , which shows that  $v$  and  $v'$  commute precisely when  $v \wedge v' = 0$  (i.e., when  $v$  and  $v'$  are parallel) and that they anticommute precisely when  $v \cdot v' = 0$  (i.e., when  $v$  and  $v'$  are orthogonal; here  $v \cdot v'$  denotes the bilinear form, or scalar product, associated to  $q$ ).

§2 is devoted to  $\mathcal{G}_2 = \mathcal{G}(E_2)$ , the geometric algebra of the Euclidean plane  $E_2$ , which for historical reasons deserves to be called *Wessel's algebra*. Two examples illustrate its applications: the derivation of a formula for the orthocenter of a triangle in terms of its vertices (Eq. (21)), and the recasting of the Levi-Civita 'spinor regularization' for the Kepler problem in  $E_2$  (Eq. (25) and Fig. 3). Remark that if we choose any two unit orthogonal vectors  $u, u'$ , then the bivector  $i = u \wedge u' = uu'$  (which represents a unit area) satisfies  $i^2 = uu'uu' = -uu'u' = -1$ , which implies that the even algebra  $\mathcal{G}_2^+ = \mathcal{G}^0 \oplus \mathcal{G}^2 = \mathbb{R} \oplus \mathbb{R}i = \mathbb{C}$  is a geometric model of the field of complex numbers, and the spinors involved in the Levi-Civita regularization are the unit complex numbers  $e^{\alpha i}$ ,  $\alpha \in \mathbb{R}$  (note also that while  $\mathbb{C}$  is commutative, its action by multiplication on vectors is not, as  $vi = -iv$ , or, more generally,  $ve^{\alpha i} = e^{-\alpha i}v$ ). In the expression for the orthocenter, a key point is that non-zero bivectors are invertible (the inverse of  $\beta i$  is  $-\beta^{-1}i$ ).

Similarly, §3 is devoted to  $\mathcal{G}_3 = \mathcal{G}(E_3)$ , the geometric algebra of the Euclidean space  $E_3$ . It is called *Pauli algebra* because Wolfgang Pauli discovered it "under the guise of  $2 \times 2$  complex matrices in his research about the spin of the electron". In this case, one gets a geometric model  $\mathbb{H} = \mathcal{G}_3^+ = \mathcal{G}^0 \oplus \mathcal{G}^2 = \mathbb{R} \oplus \mathbb{E}i$  of Hamilton's quaternions (here  $i$  is a unit volume, say  $uu'u''$  with  $u, u', u''$  unit mutually orthogonal vectors, so that we also have  $i^2 = -1$ ), which provides shrewd expressions for symmetries and rotations, and in particular for the composition of rotations (Olinde Rodrigues' formulas); a treatment of Newtonian multi-particle mechanics, including Kepler's orbits and Rutherford's scattering formula; and the celebrated Kustaanheimo–Stiefel spinor regularization of the Kepler problem in  $E_3$ . In this case, spinors are the elements of  $\mathbb{H}^\times$ .

In §4, we find "a brief overview of  $\mathcal{G}_{1,3}$  and its capacity to deal with core relativity topics, including Lorentz transformations" (IMTech Newsletter 8, Jan–Dec 2025



Figure 5: MCML, from the poster of the XXXII International Fall Workshop on Geometry and Physics (Portugal, Coimbra, 2-5 September, 2024).

tions, Maxwell's electromagnetism and Dirac's equation for the relativistic electron". This algebra "is named after P. Dirac because he discovered it in the guise of a 16-dimensional subalgebra of the algebra of  $4 \times 4$  complex matrices in his endeavor to get a relativistic theory of the electron". The Dirac spinors are the invertible even multivectors and they provide a means for describing Lorentz transformations (Theorems on page 369) and establishing their fundamental properties. In particular, the composition of two Lorentz boosts is resolved explicitly into the composition of a Lorentz boost and a rotation. Electromagnetic fields are represented as bivector fields  $F : E_{1,3} \rightarrow G_{1,3}^2$ , an electric current by a vector field  $J : E_{1,3} \rightarrow E_{1,3}$ , and the Riesz form of Maxwell's equations is  $\partial F = J$ , where  $\partial$  is the gradient operator of  $E_{1,3}$  (Theorem on page 372). The section ends with the geometric algebra form of the Dirac equation, and some of its consequences, including how spin is predicted and expressed (Eq. (73)).



Figure 6: Pictures from Dirac's biography, by Richard Dalitz, at CERN: Left, Plaque at Westminster Abbey, adjacent to Newton's grave, displaying Dirac's equation. Right: Paul Adrien Maurice Dirac, Lucasian Professor, attending a lecture.

The paper includes an appendix collecting some background facts, as for example an intrinsic construction (i.e., coordinate-free) of the Grassmannian varieties, a formula for the inner product of two decomposable  $k$ -vectors, the behavior of the grade and reversion involutions of the Grassman algebra with respect to the inner and geometric products, and the analytics of Kepler orbits.

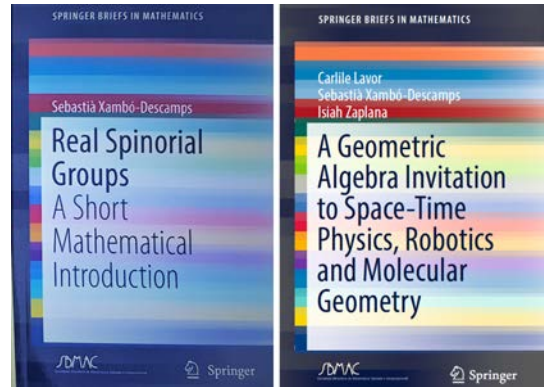


Figure 7: Book covers of references [19] and [12].

Although the geometric algebra overview of §1 deals with general spaces  $E_{r,s}$ , the applications considered in the paper are limited to  $\mathcal{G}_2$  (Wessel),  $\mathcal{G}_3$  (Pauli), and  $G_{1,3}$  (Dirac). Nevertheless, the paper may serve as an entry gate to the geometric algebra realm (represented by the twenty references provided at the end) for anyone wishing to get acquainted with it. Knowledge of mathematics should ease the understanding of its structure and facilitate the grasping of its applications to geometry and physics. The other way around may also work: knowledge of physics can be a sure hold for a finer appreciation of the mathematics involved.

## Libration point orbits: a brief journey through fundamental dynamics and applications,

by Josep Masdemont<sup>✉</sup> (DMAT<sup>✉</sup>, IMTech<sup>✉</sup>)

The collinear equilibrium points of the Restricted Three-Body Problem (RTBP) were first described by the Swiss mathematician Leonhard Euler in the 18<sup>th</sup> century. The foundational understanding of libration point orbits has its roots in the pioneering work on the continuation of periodic orbits by Hénon and others laid the groundwork for both theoretical advances and practical applications culminating in the ISEE-3 mission in 1978—the first spacecraft to exploit the dynamical properties of the  $L_1$  point in the Sun–Earth system.



The success of ISEE-3 and the recognition of libration point orbits (LPOs) as privileged locations for solar and astronomical observation led to a series of landmark missions that demonstrated the value of these regions. Halo and Lissajous orbits around  $L_1$  and  $L_2$  have since been employed in numerous missions, including SOHO, Herschel, Planck, ARTEMIS, Gaia, Chang'e 2, Queqiao, Euclid, and the James Webb Space Telescope, offering favorable geometry for both scientific operations and continuous Earth communication.

Geometrically, the dynamics near collinear equilibrium points are governed by the structure of invariant manifolds, which are normally hyperbolic, and whose stable and unstable components act as phase space conduits. These structures determine the local behaviour, organize transfers between different orbits and facilitate the design of low-energy trajectories close to heteroclinic and homoclinic connections. The intersection of invariant manifolds between different LPOs provides the framework for natural transfer trajectories with minimal control effort. When exact connections are not available due to energy mismatch or mission constraints, such transfers can be enhanced through carefully designed maneuvers.

The **Restricted Three-Body Problem** (RTBP) is the classical model in celestial mechanics describing the motion of an infinitesimal mass under the gravitational influence of two massive primaries  $M$  and  $m$  ( $M \geq m$ ). In the usual synodical reference frame they have the form, where  $\mu \in (0, 1/2]$  denotes the dimensionless mass parameter  $\mu = m/(M + m)$ . The RTBP can be formulated in Hamiltonian form with Hamiltonian  $H(x, y, z, p_x, p_y, p_z)$ ,

$$H = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + yp_x - xp_y - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{\mu}{2}(1-\mu).$$

where  $r_1, r_2$  are the distances to the primaries.

The RTBP has five equilibrium points, known as the libration points, where the gravitational and inertial forces balance. Three of them, denoted by  $L_1, L_2, L_3$ , lie on the line connecting the two primaries and are called **collinear libration points**. The remaining two,  $L_4$  and  $L_5$ , form equilateral triangles with the primaries.

