Universitat Politècnica de Catalunya ▪ BarcelonaTech

IMTech Newsletter 6

Issue 6, September-December 2023

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IMTech Newsletter

Issue 6, Sep-Dec, 2023

нет войне – No War!

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Editorial

As usual, in this first semester of the current academic term, 2023-24, we have enjoyed activities celebrating its beginning in various ways.

One was the **IMTech** Fall Colloquium on 29 November. The invited speaker was professor Maria Bruna (Univ. of Cambridge), who lectured on *Continuum models of strongly interacting Brownian particles*.We include a chronicle of that event by Gemma Huguet. Her piece annexes the introduction of Maria Bruna authored by professor Joan de Solà-Morales i Rubió.

Another was the opening of the **FME** academic term on 11 October and Jaume Franch Bullich reports on it. The keynote speaker, Joan de Solà-Morales i Rubió, just promoted to Emeritus Professor, was the first dean of the **FME** (1992-1997). His lecture, on *Mathematical Principles of Fluid Mechanics*, is included in this issue in the Outreach section. It is worth mentioning that Franch's chronicle appends the introduction of the speaker, a precious portrait of his academic track, by professor Xavier Cabré.

Following the **FME** custom, initiated in 2003-04, of dedicating each academic term to an outstanding historical figure, the choice for the current term was George E. P. Box (1919-2013). Among the activities around this decision, we include a chronicle by Marta Pézez Casany of the opening lecture of the Box year. It was delivered on 14 September by professor G. Geoffrey Vining with the title *Why do we aspire to be second-rate mathematicians when we can be first-rate scientists?* Along this line, there was a second invited lecture: On 22 November, professor Daniel Peña, who was a friend and collaborator of Box, shared his reflections on the life and contributions of that eminent statistician. In this case, the speaker's slides were morphed by Víctor Peña and Josep Ginebra, focusing on the scientific contributions of Box, into an Outreach piece with the title *Remembering G. E. P. Box: Life, Contributions, and Some Personal Experiences*. Hopefully it will help in promoting a wider recognition of Box in our community.

In the Outreach section we have included, besides the two pieces already mentioned, a new edition of Kip Thorne's lecture delivered on 25 May 2017 in his accepting the *doctor honoris causa* nomination by the **UPC**. In our view the lecture is a memorable document in itself, but we have added notes by Santiago Torres, from GAA/UPC, on the evolution of gravitational astronomy and cosmology since the discovery of gravitational waves in 2015. With this edition of the lecture we also wish to honor the memory of our colleague and friend Enrique García-Berro (1959-2017). He was the sponsor of Kip Thorne's nomination and unfortunately he died after a hiking accident in the Pyrenees just ten days before the Royal Swedish Academy announced that the 2017 prize in Physics was awarded to Rainer Weiss, Barry C. Barish and Kip S. Thorne *for decisive contributions to the LIGO detector and the observation of gravitational waves*.

In the Reseach focus section you can find three articles. Angélica Torres reports on her research with the title *Multiview varieties: a bridge between Algebraic Geometry and Computer vision*. Then Robert Cardona, Eva Miranda and Daniel Peralta-Salas write on *Undecidable trajectories in Euclidean ideal fluids*, after their paper *Computability and Beltrami fields in Euclidean space* published this year in Journal de Mathématiques Pures et Appliquées. Finally, Andrew Clarke and Marcel Guàrdia contribute with the note *Why are inner planets not inclined?* in which they describe their recent breakthrough on initial conditions that imply instabilities in solar systems.

This issue features six interviews. The people interviewed represent the current heartbeat of our research community. The first, with Eva Miranda Galcerán, was held in September, just after completing her Hardy tour (this extraordinary event was echoed in previous issues of this NL, particularly in NL05). Incidentally, we congratulate her for having been invited by the ETHZ to give the Fall 2025 FIM Nachdiplom lectures on *Singular Symplectic Manifolds*, a research topic started in 2009 in collaboration with Victor Guillemin. The second interview was with Guadalupe Gómez Melis on the occasion of being awarded The Marvin Zelen Memorial Lecture of EMR-IBSW (2023), a distinction recognizing and honoring her influence in the field of *Biostatistics*.

Xavier Cabré received a "Frontiers of Science Award" 2023, inaugural class (see ICBS) for the *Acta* paper *Stable solutions to semilinear elliptic equations are smooth up to dimension 9* (see his Research focus in NL01). Joan de Solà-Morales was interviewed on the occasion of the distinctions mentioned above: his promotion to **UPC** Emeritus professor and his key-note lecture at the **FME**. Daniel Peralta-Salas, already mentioned in his Research focus with Robert Cardona and Eva Miranda in this issue, was appointed as an EMS distinguished speaker at the 29th Nordic Congress of Mathematicians (July 2023). And for Marcel Guàrdia, mentioned above for his Research focus (in collaboration with Andrew Clarke), we also stress his appointment as scientific director of the María de Maeztu distinction awarded to the **CRM**.

As PhD highlights we include thesis summaries of Iñigo Urtiaga, Armando Gutiérrez and Jaime Paradela.

Finally, Marc Noy reviews A. Granville's paper *Accepted proofs: Objective truth, or culturally robust?* and S. Xambó, the books *Machine Learning in Pure Mathematics*, edited by Yang-Hui He, and the masterful treatise *Modern Classical Physics*, by Kip Thorne and Roger D. Blandford.

Interviews

Eva Miranda Galcerán is a Full Professor at **UPC**, a member of **IMTech**, and a member of **CRM**. She is the director of the Lab of Geometry and Dynamical Systems and group leader of the Geometry group at UPC. Distinguished with two **ICREA** Academia Prizes in 2016 and 2021, she was awarded a Chaire d'Excellence de la Fondation Sciences Mathématiques de Paris in 2017 and a Bessel Prize in 2022. She has also been the recipient of the François Deruyts Prize, a quadrennial prize conferred by the Royal Academy of Belgium, in 2022. In 2023 she was Hardy Lecturer by invitation of the London Mathematical Society.

Miranda's research is at the crossroads of Differential Geometry, Mathematical Physics and Dynamical Systems. In the last years, she added to her research agenda mathematical aspects of theoretical computer science in connection to Fluid Dynamics.

A decade ago she pioneered the investigation of b-Poisson manifolds. These structures appear naturally in physical systems on manifolds with boundary and on problems on Celestial Mechanics such as the 3-body problem.

In 2021 she constructed (jointly with Robert Cardona, Daniel Peralta Salas, and Francisco Presas) a Turing complete 3D Euler flow. This result not only proves the existence of undecidable paths in hydrodynamics, but also closes an open question in the field of computer science (the existence of "fluid computers").

Miranda's research strives to decipher the several levels of complexity in Geometry and Dynamics. She endeavors to extend Floer homology and the singular Weinstein conjecture to the singular set-up motivated by the search of periodic orbits in Celestial Mechanics.

NL. ***This NL has tried to promptly echo your research achievements and a number of concomitant recognitions since its inception in January 2021. They are summarized in the various Editorial pieces, with pointers to the details in the inner pages. For instance, the last issue (NL5) included a report on the first two parts of your Hardy Tour lectures, which by now they have been completed with the last two (19 and 21 September), but you surely have been much active in other endeavors as well. To begin with, however, we would like to go back a few years. Did it all start somehow with your ICREA Academia in 2016?***

The **ICREA** Academia has been a unique opportunity for me to focus on research at a dream level. **ICREA** has enabled me to take risks in my research which in turn yielded results that have had an important impact inside and outside mathematics. So I must say **ICREA** opened the door to what came after and I am forever grateful to have had such a great chance back in 2016. The honor coincided in time with a Chaire d'Excellence of the Fondation des Sciences Mathématiques de Paris. I was also honored with an **ICREA** Academia 2021 which has enabled me to pursue this intensification of research. In practice, this means that my teaching has been reduced to one master course and the rest of the time should be devoted to research and administration (but mostly research). I also have gathered a big group around me. Since 2016, 6 PhD students have defended their doctoral thesis under my supervision and currently I am advising 5 more. **ICREA** Academia gives me the freedom to sail my research in the direction that I want with long-term very ambitious projects.

You also got the François Deruyts prize in Geometry conferred by the Royal Academy of Belgium, which is a great achievement. Can you tell us more about it and what represents in your career?

In 2022 I got the François Deruyts prize in Geometry which is conferred by the Royal Academy of Belgium every four years. This is a unique distinction, as I am the only Spanish person in the list of awardees, and in fact the only non-Belgium awardee. The François Deruyts Prize, also known as the Prix Francois Deruyts, is presented once every four years to acknowledge advancements in the fields of synthetic or analytic superior geometry. This esteemed award was founded in 1902 by the Académie Royale de Belgique, specifically by its Classe des Sciences, and includes a monetary award. The list includes names such as Jacques Tits and Pierre Deligne or Michael Cahen and Simone Gutt. I feel very honored to be on such a list. The award ceremony took place in the Palais de l'Académie in Brussels in December 2022. This was a high moment on my career with very emotive words by the Sécretaire Perpetuelle de l'Académie Royale, Didier Viviers.

The Humboldt Foundation conferred you a Bessel prize. Can you tell us more about this prize?

Yes, I also got a Bessel Prize conferred by the Humboldt Foundation. This second recognition came with homework. I have to stay at least 6 months in Germany to foster collaboration with several institutions. In my case, it was the Universität zu Köln who nominated me for this award and this is the main institution of my stay. Other stops are Universität Augsburg, Universität Erlangen-Nürnberg, Ruhr-Universität Bochum, Universität Heidelberg, and Universität Göttingen. As I decided to use the Humboldt Prize to create a new network of collaborations, the research visits will be extended until 2025. As an offspring of the visits this year, we are applying for a Collaborative Network that involves the Universität Augsburg, the **UPC**, and **ICMAT** in Spain. My collaboration with Cologne has three different directions: that of contact geometry with Professor Hansjörg Geiges, that of Quantization with Professor George Marinescu, and that of Toric manifolds with professor Silvia Sabatini. The collaboration with the three subgroups has been fostered through several seminars and workshops.

Let us now focus on the Hardy lectures 2023. Could you assess what they represent for your career? Could you also comment on why your tour includes more lectures than in any preceding edition? How have you managed such a dense program?

To be named the 2023 Hardy lecturer is a momentous recognition of my career. This extraordinary award stands as an unparalleled milestone in my professional journey. Former Hardy lecturers include acclaimed mathematicians such as Dusa McDuff, Terence Tao, Yu Manin, Étienne Ghys, Jacob Lurie, Nalini Joshi and Peter Sarnak, just to mention a few. Again this nomination came with the commitment to deliver several lectures around the UK, In principle the prevision were six lectures, plus the one at the general meeting of the LMS, but at the end they were nine. The reason for this increase is that once the tour was announced, two more institutions asked me for additional tals: The Royal Institution of London and the University of Warwick.

We would also like to get your views on the various institutions in which the lectures have been delivered. In particular, we would like to know where and when the picture on the "Penrose way" was taken?

The tour has given me a global vision of several institutions throughout the UK. Most importantly, I have connected with many interesting individuals of diverse origin: mathematicians, physicists and computer scientists and now I am ready to start new research adventures with some of them. One of them is Roger Penrose with whom we are currently revisiting some of the basics of Twistor theory and some mysterious symmetry that breaks into the theory and was not accounted for before. With Professor Raymond Pierrehumbert we would like to understand new connections between b-symplectic geometry and the detection of exoplanets. It has been the perfect adventure in all possible ways. The picture "The Penrose way" was taken at the University of Loughborough campus while entertaining new interesting connections and adventures in mind: I want to go "the Penrose way".

In your tour you have met many people that have been involved in hosting your lectures and in organizing additional activities. Would you mind sharing your views on these aspects of the tour?

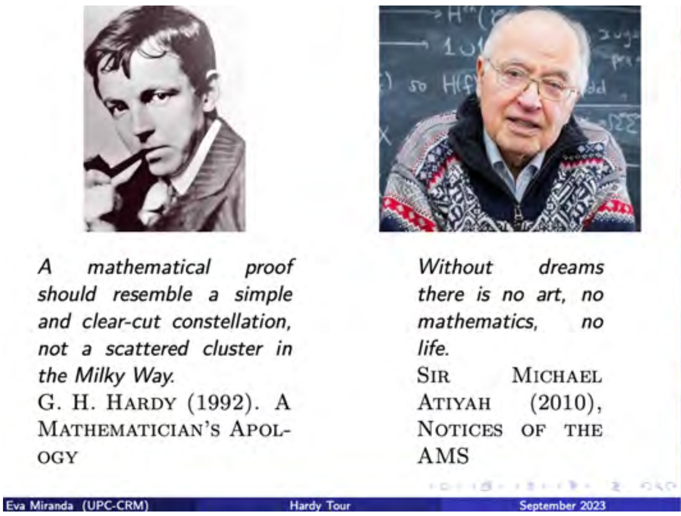
I have been absolutely delighted with my hosts. In each institution there was, besides several contacts, an official host. The hosts took care of organizing the activities and official reception at each university. At each stop there was also an official dinner. This gave me the opportunity to socialize with many different people.