The collinear points are located along the  $x$ -axis in the synodical frame and their position are determined by the so called Euler's quintic equations. Their local dynamics is described by the second order part of the Hamiltonian,  $H_2$  which admits a

symplectic change of coordinates which puts it in the normal form,

$$H_2(\mathbf{q}, \mathbf{p}) = \lambda_0 q_1 p_1 + \frac{\omega_0}{2}(q_2^2 + p_2^2) + \frac{\nu_0}{2}(q_3^2 + p_3^2), \quad (8)$$

where  $\mathbf{q} = (q_1, q_2, q_3)$ , and  $\mathbf{p} = (p_1, p_2, p_3)$  are generalized positions and momenta. So the linear flow has a *center*×*center*×*saddle* structure. The two purely imaginary eigenvalue pairs are linked to center-type motions. The center directions associated with  $\omega_0$  lies in the  $(x, y)$  RTBP plane and gives rise to oscillations associated with the planar Lyapunov family; the other corresponds to oscillations in the  $z$  direction and originates the vertical Lyapunov family. The existence of these continuous families of periodic orbits in the nonlinear system is guaranteed by the Lyapunov center theorem.

The center manifold constitutes an example of what is called a normally hyperbolic invariant manifold. The dynamics of (8), is topologically equivalent to the full RTBP nonlinear system in a limited neighborhood  $\mathcal{R}$  of the libration point. Considering a small fixed value of the Hamiltonian  $H_2 = h$ ,

$$R(h) = \left\{ (\mathbf{q}, \mathbf{p}) \in \mathbb{R}^6 \mid H_2(\mathbf{q}, \mathbf{p}) = h, |q_1 - p_1| \leq K_h \right\}.$$

The first condition fixes the energy, reducing the six-dimensional phase space to the five-dimensional energy surface  $\{H_2 = h\}$ . The second condition bounds the equilibrium region in the  $(q_1, p_1)$ -plane.

The condition  $H_2 = h$  can be written as

$$\frac{\omega_0}{2}(q_2^2 + p_2^2) + \frac{\nu_0}{2}(q_3^2 + p_3^2) + \frac{\lambda_0}{4}(q_1 + p_1)^2 = h + \frac{\lambda_0}{4}(q_1 - p_1)^2$$

which means that  $R(h)$  is homeomorphic to  $S^4 \times I_h$  where the interval  $I_h = [-K_h, K_h]$ .

Taking into account that  $q_1 p_1$ , is a first integral, for each point in the  $(q_1, p_1)$ -space, the set

$$S_h^3(q_1, p_1) = \left\{ \frac{\omega_0}{2}(q_2^2 + p_2^2) + \frac{\nu_0}{2}(q_3^2 + p_3^2) = h - \lambda_0 q_1 p_1 \right\}$$

is a  $S^3$  sphere parameterized by the center coordinates  $(q_2, p_2, q_3, p_3)$ . Because  $q_2^2 + p_2^2$  and  $q_3^2 + p_3^2$  are also first integrals, the energy  $h$  is split in the three modes (the planar and vertical oscillations and the saddle), when  $q_2 = p_2 = p_3 = p_3 = 0$  we find that the hyperbolas  $q_1 p_1 = h/\lambda_0$  bound  $R(h)$  in the  $(q_1, p_1)$  projection, and in these hyperbolas the corresponding  $S_h^3$  spheres become a point (see Fig. 8). Beyond the hyperbolas we have regions energetically forbidden (filled in salmon in the figure). Moreover, the condition  $|q_1 - p_1| \leq K_h$  creates two line segments  $p_1 = q_1 \pm K_h$  denoted by  $l_1$  and  $l_2$  which also bound  $R(h)$  in the  $(q_1, p_1)$  projection (pale green region). The area to the left of  $l_1$  and the one to the right of  $l_2$  correspond to different realm regions (interior or exterior, to the realm of a primary, depending on the  $L_i$ ).

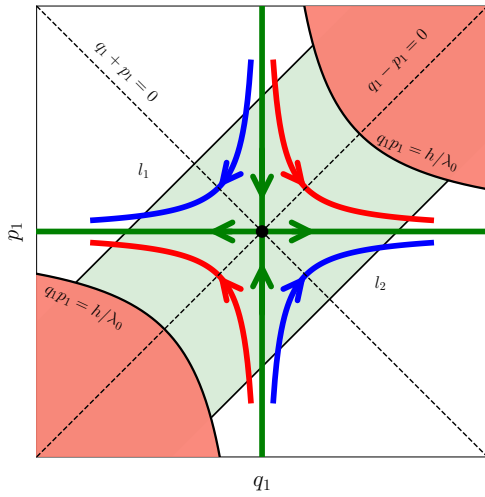


Figure 8: Topological representation of the neighborhood  $R(h)$  of an  $L_i$  point projected onto the  $(q_1, p_1)$  plane.

Due to  $q_1 p_1$  is a first integral, the orbits display an hyperbolic motion on the  $(q_1, p_1)$  projection. Of particular interest is the case  $q_1 p_1 = 0$  with the subset  $q_1 = p_1 = 0$ , defining the  $S^3$  sphere of central manifold trajectories at energy  $h$  (central black dot in Fig. 8);  $p_1 = 0, q_1 \neq 0$  defining its center-unstable manifold and  $q_1 \neq 0, p_1 = 0$  the center-stable ones (both displayed in green). Trajectories with  $q_1 p_1 \geq 0$  correspond to transit orbits from one region to the other (displayed in red) while the ones with  $q_1 p_1 \leq 0$  correspond to non transit orbits (displayed in blue) [3].

When dealing with the full nonlinear problem, considering complex variables and Lie series transformations, one can obtain a truncated normal form at high order that gives  $\mathbf{q}_1(t) = \mathbf{p}_1(t) = 0$  for all  $t$ , provided that  $\mathbf{q}_1(0) = \mathbf{p}_1(0) = 0$ . This invariance condition assures that an orbit with  $\mathbf{q}_1(0) = \mathbf{p}_1(0) = 0$  remains in the center manifold. Different normal forms, or partially normal forms, can be considered to fulfill the invariance condition. A usual one is the Birkhoff normal form which defines actions  $I_i = q_i p_i, i = 1, 2, 3$ , and then,

$$H(\mathbf{q}, \mathbf{p}) = H_2(\mathbf{q}, \mathbf{p}) + \sum_{3 \leq i+j+k \leq n} h_{ijk} I_1^i I_2^j I_3^k + R_n(\mathbf{q}, \mathbf{p}).$$

The truncated normal form is obtained removing the remainder  $R_n(\mathbf{q}, \mathbf{p})$  [4].

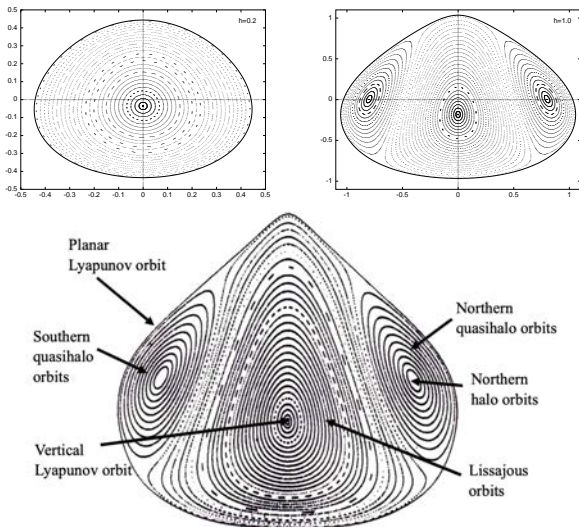


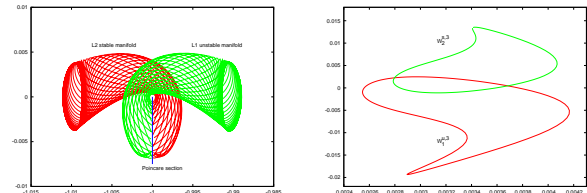
Figure 9: Example of Poincaré maps of the center manifold. Top left, before the halo bifurcation. Top right, after the halo bifurcation. Bottom, displaying different kinds of orbits.

A Hamiltonian fulfilling the invariance condition is useful to understand and visualize the whole dynamics in the center manifold. Roughly speaking, the Hamiltonian is reduced to the center manifold setting  $q_1 = p_1 = 0$  which avoids the instability, so long-term numerical integrations are possible with the reduced Hamiltonian  $\hat{H}(q_2, p_2, q_3, p_3)$ . We select the Poincaré section  $\{q_3 = 0\}$  and a fixed value  $\hat{h}$  of the Hamiltonian  $\hat{H}$ . The intersection of the constant-energy three-dimensional manifold  $\{\hat{H} = \hat{h}\}$  with the section  $\{q_3 = 0\}$  is two-dimensional and can therefore be represented in the  $(q_2, p_2)$ -plane. Given  $\hat{h}$  we choose an initial  $(q_2, p_2)$  pair and solve numerically the constraint  $\hat{H}(q_2, p_2; q_3 = 0, p_3) = \hat{h}$  for  $p_3 > 0$  (two opposite sign roots exist, of which we keep the positive one), and integrate the reduced Hamiltonian equations forward in time. Each subsequent crossing of the trajectory with  $q_3 = 0$  and  $p_3 > 0$  is plotted, producing a stroboscopic portrait of the phase space at the chosen energy (see Fig. 9).