Since for a researcher there is no resting on one's laurels, we would much appreciate if you could describe in some detail your goals for the next few years and some activities you are envisioning to achieve them.

The Fluid computer stroke my mind as a revelation but was insufficient for the purpose of finding blow-up solutions of PDE's. So in order to achieve that we need to make the theory more complete. In a way our Fluid computer was not enough: One of the aspects that has completely taken my attention the last months is the design of a new model of theoretical computer. I am currently working on a hybrid machine between Fluid and Quantum computer. I do this following Topological Quantum field theory in collaboration with Angel Gonzalez-Prieto and Daniel Peralta Salas. Another aspect is the applications of my theory to detection of escape orbits connected to several long-standing conjectures in Symplectic Topology. One of the works that I am recently pursuing goes in the direction of disproving one of these conjectures. More soon! The last couple of months I had a couple of interesting surprises. On the one hand, I have been nominated by the Universität zu Köln as the Mercator professor. This professorship has the role of ambassador of the University of Cologne in the world. This recognition is a source of immense joy for me, as it reflects my strong and interconnected international relationships. I have also been invited to teach a Nachdiplom course at ETHZ in Zurich in the Fall of 2025. More details soon!

As an IMTech member, how do you see its future? In your view, what synergies should be promoted between the various stakeholders, internal and external, in order to optimally fulfill its vision?

The **IMTech** has been consolidated in a very interesting moment for mathematics in Barcelona. It arrives in the right moment! Unlike other institutions in Catalonia, **IMTech** has a differential trait: that of gathering more interdisciplinary projects common with researchers closer to Engineering and Computer Science. Diversity is our distinct flag that makes us so special. I am thrilled to be part of it. There is a lot of work to do and it would be a good idea to organize more activities to encourage cross-fertilization among different disciplines reflected in its composition. The **IMTech** needs to position itself as an strategic "lighthouse" in this stunning city by the sea, Barcelona, that can attract international talent from all the cardinal points. Our unique character makes us so special that we are almost "irresistible" as a trademark. In my opinion we need to intensify our internationalization aspects and make our dream bigger. However, substantial dreams require tangible backing from financial institutions. **IMTech** requires further support to make our dream come true.



Guadalupe Gómez Melis is a Professor at the Universitat Politècnica de Catalunya·BarcelonaTECH (**UPC**). She leads the Research group on Biostatistics and Bioinformatics GRBIO UPC-UB. She has been visiting scientist at Harvard University (Boston, USA), Oxford University Clinical Research Unit (Ho Chi Minh City, Vietnam) and the MD Anderson Cancer Center (Houston, USA). She received the Bachelor and PhD degrees in Mathematics from the Universitat de Barcelona (**UB**) and MSc and PhD degrees in Statistics from Columbia University (NY, USA). She is president of the Consell Català d'Estadística, representative of the Catalan universities at the Consell de Salut de Catalunya, member of the UPC Ethics Committee, elected member of the Council of the International Biometric Society. Former vice-dean in the School of Mathematics and Statistics (**FME**), founder and coordinator of the Master in Statistics and Operations Research UPC-UB, coordinator of the Interuniversity Doctorate in Bioinformatics and coordinator of the Doctorate in Statistics and Operations Research, elected European Representative of the Caucus for Women in Statistics, and a recipient of The Marvin Zelen Memorial Lecture of EMR-IBS, an award recognizing and honoring her influence in the field of Biostatistics. Her main research interest is in developing methods for Survival Analysis and Clinical Trials, with an unequivocal interdisciplinary flavor, focusing especially on cancer, HIV-AIDS and lately on COVID-19. She is the PI of several funded projects of the Ministerio de Ciencia e Innovación and the Generalitat de Catalunya.

NL. ***You did your Bachelor in Mathematics at the UB. How did you decide to do a PhD and, in particular, to go to the USA to do it?***

After doing the Tesina de Licenciatura (Bachelor Thesis), directed by Prof. David Nualart, on Stochastic Processes, I realized that I loved research but, if possible, connected to real world problems. This is why I started the PhD at the UB in Statistics. I did all the courses of the two first years but when looking for thesis subjects I was not engaged by any of the research topics that I was proposed. Then, everything started as a game; I was only 23 years old and I was fearless, adventurous and a hard worker, and we (with my partner since then, Àlvar Vinacua) started to fill endless documents, send them by regular mail, waiting for the answers, and in the meantime took the required exams for the US Graduate Programs, such as the Graduate Record Examination. The acceptance at the Statistics Department at Columbia University in New York and the scholarship from the Generalitat de Catalunya did the rest and on August 1982 we landed in JFK.

When did you know that you wanted to work in Statistics?

I liked very much the theoretical aspects of Mathematics but I was attracted to real life problems and Statistics was the perfect combination.

I'm sure that during your years at the USA, in addition to the theory and practice of Statistics, you learned a lot about doing and managing research, or university organization. Could you mention some of these non-strictly-academic skills you are most grateful for?

I am very grateful to those years in New York, later in Ohio State University as Assistant professor, because I learned a lot about what a research department meant, how important was to keep a good atmosphere among professors and students, the generosity and humbleness of professors. I was lucky enough to learn the importance of collaboration and the real meaning of interdisciplinarity, how close you should be to the real problem and how much you can learn from other colleagues.

Your main research topic is survival analysis. Can you tell us how you came to this topic? What do you consider your most relevant contributions in survival?

While taking PhD courses at Columbia, I fell in love with the survival analysis course given by Professor John Van Ryzin, who became my thesis advisor. This topic gathered my two ambitions: challenging statistical problems and relevant real-life scientific problems. Along the years there are two research lines where I have made relevant contributions. I have worked a lot on interval censoring, and in this field our group is now a referent. The beauty of this research line is that we have fundamental papers on the topic of noninformativeness, along many papers developing new methods for different problems and finally highly cited contributions in the Shelf Life area (within Food Technology) where our methods have become the standard for their analysis. A second topic refers to Composite endpoints in Clinical Trials. With this topic, I approached clinical trials in a different way, and took me through different avenues, new collaborators and last, but not least, web app tools to make our methods usable by others. Both topics have been followed by many of my PhD students and thanks to them they are alive. I am very grateful to professor Steve Lagakos from Harvard University with whom I started to collaborate in 1989 and together we started the two previous topics.

On the side of teaching and teaching organization, how would you describe your main contributions and achievements?

In this aspect the Interuniversity Master in Statistics and Operations Research (MESIO UPC-UB) has been my main achievement. Back in 2006 when new regulations geared the Spanish universities to develop official Master degrees, the **FME** trusted me to lead a team to develop the new curriculum. Even with all the limitations we had, we put together an ambitious program where interdisciplinarity had to be present, students from different backgrounds were going to sit together and applications were going to have as much respect as theoretical subjects. During 10 years I coordinated this master which after 2 years became a joint program with the **UB** Universitat de Barcelona, which turned out to be a relevant and smart decision.

Over the years, you have been able to create a very active, dynamic and powerful research group. Can you summarize the evolution of this team, its greatest achievements and its projection in the medium term?

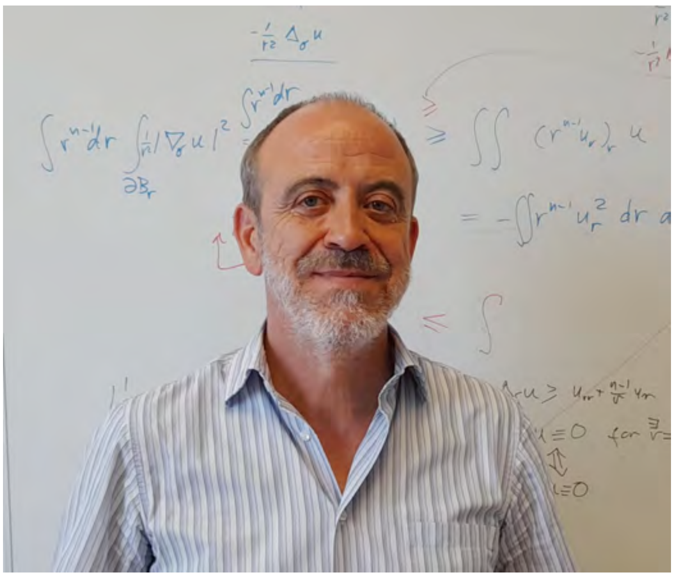
Thanks to my years in US when I returned to Catalunya I was not attached to any particular group or university and I interacted with colleagues from all the institutions. This helped a lot and was first materialized into the GRASS (Grup de Recerca en Anàlisi eStadistic de la Supervivència) that started in 1995 and became the research seed for many of today's senior researchers. Later, with the Generalitat de Catalunya call to give support to the scientific activities of the Catalan research groups, the GRASS was split, mainly by institutions, and we put together the GRBIO UPC-UB (Grup de Recerca en Biostadística i Bioinformàtica). Since 2014 we have been growing and thanks to the Generalitat funding (2014 SGR 464, 2017 SGR 622 and 2021 SGR 01421) we have grown not only in number (23 investigators and 6 PhD students) but more importantly in quality and achievements. Our biweekly seminars, our software contributions, our outreach involvement together with a fair play and mutual support has facilitated our research as well as its impact and the large number of publications. The members of GRBIO are as well members of the Catalunya-BIO node of the BIOSTATNET (Spanish network of Biostatistics). Together with seven professors from different Spanish universities we founded Biostatnet in 2010 and since then it has grown and become, together with the SEB (Sociedad Española de Bioestadística), the place of confluence for researchers in biostatistics in Spain and the place of training and the starting point of the scientific career of most of our young people. I have been very lucky to have been surrounded by a very friendly, intelligent and active biostatistics community, that has evolved into a very cohesive group; hence, I foresee a long and productive life ahead for the GRBIO and BIOSTATNET. Since July 2023 GRBIO joined the Research Centre for Biomedical Engineering (CREB UPC) and is one of the research group partners of the network Xartec Salut led by CREB UPC.

You have participated in several mentoring initiatives. Can you tell me about your experience?

I love mentoring and I think it should be a relevant part of our duties as academics. The advantages of mentoring both for the mentees and for the mentors themselves are countless. Mentoring contributes to the personal, academic, and professional growth of the mentees. It helps them build a solid foundation for a successful and fulfilling career in academia and beyond. As mentors we can share our expertise, experiences, and knowledge helping younger scholars to gain insights that might not be readily available through formal education. Witnessing the growth and success of mentees can be deeply rewarding. Mentoring young scholars is an investment in the future of academia and society at large. Among the several mentoring initiatives in which I have participated, the one we launched between the Sociedad Española de Bioestadística and the network BIOSTATNET is being very successful and has been the seed for the International Biometric Mentoring program within the International Biometric Society in which I have also collaborated to launch it.

Recently you have been involved in a research project call DIVINE, at which you have analyzed COVID-19 in Catalonia from a biostatistical perspective. Could you summarize your main findings?

The project Dynamic evaluation of COVID-19 clinical states and their prognostic factors to improve intra-hospital patient management (DIVINE) was funded under the call Pandèmies 2020 of the Generalitat de Catalunya. This project had four main goals and all of them were achieved after 18 months. Data of 5,813 hospitalized adult patients with confirmed COVID-19 in 5 hospitals (Barcelona South Metropolitan area) and corresponding to four waves of the pandemic between March 2020 and August 2021 was collected including dates to enter into different stages (severe and non-severe pneumonia; invasive and noninvasive mechanical ventilation; recovery; discharge and death) together with demographic data, comorbidities, vital signs, laboratory results, and previous medications. Using multi-state models (MSM), the generalized odds-rate class of regression models and clustering techniques, i) we identified the most clinically relevant prognostic factors and in particular that low levels of the ratio of oxygen saturation to the fraction of inspired oxygen, and high concentrations of the C-reactive protein, were risk factors for health deterioration among hospitalized COVID-19 patient; also that patients with at least one vaccination dose were slightly better-off, but mainly to prevent severity at earlier stages; ii) we developed the App MSMpred allowing the interpretation of multi-state models and prediction of the disease course of future patients; iii) we estimated the incubation time period of the COVID-19 with data from the fifth wave pandemic learning that the estimated median Sars-Cov-2 incubation period was 2.8 days (95%CI: [2.5, 3.1] days) and no statistically significant differences were found when comparing vaccinated versus unvaccinated patients; and iv) patients' profiles can be clustered between waves 1-3 and wave 5 and vaccination was crucial to distinguish among three clusters found in wave 5. The fruitful collaboration between statisticians and clinicians has been key in developing a model for the disease course of hospitalized COVID-19 patients at a higher risk of developing severe outcomes. Besides the acquired knowledge about the disease, the existing and the new developed methodology applied in this project sets the foundations for further analysis and management of hypothetical future pandemics.



Xavier Cabré Vilagut is an ICREA Research Professor at Universitat Politècnica de Catalunya (**UPC**), affiliated also to the **CRM**, in the area of Experimental Sciences & Mathematics.