**Heteroclinic or homoclinic trajectories** associated to libration point orbits provide natural transfer paths at zero propulsive cost. Such orbits are of interest in the trajectory design of missions like the NASA missions Genesis (Sun-Earth case) or Artemis (Earth-Moon case).

A simple way to introduce the concept is through periodic orbits in the planar RTBP. In the local neighbourhood of a collinear equilibrium point, and for an energy level close to that of the equilibrium, there is only one periodic orbit. A typical example is given by heteroclinic orbits linking planar Lyapunov orbits around  $L_1$  and  $L_2$  in the Sun-Earth problem.

The main procedure to detect such connections begins by defining a suitable surface of section  $\mathcal{S}$ . The stable and unstable manifolds of the reference periodic orbits are then numerically integrated and their intersections with  $\mathcal{S}$  are recorded. In the simplest consideration, one obtains closed curves and their intersections give heteroclinic trajectories, due to the fact that the planar restriction is a two degrees of freedom Hamiltonian where orbits and manifolds are in the same energy level.



In the 3D case, and again for a fixed Jacobi constant of the RTBP, the propagation of the manifolds to  $\mathcal{S}$  produces  $S^3$ -spheres, so their transversal intersection inside  $\mathbb{R}^4$  (the six dimensional space is constrained by energy and section  $\mathcal{S}$ ) topologically is an  $S^2$ -sphere instead of a finite set of points.

It is worth noting that the  $S^2$ -sphere family of heteroclinic connections does not occur entirely between the same pair of 2D tori (i.e., the same Lissajous orbits), but rather spans a range of them. The stable manifold of a given Lissajous orbit connects heteroclinically to a set of Lissajous orbits on the opposite side, and in the case of homoclinic connections (understood here as homoclinic to the region, not to a single orbit), to a range of tori in its vicinity. This structure provides a clear example of Arnold diffusion mechanisms within  $W^c$  [2].

Most studies exploit the classical RTBP, taking advantages of its autonomous characteristics. However, Earth-Moon RTBP neglects the Sun's direct gravity influence on the spacecraft and indirect influence on the Earth and Moon. As the missions become more complicated, higher fidelity non-autonomous

four-body problems become more necessary. The Bicircular Problem, **Quasi-bicircular Problem (QBCP)**, and Hill Restricted Four-Body Problem are the most extensively studied models, they are all periodic-perturbed systems. The QBCP considers the Sun, Earth and Moon in a solution of the general planar three-body problem and one can consider Sun-Earth Moon perturbed or Sun-Earth to Earth-Moon heteroclinics.

However computing tori and their manifolds in non-autonomous system is more complex than in autonomous system because the problem adds the set of frequency perturbations of the non-autonomous one. 2D invariant tori in CRTBP become 3D invariant tori in QBCP. In these settings, an additional difficulty appears: the stable and unstable manifolds must intersect at the same phase of the Sun-Earth-Moon system.

The quasi-bicircular problem is a planar approximation for the real Sun-Earth-Moon-spacecraft four-body system. It considers the primaries revolve in planar quasi-bicircular periodic trajectories of the general three-body problem. the dynamics of a massless fourth particle  $P$ , is expressed in two synodical reference frames, the Sun-barycenter  $S^{\text{SE}}$  and the Earth-Moon  $S^{\text{EM}}$ . Both formulations share the same formal expression for the Hamiltonian,

$$H(\mathbf{x}, \theta) = \frac{1}{2} \alpha_1(t) (p_x^2 + p_y^2 + p_z^2) + \alpha_2(t) (p_x x + p_y y + p_z z) + \alpha_4(t) x + \alpha_3(t) (p_x y - p_y x) + \alpha_5(t) y - \alpha_6(t) \sum_{b \in \mathbb{B}} \frac{m_b}{\|\mathbf{r} - \mathbf{q}_b(t)\|},$$

where  $\alpha_i(t)$ , are  $T$ -periodic coefficients,  $\mathbb{B} = \{S, E, M\}$  denotes the set of primaries (Sun, Earth, Moon) and  $\mathbf{q}_b(t)$  their corresponding position vectors.

The **parameterization method** is one way to represent invariant manifolds associated with invariant objects by means of high-order expansions [6]. For periodic non-autonomous systems, the manifolds associated to  $L_i$  dynamical equivalents (resonant periodic orbits) are parameterized in Fourier-Taylor series [1]. The method constructs a conjugacy between an  $n$ -dimensional invariant manifold  $\mathcal{M} \subset C^n$  of the  $T$ -periodic vector field  $\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z}, t)$  and a lower-dimensional model manifold  $\mathcal{S}$  with coordinates  $\hat{\mathbf{s}} \in \mathbb{C}^d$ , governed by a reduced vector field  $\dot{\mathbf{s}} = \mathbf{g}(\mathbf{s}, t)$ . The manifold  $\mathcal{M}$  is parameterized by a map  $\mathbf{z} = \mathbf{w}(\mathbf{s}, t)$ , which is obtained by means of the invariance equation

$$\mathbf{F}(\mathbf{w}(\mathbf{s}, t), t) = \frac{\partial \mathbf{w}(\mathbf{s}, t)}{\partial \mathbf{s}} \mathbf{g}(\mathbf{s}, t) + \frac{\partial \mathbf{w}(\mathbf{s}, t)}{\partial t}.$$

Both, the parameterization  $\mathbf{w}$  and the reduced dynamics  $\mathbf{g}$ , are recursively computed and expressed by truncated multivariate Fourier-Taylor expansions,

$$w_p(\mathbf{s}, t) = \sum_{k=0}^K \sum_{\mathbf{r} \in \mathcal{R}_k} \sum_{j=-J}^J w_p^{\mathbf{r}, j} e^{ij\omega t} \mathbf{s}^{\mathbf{r}} + \mathcal{O}(|\mathbf{s}|^{K+1}),$$

$$g_p(\mathbf{s}, t) = \sum_{k=0}^K \sum_{\mathbf{r} \in \mathcal{R}_k} \sum_{j=-J}^J g_p^{\mathbf{r}, j} e^{ij\omega t} \mathbf{s}^{\mathbf{r}} + \mathcal{O}(|\mathbf{s}|^{K+1}),$$

where  $\mathbf{r}$  is a multi-index and  $\mathcal{R}_k = \{\mathbf{r} \in \mathbb{N}^d : |\mathbf{r}| = k\}$ .

Using the parameterization method one can obtain accurate local representations of libration point orbits and their manifolds which can be propagated by numerical methods. The heteroclinic connections between the  $L_1$  and  $L_2$  orbits of the Sun-Earth system can be strongly perturbed when the Moon is included in the model. As a result, it becomes possible to obtain connections between orbits with very different amplitudes [5] (see Fig. 10).

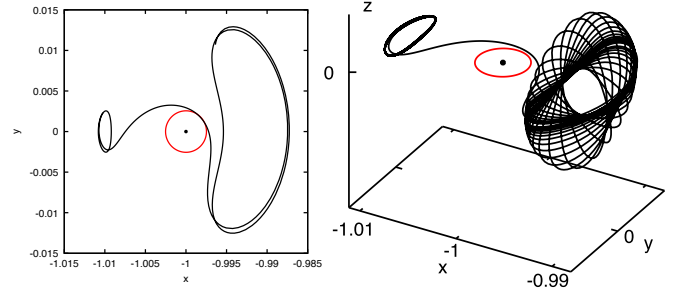


Figure 10: Examples of planar (left) and 3D (right) heteroclinic connections in the Sun-Earth system, strongly perturbed by the Moon. The Earth is shown as a black dot near the center, and the red circle indicates the Moon's orbit.

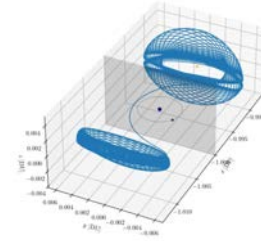


Figure 11: Sun-Earth  $L_1$ - $L_2$  heteroclinic between Lissajous orbits.

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## PhD highlights

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José Lamas Rodríguez<sup>✉</sup> submitted his PhD thesis<sup>✉</sup> *Oscillatory motions, parabolic orbits and collision orbits in the planar circular restricted three-body problem*, supervised by Marcel Guàrdia Munárriz<sup>✉</sup> and Tere M-Seara Alonso<sup>✉</sup> at the Universitat Politècnica de Catalunya (April 2025).