Born in 1966 in Barcelona, he holds a PhD in Mathematics from the Courant Institute, New York University, 1994. The title of his thesis, developed under the supervision of Louis Nirenberg, was *Estimates for Solutions of Elliptic and Parabolic Equations* and it was awarded the Kurt-Friedrichs Prize in 1995. He was a member of the Institute for Advanced Study (**IAS**), Princeton, 1994-95. In 1998 he was conferred the "Habilitation à diriger des recherches" by the Université Pierre et Marie Curie-Paris VI. In the period 2001-2002 he was a Harrington Faculty Fellow of The University of Texas at Austin, and in 2002-2003 he held a Tenure Associate Professor position at the same university. Since 2003 he is an ICREA Research Professor at the **UPC**, and since 2013 he is a Fellow of the American Mathematical Society (**AMS**), inaugural class. In 2021 he was plenary speaker at the 8th European Congress of Mathematics (see also NL01, p. 3). He has received a "Frontiers of Science Award" 2023, inaugural class, ICBS.

His research field is the mathematical analysis of Partial Differential Equations. These equations arise in mathematical physics, differential geometry, finance, and biology. His focus is on elliptic and parabolic equations, and on the analytical understanding of the regularity, symmetry, and other qualitative properties of their solutions. This often involves the use of geometric tools such as isoperimetric inequalities, whose study is an important part of his research. A main current project of him concerns a recently flourishing area: reaction problems for fractional diffusions associated to jump or Lévy processes. These are the so called 'anomalous diffusions', well noticed in the last decades in some reaction and biological fronts, as well as in mathematical finance.

NL. ***In the last months you have participated in various scientific activities abroad. Would you mind telling us the where and why of them?***

I attended the Abel Prize 2023 ceremony in Oslo (since the awardee, Luis Caffarelli, is one of my mentors, besides a collaborator and friend), a meeting in Pisa in June for the 60th anniversary of Luigi Ambrosio (a collaborator of mine; I gave a talk), the International Congress on Basic Science in Beijing in July (to receive a prize), and another meeting for the 60th aniversary of Luigi Ambrosio this time at the ETH-Zurich in September (also to collaborate with Joaquim Serra and Alessio Figalli).

What impressions did you gather from the Abel Prize 2023, awarded to Luis Á. Caffarelli, ceremony?

It was a beautiful ceremony, very emotional. Formal (the king of Norvay gave the prize to Luis Caffarelli) but not too pompous; just up to right amount. This is a very important prize and, of course, one could feel the emotion of the awardee, L. Caffarelli.

And about the lectures that followed?

The three lectures were given by top experts on the research topics of L. Caffarelli (Sylvia Serfaty, Alessio Figalli, and Luis Silvestre), who gave precise and beautiful presentations.

In your opinion what are his main contributions to mathematics?

His ideas have modelled and conducted the developments on the last fifty years on the topic of Free Boundary Problems. This is a large area in the theory of PDEs with many applications within Math and to Technology and Industry. In addition, his contributions to Fully Nonlinear Elliptic Equations and to Optimal Transport Problems are also central. These are two theories who saw main developments in the eighties and from the nineties, respectively. Besides the concrete topics, Luis Caffarelli has had an enormous influence in the field of PDEs by having many PhD students and collaborators, as well as by his very personal, rather geometric, understanding of Analysis. This may be related, partly, to the influence that the Catalan mathematician Lluís Santaló had on him. Indeed, Caffarelli told me once that he had him as professor in his Buenos Aires Math undergraduate and that Santaló was the professor who most impressed him. One may read, in Caffarelli's own words about Santaló and other mentors of him: "As a student I was heavily influenced and inspired by Luis Santaló [1911–2001], Manuel Balanzat [1912–1994] and Carlos Segovia [1937–2007]. Santaló and Balanzat were both Spanish mathematicians who moved to Argentina as a consequence of the Spanish Civil War. Santaló made important contributions to integral geometry and geometric probability, while Balanzat worked in functional analysis. They built, jointly with Rey Pastor [1888–1962] and Pi Calleja [1907–1986], a superb undergraduate and graduate mathematics program at the University of Buenos Aires, generating a very strong group in analysis, geometry and algebraic geometry. The harmonic analyst Segovia was a prominent graduate from Universidad de Buenos Aires who did his PhD at the University of Chicago in 1967 with Alberto Calderón [1920–1988]. While closer to me in age than Santaló and Balanzat, Segovia was always a strong support."

We are also eager to hear a first hand account about your attending the July 2023.07.16–28 International Congress on Basic Science, held in China, and in particular about you receiving one of the Frontiers of Science Awards.

The ICBS congress has been conceived by the North American and Chinese mathematician Shing-Tung Yau. I attended it to receive a Best Paper Award (Frontiers of Science Award 2023, inaugural edition) for my joint paper *Stable solutions to semilinear elliptic equations are smooth up to dimension 9* with Alessio Figalli, Xavier Ros-Oton and Joaquim Serra (Acta Math. 224 (2020), 187–252). In the prize ceremony (of twelve hours at the magnificent "Great Hall of People" in Beijing), awards were delivered to papers in 22 sections of Mathematics, plus a smaller number of awards in Physics and Computer Science. The selection of papers was made by an international committee. Before giving each prize, a very informative abstract of the corresponding paper, written by the committee, was displayed and read. This made the ceremony very interesting from the research viewpoint: a great account of major developments in Math in the last years. From what I heard (I believe this is not yet confirmed), the prizes will be given yearly: one awarded paper (published the previous year) for each of the 22 Math sections designed for the prize.

Recently you have had a research visit to the ETH in Zurich. How did it go?

It went great. I described a project that I started conceiving in the last months to Alessio Figalli and Joaquim Serra, both at ETH. Fortunately it was interesting enough to bring their attention and thus start, perhaps, a new joint work.

What lessons would we do well in learning from that eminent institution?

An institution benefits a lot from having its top researchers have an important role on its hiring and teaching strategies.

Going back a few years, how do you value the ICREA program in terms of how it has helped you to develop your potential?

It is being essential to be an ICREA researcher to keep up my motivation and ambitions, as well as for having more time for research.

As an IMTech member, how do you see its future? In your view, what synergies should be promoted between the various stakeholders, internal and external, in order to optimally fulfill its vision?

I believe its future should be linked to a common Institute or Center for the three or four main universities in Barcelona.

Joan de Solà-Morales i Rubió (**DMAT**, **IMTech** and **BGSM**) has been Full Professor of Applied Mathematics at the **UPC** from 1989 till 2023, having now been promoted to Emeritus Professor.

He earned his PhD degree in 1983 from the **UAB** with the thesis *Les equacions de Navier-Stokes en un canal amb obstacle* (The Navier-Stokes equations in a channel with an obstacle) supervised by Carles Perelló (1932-2021). Since then, his research areas have been Partial Differencial Equations, Infinite-dimensional Dynamical Systems, and Modeling in Industrial Mathematics. He has been visitor at Georgia Tech, Universidade de São Paulo, Leiden Lorentz Center, Universidad Complutense de Madrid, and OCIAM at Oxford.

Solà-Morales has been Dean of the Facultat de Matemàtiques I Estadística (1992-97, founding period), Director of the Department of Applied Mathematics 1 of the **UPC** (2002-05), Deputy director of the **CRM** (2007-10), President of the SCM (2010-14), and since 2012 he is Member of the **IEC**.

NL. ***Your father and two eldest brothers of yours have been prestigious architects. Thus it may be surprising to some that you chose to pursue mathematics. How did this occur? Why were mathematics more appealing to you than other careers?***

I think I don't really remember exactly why. I remember very well that I was attracted to mathematics and also that I did not want to be influenced very much by my family. I wanted to be as independent ans possible. Maybe my father didn't like my choice very much at the beginning because if I had chosen architecture like himself or my brothers, they could have substantially helped me. But I remember that he spoke then to a friend of his (Jordi Dou, mathematician and architect) who told him that being a mathematician had given to him much more satisfaction than being an architect. That helped my father to understand my decision.

What memories do you have of your undergraduate studies?

I had very good teachers and very good colleagues, and that helped me very much in keeping my mathematical interest always alive. Those were times of quite strong political events, and I was often engaged in that. I also had some other interests, outside the university. I have to recognize that all of this perhaps didn't help me very much in mathematics, but as a whole, it was a fruitful and happy time.

Now we would like to know how was your initiation to research and, in particular, what people were influential in your decisions and what topics did you find attractive to work on.

My initiation in research was in a modest work (at that time called "Tesina"), on geodesics of revolution surfaces, under the direction of Carles Perelló. This is an integrable Hamiltonian system, and we also considered perturbations of it in the spirit of the KAM theory. Professor Carles Simó had also some influence on this work because the the type of results I was looking for were very well within the scope of Simo's interests of that time. Something I can notice is that both Carles Perelló and Carles Simó were graduated as mathematicians and engineers, and they shared an applied viewpoint that was always present in their scientific aims. I think I inherited somehow this viewpoint from them.

Your PhD thesis was on a problem in fluid mechanics, and your advisor was Carles Perelló. What recollections would you like to share about those years?

Those were quite optimistic times. The Dynamical Systems approach to the Navier-Stokes equations seemed to be the key point in explaining two important phenomena: transfer of stability between different regimes and also transition to turbulence. My PhD thesis espoused enthusiastically this approach. Perhaps the results were not as important as expected, but along the way, I learned many things, in a direction that at that time became popular and fruitful, perhaps when applied to other equations. And I also learned Fluid Mechanics, at least from the mathematical point of view, something very beautiful and still challenging.

In relation to your research track, could you describe the main topics that have been the focus of your endeavors?

I think that the Dynamical Systems approach for Partial Differential Equations has always been present in my research. Also, something that is very important to me, to work in problems whose origin I can understand, often because they come from Physics or Engineering. In this sense, I've tried to work keeping a watch on modeling. Understanding models has been always relevant for me. There is an exception, that is a subject perhaps mainly of a theoretical interest: the work I've done on linearization and stability for infinite-dimensional systems.

With what researchers have you collaborated more closely?

Apart from Carles Perelló, there was a person that helped me a lot, and whom I'm very much indebted to: Jack Kenneth Hale, from Georgia Tech, in Atlanta. He was one of the pioneers in the subject of infinite-dimensional Dynamical Systems defined by Partial Differential Equations. He had been the PhD advisor of Carles Perelló, and perhaps because of this fact I connected very well with his interests and approach. Thinking of direct collaborators I want to mention Xavier Mora and Àngel Calsina, from the UAB, and also my PhD students Marta Velència, Neus Cònsul, José Antonio Lubary and Marta Pellicer. And Xavier Cabré, whom I met later on and has been very influential on me ever since. I have had also other collaborators, all of them very important to me, like Maria Aguareles, Jaume Haro, Mar González and Joaquin Menacho. And other people, several other people. Some of these people are joint authors of some of my papers, but others, sometimes even more influential, have been people from whom I learned many things, just through mathematical conversations.

A mention apart is deserved by my close collaborator Hildebrando Munhoz Rodrigues, from the Universidade de São Paulo in São Carlos (SP, Brazil). My relation with him started trough Jack K. Hale. We have worked together in infinite-dimensional linearization and stability and have written several papers, and we have visited each other many times. It is the moment to say the sad news that he passed away recently (November 2023). At the moment of answering this interview I'm still shocked by these news. With him, I have lost a good friend and a powerful mathematical collaborator.

On the academic service side, you were the first dean of the FME. What reminiscences do you have about how the center was con-

ceived and launched? What features distinguished the new degree in mathematics? And the ensuing academic environment?

The **FME** and the degree started under the initiative and influence of Jaume Pagès, first as a vice-rector and later as the rector of the university. Jaume Pagès is a good example of the interest of engineers on mathematics, and I think that for him the birth of the **FME** was the accomplishment of something important. I had the essential help of two colleagues, Marta València and Josep Grané, and later the help of Jaume Barceló, who was already head of studies for the statistics diploma. But it was also very important that other **UPC** professors joined the project. Many of them collaborated by teaching, and others collaborated without doing so. The dedication of all of them created a good atmosphere at the **FME**, also favored by its small size.

This year the FME celebrated its thirtieth birthday and you have been promoted to Emeritus Professor. How do you assess its evolution and accomplishments during these three decades?

As a whole, I think that the FME has been a very successful experience. But we shall not forget that the success comes from the students. Having the confidence of good students is the essential key point. In this we have had the help of the CFIS, a center of selected students that have always taken mathematics into account. At first, I was particularly interested in the more applied features of the studies. I believed that they would open many doors for us, and that they would allow us to create a clear-cut identity. But it is true that along these years mathematics has evolved towards applications that at that time we would not have thought important. My conclusion is that we should continue to bet on applicable mathematics, but I admit that many people have seen that almost all parts of mathematics are applicable, and with success.

You delivered the opening lecture of the academic year 2023-2024. Its contents is echoed in the Outreach piece Mathematical principles of fluid mechanics that you have authored for this issue. It reflects your concern for strengthening the interconnections between subjects, provides some hints on your didactical stance and on your regard over the applied side of mathematics, and points out the living relevance of teaching for research. Could you comment on these and related traits?