Celestial mechanics is a classical meeting point between physics and mathematics. Its starting point is deceptively simple: write Newton's laws for bodies that attract each other gravitationally and try to predict their motion. Yet, as soon as three bodies interact, long-term behavior becomes difficult to control, as already emphasized in Poincaré's work [9]. Close encounters can produce large changes in a trajectory, and small uncertainties in the initial data may grow over time. For this reason, a modern approach does

not aim primarily at explicit formulas for the motion, but at understanding which types of trajectories are possible, how they can transition from one type to another, and which geometric structures in phase space organize this complexity.

A standard model to study these questions is the planar circular restricted three-body problem (PCRTBP). In this setting, two primaries (the Sun and Jupiter) move on circular coplanar orbits, while a third body of negligible mass (for instance, an asteroid) evolves under their gravitational field. In rotating coordinates the primaries are fixed, the system is Hamiltonian, and trajectories are either defined for all time or end in collision with one of the primaries. For trajectories defined for all time, Chazy classified the possible asymptotic behaviors (final motions), which include unbounded motions (hyperbolic and parabolic), bounded motions, and oscillatory motions [1]. The main goal of this thesis is to incorporate the collision set into this picture and to understand how final motions and collision dynamics fit together in a single geometric framework.

The first part focuses on the dynamics close to the massive primary (the Sun), in a regime adapted to a small mass ratio. The main result is that every combination of past and future final motions can be realized by trajectories that pass arbitrarily close to collision with the Sun. Beyond this, the thesis constructs ejection-collision trajectories with the Sun that make arbitrarily large excursions before returning, and it builds hyperbolic invariant sets whose closure intersects both the collision set and infinity. These sets provide a precise form of chaotic dynamics: they contain in their closure ejection and collision orbits to the Sun, together with trajectories displaying prescribed asymptotic behavior, and their dynamics can be encoded by sequences of symbols [3, 10].

The techniques in this part are geometric and rely on regularization and invariant manifolds. Near the Sun, collision is regularized using McGehee coordinates: the singularity is replaced by invariant tori containing normally hyperbolic cir-

cles, each with stable and unstable manifolds. On the other end of phase space, "parabolic infinity" is compactified into an invariant object that also has stable and unstable manifolds. A central step is to bring these manifolds to a common transverse section and compare them by computing a distance function. Following the ideas in [5], this distance is analyzed through a Melnikov-type computation, which yields transverse intersections between the manifolds associated with collision and infinity. Combining these intersections with the classical description of the dynamics near infinity (in the sense of Moser [8]) produces the large ejection-collision excursions. A further local analysis close to the regularized collision set yields additional intersections near collision, and at a suitable energy level a more refined configuration leads to hyperbolic dynamics accumulating simultaneously on collision and on infinity.

The second part extends the analysis to the smaller primary (Jupiter) and to the interaction between both primaries. The conclusions mirror those near the Sun: all combinations of past and future final motions can be realized by trajectories that accumulate on collision with Jupiter; there exist ejection-collision orbits with Jupiter that perform arbitrarily large excursions; and there are hyperbolic invariant sets whose closure intersects Jupiter's collision set and supports symbolic dynamics. Building on these constructions, we also prove the existence of ejection-collision orbits that connect the two primaries.

The methods differ because the local geometry near Jupiter is not the same as near the Sun when the mass ratio is small. To obtain a meaningful description as the mass parameter tends to zero, the analysis splits phase space into two regions: far from Jupiter, the motion is treated as a nearly integrable perturbation of the Kepler problem with the Sun; close to Jupiter, one zooms into a neighborhood whose size is determined by the mass ratio. In this near region, collision is regularized using Levi-Civita coordinates [4], which replace the singularity by a regular circle and allow ejection and collision trajectories to be treated as regular orbits passing through the transformed collision set. A local analysis then controls the passage near Jupiter (adapting ideas in the literature [6]), and the next step is to compare these near-Jupiter trajectories with the invariant manifolds associated with infinity by extending parameterizations to a transverse section near Jupiter. Since these manifolds are Lagrangian, the extension is carried out using Hamilton-Jacobi parameterizations together with complex-analytic estimates [2]. Finally, a Poincaré-Melnikov-like computation produces transverse intersections, from which the large-excursion ejection-collision orbits, the symbolic-dynamics structures, and the connecting trajectories between primaries are obtained.

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Rafel Perelló i Ribas<sup>✉</sup> submitted his PhD thesis<sup>✉</sup> *Data assimilation for real-time dynamic prediction of wind-induced forces in vehicle platooning*, supervised by Antonio Huerta<sup>✉</sup> and Sergio Zlotnik<sup>✉</sup> at the Universitat Politècnica de Catalunya (November 2025).



Nowadays, in the field of automotive aerodynamics, there is a big concern on concepts such as safety and energy efficiency. This binds with the emerging field of autonomous vehicles where it is critical to ensure safe control and stability of the vehicle under all conditions that might be encountered in realistic situations [1–3]. Moreover, it allows the possibility to explore innovative solutions such as platooning techniques to improve energy efficiency and in-

crease road capacity [4, 5].

The increase of computational power over recent years has allowed engineers and scientists to study the problems of parametric Partial Differential Equations (PDE). That is, to study how the solution of a PDE depends on some parameters defining material constants, body forces, boundary conditions, domain geometry, etc. [6, 7]. However, in many engineering applications, the computational cost of finding a solution of a PDE for a single combination of the parameters is still very high. This is particularly true in Computational Fluid Dynamics (CFD) [8], thus it is unfeasible to predict the aerodynamic effects on vehicles under a wide range of conditions using standard techniques.

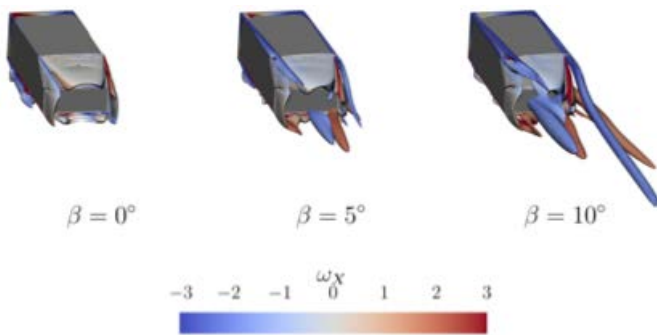


Figure 13:  $Q$  criterion isosurface ( $Q = 8000 \text{ s}^{-2}$ ) coloured by vorticity  $\omega_X$  and for different yaw angles  $\beta$ .

Surrogate modelling techniques are introduced to obtain a surface response of parametric PDEs for problems where traditional methods are unfeasible. An interesting family of surrogate methods are those based on multifidelity, i.e., methods

that combine information obtained from low and high-fidelity simulations [9, 10]. Multifidelity methods do not simply use low fidelity simulations to obtain cheap approximations and then switch to high fidelity if higher accuracy is desired. In contrast, they use simultaneously information obtained from low and high-fidelity simulations to obtain a single surrogate.

In this work we devise surrogate techniques based on multifidelity extensions of the Smolyak approximation method [9, 11] to predict the aerodynamic forces acting on a vehicle in platoon conditions on realistic road conditions. We consider the geometry of the leading vehicle, the spacing between vehicles, the speed of travelling and the presence of cross-wind as parameters of our model and show that the devised methodology is able to construct an accurate surrogate model while reducing considerably the computational cost required by standard techniques.

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Pradeep Kumar Bal<sup>✉</sup> submitted his PhD thesis<sup>✉</sup> *Mathematical and computational modeling of the active mechanics of multicellular systems: from cell-cell adhesion to epithelial reshaping*, supervised by Marino Arroyo<sup>✉</sup> at the Universitat Politècnica de Catalunya (September 2025). He is currently expanding his doctoral research as a postdoctoral researcher with Prof. Marino Arroyo.



This thesis develops theoretical and computational frameworks to model two fundamental mechanical functions of multicellular tissues: cell-cell adhesion and epithelial reshaping. Although these processes are governed by subcellular dynamics, they manifest at mesoscopic scales, posing a challenge for existing modeling approaches. The overarching goal is to bridge this gap by providing models that capture the essential physical and biological mechanisms across scales. The work is structured in two parts, each addressing a distinct aspect of tissue mechanics, while sharing a common theoretical foundation based on irreversible thermodynamics and active gel theory.

Part I of the thesis focuses on the dynamic formation and organization of cell-cell adhesions, with particular emphasis on cell doublets as the simplest multicellular unit. A mesoscale theoretical framework is developed that couples the mechanics of the cellular surface with the chemical kinetics of adhesion molecules, their lateral diffusion on the membrane, and their feedback with the actomyosin cortex. The formulation is grounded in Onsager's variational formalism to ensure thermodynamic consistency and is implemented computationally using both axisymmetric and fully three-dimensional finite element methods.

Part II focuses on the development of continuum models to study epithelial reshaping, a central driver of morphogenesis. At larger length scales, epithelial sheets represent a prototypical class of cellular tissues in animals, composed of one or a few layers of tightly connected cells forming cohesive barriers that often separate a cavity or free surface from underlying stromal tissue [4]. During development, these tissues envelop embryos and undergo extensive reshaping while performing essential physiological functions such as secretion, absorption, filtration, and protection of underlying tissues.

The functional role of epithelial tissues in development and homeostasis depends critically on their three-dimensional geometry. Epithelial folding underlies many morphogenetic processes, including *Drosophila* gastrulation and wing development [5], as well as placode invagination in organs such as salivary glands and hair follicles. Out-of-plane deformations arise from apicobasal asymmetries in contractility or from mechanical instabilities such as buckling induced by lateral compression [6].

At cellular and subcellular scales, epithelial reshaping emerges from the interplay of multiple mechanisms, including cell division, apoptosis, and neighbor rearrangements, as well as actively generated forces that drive bending through contractility asymmetries or promote folding via compressive stresses [7]. Morphogenesis often results from the coordinated action of these mechanisms rather than a single dominant process [6].

Existing models, however, fail to connect subcellular cytoskeletal dynamics with the large-scale reshaping of epithelial monolayers, which behave as active continuum shells. The first aim of Part II is thus to develop a continuum shell theory derived from active gel descriptions of the actin cortex. We present a fully nonlinear, viscoelastic, active theory that homogenizes a 3D vertex-like model of individual cell surfaces (see Figure 15 (I)). Each cell surface is treated as an active gel patch, yielding a continuum surface model that captures the collective behavior of viscoelastic active gels undergoing turnover. The theory accounts for geometric and mechanical anisotropy of lateral junctions, as well as apicobasal asymmetries, by relating the deformation of apical, basal, and lateral surfaces to the mid-surface of the tissue. Two formulations are introduced: a Kirchhoff shell theory, with lateral junctions perpendicular to the mid-surface, and a Cosserat theory, allowing junctional tilt. Using a variational formalism, microscopic Rayleighian functionals for each cellular surface are coarse-grained to yield continuum governing equations, which are implemented using finite element methods. Comparison with 3D vertex simulations demonstrates excellent agreement between the continuum and discrete models, and between the Kirchhoff and Cosserat theories (Figure 15 (II)).

The second aim is to develop a finite element approach to approximate this shell theory. The third aim is to apply these models and methods to understand how apicobasal asymmetries and buckling control the reshaping of epithelial shells. More specifically, we aim to elucidate the mechanisms underlying epithelial wrinkling and pattern formation in experiments involving rapidly deflated epithelial shells (see Figure 15 (III) and

zation of activated bonds, drive the formation of large, mature adhesion patches enriched in trans bonds (Figure 14c). These results reproduce key experimental observations of adhesion maturation and provide a mechanistic framework for understanding how microscopic activity gives rise to emergent adhesion structures.

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IV)) [8]. The continuum model demonstrates how cortical viscoelasticity, viscous drag from the surrounding medium, and curvature anisotropy determine the morphology and spatial patterning of wrinkles in epithelial shells. We then study wrinkling of compressed and asymmetrical epithelial sheets (see Figure 15 (V)), as investigated by [9] using a 1D beam-like continuum model that accounts for tilt and coarse-grains a 2D lateral vertex model, thereby providing a verification of our Cosserat theory. Finally, we examine the effect of apicobasal asymmetry on free-standing tissues by simulating the experiments of [10], finding that the tilt emerging from the Cosserat theory is essential to achieve stable steady states (see Figure 15 (VI)).

The continuum model offers several advantages over detailed cellular models: it enables efficient simulations, is amenable to mathematical analysis, and identifies essential parameters. While this work focuses on the actomyosin cortex, the coarse-graining approach can incorporate additional elements, such as apical belts, intermediate filaments, or adhesion dynamics from Part I. It can also be coupled to deformable substrates or adapted to plant tissues. Although our theory assumes that tissue architecture is fixed, it provides a natural framework to extend to evolving architectures due to cell division, extrusion, or junctional rearrangements (Lemke and Nelson, 2021). Finally, the approach can be combined with models of mechanosensitive signaling (Bollenbach et al., 2007; Hidalgo et al., 2019) to explore the interplay between mechanics and morphogen-mediated regulation, offering a powerful tool to understand epithelial tissue mechanobiology.

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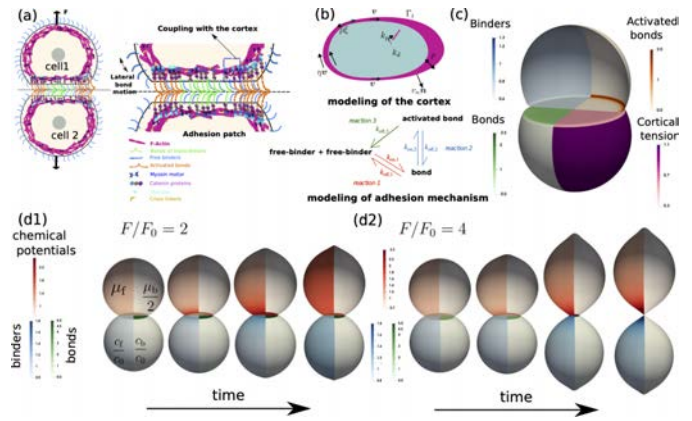


Figure 14: (a) Schematic of the cell–cell adhesion mechanism. (b) Active-gel model of the cell cortex and reaction network underlying the adhesion mechanism. (c) Out-of-equilibrium steady state observed during cell–cell adhesion maturation, obtained from FEM simulations. (d) Forced decohesion of two adhered vesicles under different applied forces: (d1)  $F/F_0 = 2$ , for which a stable adhesion patch is observed; (d2)  $F/F_0 = 5$ , for which complete separation occurs.

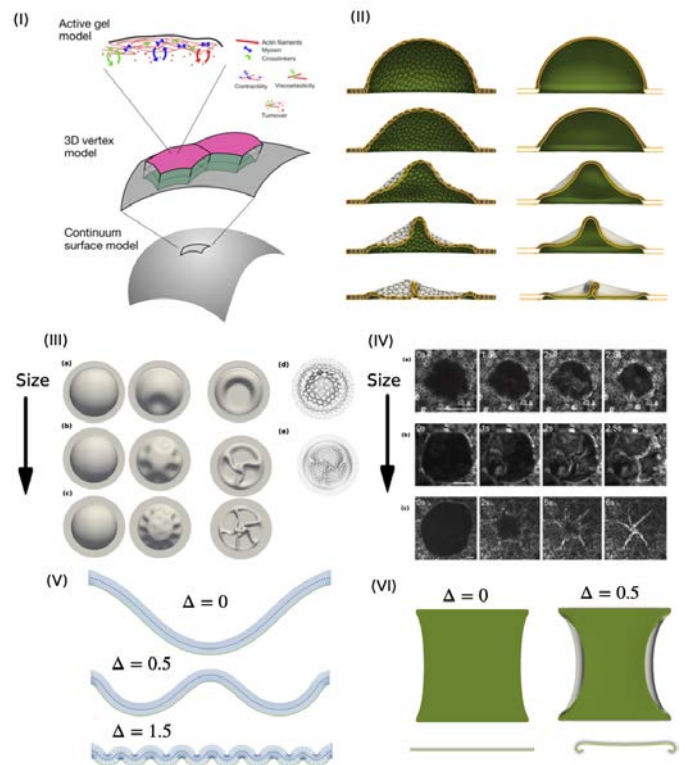


Figure 15: (I) Schematic of the continuum active bilayer model of epithelial tissue, derived from subcellular descriptions of the actin cortex as an active gel. (II) Comparison with 3D vertex model simulations showing rapid deflation of an epithelial dome, which induces buckling. (III) Wrinkling dynamics in epithelial shells studied using both the continuum active bilayer model and the 3D vertex model, for epithelial domes of different sizes subjected to rapid deflation. The aspect ratios of the domes are (a) 13, (b) 26, and (c) 70. (IV) Experimental investigation of epithelial dome wrinkling dynamics using the “MOLI” (Mono-Layer Inflator) setup developed in Xavier Trepats’ group, for domes with aspect ratios (a) 13, (b) 26, and (c) 70. (V) Wrinkling instability in an unsupported epithelium under compression due to apicobasal asymmetry. (VI) Curling of a tissue under tension resulting from apicobasal asymmetry.

Sergi Pérez<sup>✉</sup> submitted his PhD thesis<sup>✉</sup> *Finite Element computational modeling of flexoelectricity and flexo-photovoltaics*, supervised by Sonia Fernández<sup>✉</sup> and Irene Arias<sup>✉</sup> at the Universitat Politècnica de Catalunya (July 2025). He is currently expanding his doctoral research as a postdoctoral researcher at Barcelona Supercomputing Center (BSC).



The PhD focuses on the numerical modeling of flexoelectricity and flexo-photovoltaics, both at infinitesimal and finite deformations, using standard  $C^0$  Finite Element (FE) approximations.