Theoretical Fluid Mechanics could be considered very well as a part of Mathematics. Of Applied Mathematics, if you wish. Also the contribution of Scientific Computation and Numerical Methods has been, and still is essential to Fluid Mechanics. The same happens with other parts of Physics and Engineering, like Electromagnetism, Quantum Mechanics, Relativity Theory, but I confess that I prefer Fluid Mechanics. We mathematicians are used to working on the thinks that we prefer, that we like. This is the freedom of the scientist, that is perhaps wider in mathematics. We have to try to preserve this possibility. Of course there is a counterpart: academic authorities and supporting government agencies possibly prefer fully oriented research, and also practical teaching, as in other sciences. But it will be better for all if we resist, under reasonable limits, to these interests.

We would also like to have your views on the Study groups, that initiative of yours to set up cooperation ties between academic researchers and industry actors through problem-solving sessions focused on questions presented by the latter.

Study Groups are a worldwide initiative to put together companies and mathematical researchers. Here we have done our best, in this direction. I'm deeply indebted to Prof. Tim Myers, in this respect. But there is still much to be done, especially at the local level. I mean local researchers and local industries. We need that collaboration, or, at least, this communication. We truly need that.

In the period 2007-2010 you were deputy-director to the CRM. We would like to know your thoughts about this research institution, not only for the said period, but in general concerning its evolution and, perhaps more importantly, about its potential for the coming years.

We should be open to changes that today can be unexpected, but from the point of view of the research in Catalonia I think we will always need an institution like this, that has two sides. The first side is to gather researchers from all the Catalan universities in order to undertake large projects, that are not possible at a smaller scale. The other side is to maintain a group of researchers hired by the center devoted to specific research subjects. This group is the core of a standard research center. But in mathematics we will always need also the first group, the group that is scattered in universities, because everywhere in the world mathematics is made by people distributed in perhaps small departments of many universities.

In the period 2010-2014 you were president of the SCM, an IEC society whose membership represents a great variety of professional profiles at all levels and across many institutions. What initiatives did you promote from that position and what achievements did ensue from them?

The SCM is a very lively institution. As any other society, its main goal is to put into contact people belonging to different institutions or doing different activities. This is especialy necessary among secondary school teachers, where the identity of the teacher of mathematics is nowadays somewhat diffuse. But it is also necessary at the university and research level, because at the end the universities tend to isolate themselves from external relations. Among the things we started during the period that I was the president of the SCM I remember the association with the societies from Valencia "Al-Kwarizmi", and from Balearic Islands "Xeix" into the *Jornades d'Ensenyament de les Matemàtiques*, birth of the secondary school contest "Copa Cangur", of the journal "reports@scm", and of the series of international conferences "Barcelona mathematical days".

Since 2012 you are an ordinary member of the IEC. As a mathematician, you belong to the Science and Technology Section. What are the main lines of work you are pursuing from that capacity?

Concerning the **IEC**, my opinion is that it is hard to believe in something like "Catalan mathematics". Mathematics, and other sciences, are universal. In this direction I have always focussed more on the idea of giving help to what we have called "mathematics in Catalan", instead of "Catalan mathematics". The **IEC** should be committed on supporting mathematical initiatives in the Catalan language, even if we have to accept that they would be less on research and more on teaching, at all levels or on public dissemination.

A final question related to the first: Since very early, you breathed architecture and art from your father and your brothers. Did this exposure influence in any way your style as a professional mathematician, be it as a teacher, a communicator, or a researcher?

It is not the same to be a Science or to be a Profession. Mathematics is a Science, and Architecture a Profession. I always had the idea that we have at least to try that Mathematics becomes a Profession, not only a Science. I've always compared the social presence of architecture, that is so wide, with that of mathematics, that it is perhaps scanty. This is something that I learned perhaps because of my family experience, this is true.

NL. Thank you very much for devoting your time to attend us.

Daniel Peralta Salas is a senior scientific researcher at the Institute of Mathematical Sciences (**ICMAT**) in Madrid and Chair of the Group Differential Geometry and Geometric Mechanics. He got a PhD in Mathematical Physics at Complutense University in 2006 (*Invariant Sets And First Integrals of Dynamical Systems*) and joined the **ICMAT** in 2010. He has published about 100 research articles in high profile journals, such as *Annals of Mathematics*, *Acta Mathematica*, *Proceedings of the National Academy of Sciences*, or *Duke Mathematical Journal*, and has been an invited speaker in more than 100 international conferences, seminars and courses.

Among his main distinctions we highlight the Plenary Lecture at the 7ECM (European Congress of Mathematics 2016, Berlin, Germany), the Barcelona Dynamical Systems Prize (2015), the Floer Lectures at the Floer Center of Geometry in 2019 (Bochum, Germany) and the MINT Distinguished Lectures (Tel Aviv, Israel) in 2020. Recently he was appointed as an EMS distinguished speaker at the 29th Nordic Congress of Mathematicians.

During the period 2014-2019 he was the PI of the Starting Grant from the European Research Council (ERC) *Invariant manifolds in Dynamical Systems and PDE*. The research lines of Peralta-Salas concern the connections and interplay between dynamical systems, partial differential equations and differential geometry. This includes different topics in fluid mechanics, spectral theory, conservative dynamics and geometric analysis.

NL. ***Let's start with a general question about your academic journey. We would like to know what have been its more salient highlights and also how did you become interested in the interplay between dynamical systems, partial differential equations (PDEs), and differential geometry?***