On one side, an alternative formulation to the drift-diffusion semiconductor modeling equations, relying on adimensionalized logarithmic quantities, is developed. The FE implementation of both formulations in an in-house

MATLAB code is able to reproduce benchmark problems, demonstrating the benefit of the logarithmic formulation in convection-dominated scenarios, where coarser meshes are able to provide solutions without spurious oscillations. Furthermore, two non-linear solvers are assessed and compared: a monolithic Newton-Raphson method and the Gummel method. On the other side, the focus is placed on the development of the extension of  $C^0$  Interior Penalty formulations for the solution of the partial differential equations (PDE) modeling linear flexoelectricity, including additional converse flexoelectricity and gradient dielectricity effects, and on the development

of a combined  $C^0$  Interior Penalty Newton-Raphson method for the solution of the non-linear PDE modeling flexoelectricity at finite strains. The proposed schemes are able to avoid the drawbacks of  $C^1$  approximation spaces or mixed formulations, enabling the solution of fourth-order PDE by standard FE approximations. The computational implementation of the developed numerical schemes has shown the expected high-order convergence of the methods, and is able to reproduce benchmark problems. Moreover, the developed frameworks are extended, incorporating generalized periodicity boundary conditions, to reproduce apparent piezoelectric metamaterials at large deformations.

Finally, the coupling of flexoelectricity and semiconductor modeling is carried out. Proof of concept experiments, simulated with the FE solution of the proposed coupled continuum model at infinitesimal deformations, are reported, comparing the obtained results with standard photovoltaic simulations. In addition, the thesis provides a continuum modeling approach for the flexo-photovoltaic effect at finite deformations.

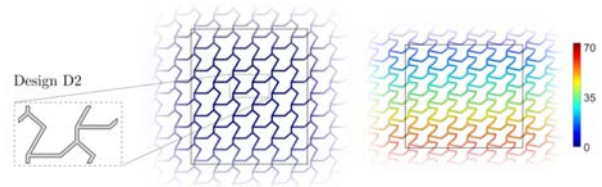


Figure 16: Metamaterial geometry: on the left the unit cell, on the center, the undeformed lattice, and on the right, the material under a 25% compression. Color represents the electric potential generated due to the flexoelectric effect.

### Mathematical formulation of contaminant removal in adsorption columns by Marc Calvo-Schwarzwalder<sup>✉</sup> (PDEs Group<sup>✉</sup>, IMTech<sup>✉</sup>)

Column sorption is a practical and widely used method for removing contaminants from carrier fluids and has attracted considerable attention in recent years. The process consists of passing a fluid through a column packed with a material capable of selectively capturing specific components. The mathematical modelling of this process, together with the underlying chemical and physical mechanisms, has been a central focus of an ongoing interdisciplinary collaboration involving multiple research centres and universities across Europe.

This work extends beyond mathematics alone, relying on close collaboration with chemical and environmental engineers as well as experimental scientists. Participating institutions include UPC, with researchers from the Departments of Mathematics, Fluid Mechanics, and Chemical Engineering; the Universitat de Girona; LEQUIA (Laboratori d'Enginyeria Química i Ambiental); IQS (Institut Químic de Sarrià); ICIQ (Institut Català d'Investigació Química); the University of Leeds; the University of Oxford; and, more recently, the University of British Columbia (Vancouver, Canada). Interdisciplinary collaborations of this kind are essential for the development of new technologies and the advancement of existing ones.

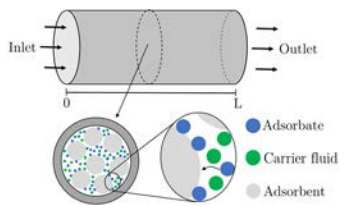


Figure 17: Schematic of the experimental setup. A gas mixture flows through a column of length  $L$  which is filled with an adsorbing material. The contaminant molecules attach to these particles as the mixture moves towards the column outlet.

Mathematically, the removal of trace-amounts of a single contaminant in a column of length  $L$  typically consists of one diffusion-advection-reaction equation of the form

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} - \alpha \frac{\partial q}{\partial t}, \quad (9)$$

with  $c(x, t)$  being the cross-sectional average of the molar concentration of contaminant at time  $t > 0$  and at some point  $x \in (0, L)$  of the column;  $u$  is the interstitial velocity of the mixture flowing through the column, which remains approximately constant in the case of trace amounts;  $D$  is a dispersion coefficient and  $\alpha \dot{q}$  is a sink term describing the rate at which the contaminant is being removed. The quantity  $q(x, t)$  quantifies the amount of contaminant that is being captured per unit mass of adsorbent material. In addition, one must consider the so-called kinetic equation

$$\frac{\partial q}{\partial t} = \Phi(c, q),$$

where the form of  $\Phi$  depends on the physical and/or chemical mechanisms. A common model used at this point is the

one developed by Langmuir [1], which allows a contaminant to adsorb and desorb from the adsorbent at rates  $k_a$  and  $k_d$ ,

$$\Phi_L(c, q) = k_a c(q_m - q) - k_d q, \quad (10)$$

with  $q_m$  being the maximal amount of contaminant that can be removed by the adsorbent. This mathematical formulation has been widely studied for the case when both trace or large amounts of one contaminant are being removed [2–8].

In practise, a carrier fluid typically contains more than one contaminant, which results in significantly more complicated kinetics. Mathematically, the process is also much more complicated since competition among the different species must be accounted for. When multiple contaminants are being removed, the different species compete among them to occupy the available sites on the surface of the adsorbent. For instance, a free molecule of one species can remove an adsorbed molecule of another species and occupy the site instead, see Figure 18. Besides the individual adsorption and desorption processes, which are described similarly to the single contaminant case, these interactions must be included in the mathematical formulation.

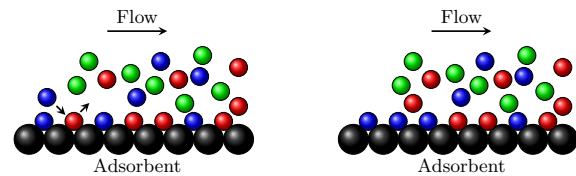


Figure 18: A carrier gas (green molecules) contains two contaminants (blue, red molecules) which attach to the adsorbent. Here, the blue molecule removes an already adsorbed red molecule and ends up taking its place.

Recently [9, 10], we have extended the single-contaminant model to a system where two species compete against each other to be adsorbed, eventually replacing each other on occupied adsorption sites. The molar concentration  $c_i$  of each species satisfies an equation that is similar to Eq. (9), with the particularity that the removed amounts are now quantified by  $\theta_i$  rather than  $q_i$ , which we refer to as the fractional coverage of each species. Both quantities are related via  $\theta_i = q_i/q_{m,i}$ . For each contaminant, the advection-diffusion equation now reads

$$\frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial x} = D \frac{\partial^2 c_i}{\partial x^2} - \alpha_i \frac{\partial \theta_i}{\partial t},$$

whereas the kinetic equations take the form

$$\begin{aligned} \frac{\partial \theta_i}{\partial t} = & \underbrace{k_{ad,i} c_i \left( 1 - \sum_j \theta_j \right)}_{\Phi_i} - k_{de,i} \theta_i \\ & + \underbrace{\left( \sum_{i \neq j} k_{i,j} \theta_j \right) c_i - \left( \sum_{i \neq j} k_{j,i} c_j \right) \theta_i}_{\Psi_i}, \end{aligned} \quad (11)$$

where  $\Phi_i$  and  $\Psi_i$  respectively account for individual adsorption mechanisms and interactions among species. In Eq. (11),

we have used the Langmuir model to describe individual adsorption. This general form allows generalising the model to any number of contaminants  $N$ , which we have done in a follow-up paper that is currently being reviewed [11].

After introducing sensible non-dimensional variables and discussing the relative importance of the relevant adsorption mechanisms, we have been able to develop analytical solutions for the breakthrough concentration - the concentration that is being measured at the column outlet - of each species in a 2-component model [9,10] and, in  $N$ -contaminant system with arbitrary  $N$ . This had not been done before and represents an important step forward in understanding competitive adsorption of multiple contaminants.

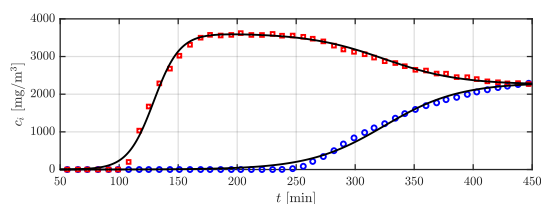


Figure 19: Evolution of the concentrations at the column outlet. Symbols refer to experimental data obtained at LEQUIA and solid lines are the solutions to the 2-component model.

Our current research focuses on several interconnected directions: investigating the role of humidity in adsorption processes, which can be viewed as a particular class of multi-component systems; developing more general analytical solutions for multi-component models, for example by relaxing restrictive assumptions on model parameters; and exploring adsorption mechanisms beyond the Langmuir model. The latter is of particular interest for systems involving the removal of metallic species from liquid phases, among other applications.

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### From bulk to surface: The emergence of surface effects in nanoscale dielectrics by David Codony<sup>✉</sup> (LaCàN Group<sup>✉</sup>, IMTech<sup>✉</sup>)

**The flexoelectric effect.** In the world of smart materials, electromechanical transduction—the ability to convert mechanical force into electricity and vice versa—is ubiquitous, and represents the backbone of modern devices: from consumer electronics (smartphone haptics, inkjet printers, voice recognition devices) to medical devices (ultrasound imaging, precision surgery) and automotive industry (fuel injectors, parking sensors), to name a few. Most of these devices rely on *piezoelectric* transduction—physical coupling between uniform deformation and electric field—which is restricted to specific, non-centrosymmetric crystals only. Our research group is exploring an alternative, yet elusive electromechanical transduction mechanism: *flexoelectricity*, which is a universal property of *all* dielectric materials.



Flexoelectricity refers to the two-way coupling between strain *gradients* (non-homogeneous deformations like bending) and electric fields: bend it, and it kicks out voltage; zap it, and it bends (20).

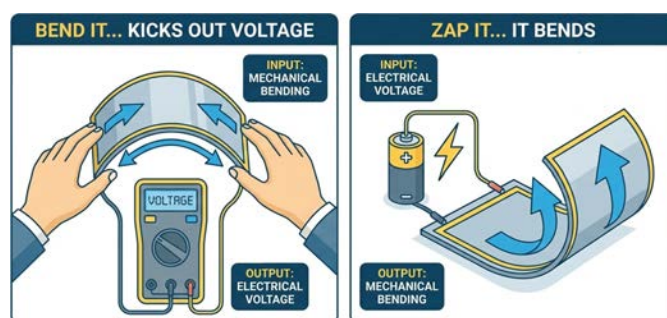


Figure 20: Two-way coupling of flexoelectricity: Sensing (left), actuation (right).

Mathematically, it is governed by a coupled system of fourth-order partial differential equations written in terms of mechanical displacements  $\mathbf{u}$  and electric potential  $\phi$ —or more precisely, the strains  $\boldsymbol{\varepsilon} = \text{sym}(\nabla\mathbf{u})$  and electric fields  $\mathbf{E} = -\nabla\phi$ —as

$$\begin{aligned}\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} &= \mathbf{0}, & (\text{Mechanical balance of momentum}) \\ \nabla \cdot \mathbf{D} - q &= 0, & (\text{Gauss's law for electrostatics})\end{aligned}$$

with the constitutive equations  $\boldsymbol{\sigma}(\boldsymbol{\varepsilon}, \nabla(\nabla\boldsymbol{\varepsilon}), \mathbf{E}, \nabla\mathbf{E})$  and  $\mathbf{D}(\mathbf{E}, \nabla(\nabla\mathbf{E}), \boldsymbol{\varepsilon}, \nabla\boldsymbol{\varepsilon})$ , both depending on low- and high-order mechanical and electrical kinematic variables.

Since the flexoelectric effect is governed by the change in deformation over a distance, it only becomes powerful enough at micro- and nano scales, where strain gradients are inherently large. At such small scales ( $\ell$ ), bulk effects—including flexoelectricity—compete with other surface effects, which become more and more dominant at smaller scales, simply because area ( $\ell^2$ ) reduces at a slower rate than volume ( $\ell^3$ ) upon device miniaturization ( $\ell \rightarrow 0$ ).

**Recent Results: The Emergent “Piezoelectric Skin”.** Our recent work has shifted the focus from the bulk of the material to its *surfaces*. We have noticed that in purely flexoelectric samples, the simple presence of a free surface breaks the material’s symmetry [3], causing boundary layers to *naturally* emerge in the strain and electric field profiles, even if bulk effects *only* are accounted for (2). The behavior of the material “skin” resembles that obtained with models that explicitly include surface piezoelectricity and other surface effects.

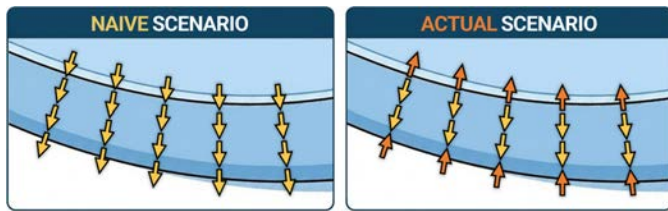


Figure 21: Flexoelectricity-induced boundary layers in the electric field.

Upon analytical [2,3] and numerical [2] studies, we showed that these boundary layers grow exponentially near the surface, with specific characteristic lengths depending on mechanical, dielectric and flexoelectric material properties. Furthermore, we have demonstrated that in lattice metamaterials—structures with very high surface-to-volume ratios—these surface effects can significantly enhance or even dominate the overall electromechanical macroscopic response.

**Computational Challenges: Solving the “Sharp + Smooth” Physics.** The numerical modeling of these effects presents significant computational hurdles. On the one hand, traditional numerical methods, like standard finite elements, struggle with flexoelectricity equations because high-order continuity across elements is required. Typically, smooth alternatives are considered, including B-spline-based approaches [1] or finite element formulations enforcing weak  $C^1$  continuity across elements [4], among others.

On the other hand, smooth approximants struggle when it comes to capturing sharp boundary layers, often requiring specialized mesh refinement near boundaries. The presence of sharp layers may also require addressing the numerical instabilities of the computations.

**Upcoming Research: Surface-aware Analytical Modeling.** Despite the demonstrated relevance of surface effects in flexoelectric materials at high surface-to-volume scenarios, they are frequently overlooked, particularly in analytical modeling. This is the case of one-dimensional models, like Euler-Bernoulli flexoelectric beam theories. These theories typically assume constant strain gradients and electric fields along the cross section, thereby ignoring their naturally-emerging transversal boundary layers.

To address these challenges, we are working on a high-order formulation for flexoelectric beams that intrinsically accounts for boundary layers. This formulation, validated against numerical computations, enables in turn an accurate characterization of the non-vanishing longitudinal electric fields—also typically neglected—which are crucial for the electromechanical operation of architected flexoelectric devices based on truss structures.

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## Chronicles

### IMTech scholarships to support research groups

by Gemma Huguet<sup>✉</sup> (DMAT<sup>✉</sup>, IMTech<sup>✉</sup>)

During the course 2024-2025, **IMTech** awarded scholarships to support research groups<sup>✉</sup> to UPC master students that developed a research project within the research groups of the **IMTech**.

The awarded candidates were Rosa Delicado, Maria Morella and Marc Torrecillas, who were students of the UPC Master in Advanced Mathematics and Mathematical Engineering (MAMME)<sup>✉</sup> and Emad Dehghani, who was student of the

Master’s degree in Numerical Methods in Engineering<sup>✉</sup>.

**Rosa Delicado** holds a Bachelor’s degree in Mathematics and Telecommunications from the **Universitat de les Illes Balears** and she was enrolled in the MAMME. Currently she is working for a PhD at the **Universitat de les Illes Balears** in the field of Mathematical Modeling in Neuroscience.

Her master thesis *Modeling neural oscillations and synchronization in brain networks*<sup>✉</sup> was supervised by Professor Pau Clusella<sup>✉</sup> (DMAT), and Professor Gemma Huguet<sup>✉</sup> (DMAT, IMTech, CRM) and was defended in June 16, 2025.

**Summary:** Understanding the dynamics that emerge from

large-scale brain models is challenging due to the high complexity of the system. In this project, we build upon the study in (Clusella et al, PCB, 2023) to investigate the emergence of complex spatiotemporal patterns in a network of 90 interconnected brain regions, each modeled as an excitatory-inhibitory (E-I) network whose dynamics are described by next generation neural mass models. We analyze the homogeneous oscillatory state of the system and study its stability under uniform perturbations. To assess its stability against non-uniform perturbations, we apply the Master Stability Function formalism, which allows us to characterize the emergence of complex spatiotemporal patterns from the unstable directions of the homogeneous state.

**Maria Morella** holds a Bachelor's degree in Mathematics from the [Universitat Politècnica de Catalunya](#) and was enrolled in the master [MAMME](#). Currently she is working for the company KPMG España.

Her master thesis [Analytic information encoded in the Bernstein-Sato polynomial of genus-one plane branches](#) was supervised by [Maria Alberich](#) (DMAT, [IMTech](#), IRI) and was defended on October 2025.

**Summary:** One of the central challenges in singularity theory is to determine what analytic information is encoded by a given analytical invariant. In this work, we focus on the Bernstein-Sato polynomial, a discrete analytic invariant whose roots vary discontinuously with parameters, changing in a jumping way. For plane branches, Blanco has recently identified a set of topological roots, namely those that remain constant under deformations within the same topological class. It remains an open problem to understand what analytic information is carried by the remaining roots. In this thesis, we show that for branches of genus one (i.e., with a single characteristic exponent), these roots determine the first non-topological value of the Kähler differentials, which corresponds to a shift of the Zariski exponent. Moreover, we provide an example demonstrating that no further non-topological values of the Kähler differentials can be recovered from the Bernstein-Sato polynomial.

**Marc Torrecillas Castelló** holds a double Bachelor's degree in Mathematics and Engineering Physics from the [Universitat Politècnica de Catalunya](#) and was enrolled in the master [MAMME](#). Currently he is about to start a PhD in Mathematics at [Universitat Politècnica de Catalunya](#).

His master thesis [Microlocal algorithms for the Gauss-Manin](#)

[connection of ICIS](#) under the supervision of Professor [Josep Álvarez](#) (DMAT, [IMTech](#), CRM) and [Guillem Blanco](#) (DMAT, [IMTech](#), CRM), which was defended in June 25, 2024.

**Summary:** The microlocal structure of the Brieskorn lattice of an isolated hypersurface singularity has been used to develop efficient algorithms for the computation of singularity invariants, such as the cohomological monodromy or the singularity spectrum. In this master thesis, we partly generalize these ideas to Isolated Complete Intersection Singularities. We study the microlocal structure of the Brieskorn lattice for smoothings of ICIS of positive dimension such that the domain has rational singularities and, under these hypothesis, we present an algorithm to compute the generic monodromy, defined as the monodromy of any  $\mu$ -minimal smoothing.

**Emad Dehghani** holds a Bachelor's degree in Mechanical Engineering from [Yazd University](#) (Iran) and was enrolled in the master MNE at [UPC](#). He will soon begin a PhD thesis at [CIMNE](#) in the field of Computational Science and Engineering.

His master thesis [Optimal transport for computational engineering](#) was supervised by Professor [Matteo Giacomini](#) (DECA, [IMTech](#), [CIMNE](#)) and Professor [Pedro Díez](#) (DECA, [IMTech](#)), and defended on July, 22, 2025.

**Summary:** This thesis explores transformation maps based on optimal transport theory, with a particular emphasis on their application to computational engineering. A computational framework is developed for constructing intermediate probability distributions via the Kantorovich formulation of optimal transport, combining linear interpolation and quadrature-based numerical integration given an optimal cost function. The methodology is implemented in both one and two dimensions, utilizing the Earth Mover's Distance (EMD) from the Python Optimal Transport (POT) library. In this context, a key limitation of classical interpolation schemes is their tendency to exhibit spurious oscillations, especially under mesh refinement. To address this, an adaptive smoothing post-process is proposed. This method systematically removes high-frequency artifacts while conserving mass and maintaining interpolation fidelity. It further adapts to mesh resolution, ensuring consistent behavior across discretizations. Comprehensive numerical experiments in 1D and 2D validate the proposed method's accuracy, robustness, and general applicability to both unimodal, multimodal, and other generic distributions.

▷ Editorial

[ICMNS-2025](#) by [Toni Guillamon](#) (DMAT, [IMTech](#) and [CRM](#))

Received December 22nd, 2025

The [ICMNS-2025](#) ([International Conference on Mathematical Neuroscience 2025](#)) was held at the [PRBB](#) (Parc de Recerca Biomèdica de Barcelona) on the week June 17–20, 2025.

The ICMNS is an inter-disciplinary conference series, bringing together theoretical/computational neuroscientists and mathematicians. The conferences are aimed at scientists interested in using or developing mathematical techniques for neuroscience problems. ICMNS 2025 has been the tenth annual conference. ICMNS 2024 was held in Dublin, whereas previous editions were held in Copenhagen, Joan dei Pins and Boulder (Colorado).



The conference brought together 174 scientists from both local and international communities, including renowned experts in the field of mathematical neuroscience, making it the ICMNS edition with the highest participation to date. Undoubtedly, the strong activity in mathematical and computational neuroscience in Catalonia (articulated around the [Barccsyn](#) platform) has been a decisive factor in attracting such a high level of participation

The conference featured 3 plenary lectures, 8 invited lectures, 44 contributed talks, and 80 posters. The communications presented at the conference addressed a broad range of topics in neuroscience, including brain connectivity, cognitive modeling (perception, decision-making, attention, reasoning, learning, memory, ...), neural coding, and neural plasticity. A unifying feature of all the presented work was the use of mathematical modeling and its theoretical and computational analysis, drawing primarily on dynamical systems, probabilistic, data-driven, and machine-learning approaches.

The plenary talks were delivered by [Elad Schneidman](#), from the Weizmann Institute of Science, who lectured on *Learning the code of large neural populations using sparse random projections*, [Tatyana Sharpee](#), from the Salk Institute, who lectured on *How neural manifolds change with learning*, and [Tatjana Tchumatchenko](#), from Universität Bonn, who lectured on *Energy minimization as an organizing principle for neural proteins*.

The invited speakers were: [Rafal Bogacz](#), from Oxford University, [Áine Byrne](#), from University College Dublin, [Alexis Dubreuil](#), from University of Bordeaux (CNRS), [Soledad Gonzalez Cogno](#), from Kavli Institute (NTNU), [Stephanie Jones](#), from Brown University, [Anna Levina](#), from University of Tübingen, [Sukbin Lim](#), from NYU Shanghai, and [Jonathan Touboul](#), from Brandeis University.

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The conference was organized by the [Centre de Recerca Matemàtica \(CRM\)](#) and received financial support from both the [European Society of Mathematical and Theoretical Biology \(ESMTB\)](#) and the [Society for Mathematical Biology \(SMB\)](#). Two members of the Organizing Committee (T. Guillamon and G. Huguet) belong both to the [IMTech](#) and to the [UPC Dynamical Systems Group](#).

[▷ Editorial](#)

#### IMTech Colloquium 26/03/2025

by [Gemma Huguet](#) (DMAT, [IMTech](#))

On March 26, 2025, Professor [Anup Rao](#) (University of Washington) delivered the [IMTech Colloquium Lecture](#) at the Faculty of Mathematics and Statistics (FME). He is Professor in the Department of Computer Science and Engineering at the University of Washington (Seattle). He received his BS from the Georgia Institute of Technology, and his PhD from the University of Texas at Austin. He spent two years as a postdoctoral researcher at the Institute for Advanced Study. His research focuses on theoretical computer science, particularly on aspects related to computational complexity theory.

The title of his lecture was *Computation and Conversation*.

Abstract: Every computational process seems to involve two or more entities that are engaged in a conversation with each other. Communication complexity is the study of the complexity of such conversations. In this talk, I will give a broad overview of this connection between computation and conversations, including some nice consequences for computational models like Turing machines and data structures. I will describe some of the big open questions in the area, which are clean mathematical questions in their own right. No background will be assumed.

The video recording of the talk is available at [Zona video](#).



[▷ Editorial](#)

## Events

### 2a Jornada IMTech 13/II/2025

by Gemma Huguet [↗](#) (DMAT [↗](#), IMTech [↗](#))

On November 13th, 2025, took place the [2a Jornada IMTech](#) at the Faculty of Mathematics and Statistics (FME [↗](#)). The second IMTech meeting gathered around 60 participants, including master and PhD students as well as several members of the mathematical community at UPC.

The morning featured the Keynote Talk by [Alain Goriely](#) [↗](#) (University of Oxford [↗](#)), who is member of the [scientific advisory board](#) [↗](#) of IMTech. His talk was on [Modeling dementia](#).

A dynamic Poster Blitz session allowed presenters to introduce their work before the Poster Session, which showcased 12 research posters from master and PhD students at UPC covering a wide range of topics. Lively discussions continued over coffee, providing attendees with the opportunity to engage with presenters and deepen their understanding of the work on display.

The event continued with two invited talks by members of IMTech. [Jezabel Curbelo](#) [↗](#) (DMAT [↗](#)) talked about [Lagrangian Techniques for Analyzing Atmospheric and Oceanic Dynamics](#), offering a compelling look at powerful techniques to analyze transport and mixing in the atmosphere and the ocean. Following this, [Rodrigo Silveira](#) [↗](#) (DMAT [↗](#)) gave the talk [Continuous Graphs: Connecting the dots, continuously](#), where he discussed how well-known graph problems transform in the continuous setting.

The meeting concluded with a Prize Awards ceremony for the best posters, which recognized the contributions of Luis Álamo Rodríguez and Eduard Jove, PhD students at UPC, followed by a networking lunch at the FME garden, allowing participants to exchange ideas in a more informal setting.

The abstracts of the talks are available at [Talks](#) [↗](#) and the list of posters and presenters is available at [Posters](#) [↗](#). [▶ Editorial](#)



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The image in the cover corresponds to roots of  $10x^3 - y^{23} - a = 0$ . [Imaginary. Una mirada matemàtica](#) [Gallery of Singular Algebraic Surfaces](#), is one of those included in the [Imaginary](#) exhibits.