Since I was a kid I wanted to devote my life to scientific research. During my high school studies I realized that my major interests were in Physics and Mathematics. I studied the degree of Physics at Complutense University; I was a big fan of Roger Penrose and Edward Witten, and I wanted to become a theoretical physicist working on quantum gravity and string theory. In the second year I started a collaboration with my teacher of Classical Mechanics, Prof. Francisco González-Gascón, who ultimately was my PhD thesis advisor. This was my first contact with the theory of dynamical systems, specifically with integrability and symmetries. I published my first article when I was 21, in the *Journal of Mathematical Physics*. At that time, I was very proud of this result (a relation between some properties of the first integrals of a vector field and the stability of the field), but now I see it as almost a triviality. Anyway, I continued my collaboration with Gascón and we produced several articles before I had finished the degree of Physics. This was an excellent opportunity to realize that I can understand and work better with mathematics than with physics; I liked precise definitions, rigorous proofs, well developed theories, and except in the context of "classical physics" (which includes relativity and quantum mechanics), this was not the case of modern physical theories. When I finished my degree it was very clear to me that I wanted to do a PhD with mathematical content, although somehow related to Physics. Still the mathematics that I enjoy the most is that related to problems in theoretical physics. My interest in the interplay between PDEs, dynamics and geometry arose in 2001, after a course I followed the last year of the Physics degree on General Relativity. Prof. Francisco Javier Chinea introduced to us the problem of the equilibrium shapes of self-gravitating fluids. It was completely amazing to me how a geometric property, like an equipotential surface having constant curvature, could arise from the intricate properties of the solutions to a certain (overdetermined) PDE, and how some of the proofs involved the use of dynamical systems. Actually, many remarkable objects in Physics, like stream lines of a fluid flow, equipotential surfaces of a classical potential, or magnetic lines, are geometric objects that emerge from solutions to classical physics PDEs. A huge part of my research has been devoted to understanding (I am afraid that still not much) about these structures. To finish this (sorry, too long!) answer, I want to mention two extremely important highlights in my academic journey. One is my long-term collaboration with Alberto Enciso, whom I met in 2003, while we were PhD students in the same department (contrary to what many people think, I am only 2 years older than him!) … 20 years after this we have produced about 50 joint articles, some of them I think that real breakthroughs. The second is that I learnt a lot of geometry and topology from Prof. Gilbert Hector in Lyon, whom I visited many times during my postdoctoral life, and many of these ideas were important a few years later to approach some geometric problems in PDEs.

Now we would like to inquire a bit more closely on various aspects of your research. What main challenges have you encountered in working at the intersection of dynamical systems, PDEs, and differential geometry? How have you faced these difficulties?

The main challenge is that there are not many tools that allow you to obtain geometric information from a PDE. In some sense, they are different worlds, which speak different languages sometimes, although the objects are there, in the guts of the solutions to the differential equations. For example, from the analytic viewpoint it is natural to ask about the regularity of the solutions, their "size" or even some asymptotic properties, but things become much more complicated if you want to understand something very basic as, what do the level sets of the solutions look like? Which patterns can the stream lines of the solutions form? You are asking questions that involve a next order study, whose objects are not "the natural ones" for the PDE. But they are there, they have physical significance (actually you can measure many of them in the laboratory) and is the duty of a mathematician to study these problems. Generally each equation and object of study requires techniques specifically tailored for that. In collaboration with Alberto, we were very fortunate, because we discovered a general strategy that turns out to be quite succesful to study some geometric properties as far as the equations are linear and satisfy some additional assumptions. We applied this to construct steady fluid flows with knotted vortex lines and tubes, solutions to the heat equation with a prescribed path of hot spots and solutions to the nonlinear Schrödinger equation which exhibit vortex reconnections of arbitrary topological complexity. To many mathematicians these results are amazing (and I believe that very few can understand the proofs), but the general approach and ideas were always very natural to me. Simply you try to adapt to the PDE what you would do in differential topology or dynamical systems, but taking into account the PDE constraint; this forces to change some concepts in a natural way, e.g., instead of talking about extending a function, you talk about approximating a function, and this works many times.

As you continue your work, what are the emerging questions or areas that you find particularly intriguing? Are there new developments or directions you are excited to explore in the coming years?

I am very interested in many problems, but my favourite ones are always those related to the equations of classical physics. I will mention three directions that I am exploring now, with my collaborators and students, and I will continue to do so in the following years. One is related to plasma physics and magnetohydrodynamics. There are important mathematical problems there, which may even have some impact in applications and the understanding of physical phenomena. One is Grad's conjecture on the structure of magnetohydrostatic equilibria, and the other is Parker's hypothesis on topological relaxation of generic magnetic fields. Both questions are related to solutions to the magnetohydrodynamics PDEs, but the objects you are interested in are geometric, this makes these problems very hard. Our current PhD student, Javier Peñafiel, is doing an excellent work developping convex integration techniques that I think will be useful to shed some light on these questions. A second direction concerns the incompressible Euler equations from a dynamical systems viewpoint. I would like to understand the finite dimensional dynamics that can be embedded into the infinite dimensional dynamics of the Euler equations in Euclidean space. My former student Francisco Torres de Lizaur proved recently that essentially any dynamics can be embedded provided that you choose the Riemannian manifold where the Euler equations are defined, but unfortunately his techniques do not give any insight of what happens in Euclidean space. A final direction that I find exciting is a conjecture of Berry related to quantum chaos in spectral geometry. Our current student Alba Garcia-Ruiz has discovered that if Berry's conjecture holds true, it implies a property that Alberto, Francisco and myself discovered in 2015 and now we call "inverse localization"; I think this connection may yield major results in the understanding of Berry's conjecture.

What are your overall thoughts on the current state and future prospects of the interdisciplinary research landscape, particularly in the realms of dynamical systems, PDEs, and differential geometry?

The problems that involve different mathematical disciplines are usually very hard, they require the use of different techniques and ideas, as the problems that I mentioned above. But they are very natural in Physics, and many questions and physical phenomena are related to them. Now I am 45, and I am not sure I will have new ideas as good as I had a few years ago, but of course I keep trying (and personally I am very motivated and do not feel weak). Anyway, there are younger people in Spain who are doing an excellent job in this direction and I feel that my way of doing mathematics and approaching some problems have influenced them a bit. I am thinking of Francisco Torres de Lizaur (in Sevilla) and Robert Cardona (a former student of Eva Miranda, in Barcelona), I have great faith in them.

Can you tell us about any specific collaborations or joint projects you have undertaken with members of IMTech, particularly with Eva Miranda? How have your research interests been potentiated by them?

I started my collaboration with Eva and Robert Cardona in 2017. Robert got a JAE Intro scholarship at the ICMAT and this was the beginning of our first article where we revisited Arnold's structure theorem of steady fluid flows from the viewpoint of b-symplectic geometry (a subject where Eva is a leading expert). However, our more important results came later, some of them in collaboration with my colleague Francisco Presas at the ICMAT. We proved the universality and Turing completeness of the Beltrami flows on Riemannian manifolds (including the Euclidean space), and also constructed time-dependent solutions to the Euler equations on high-dimensional Riemannian manifolds that are Turing complete (from an Eulerian viewpoint). This was a very stimulating project, and somehow tackled Moore's conjecture on the computational power of hydrodynamics (it was never stated in a very precise way). Now with Eva, Cédric Oms (her former student) and brilliant master student Josep Fontana, we are studying the dynamics of Reeb fields in b-contact geometry, how it is related to the celebrated Weinstein conjecture, and how it potentially may lead to outstanding results in celestial mechanics. This last part is particularly exciting, and since I am not really an expert in these things, I am very fortunate that Eva, Cedric and Josep allow me to participate in these studies.

Let us now turn to the recognition you have received and its significance. You were one of the few Spanish mathematicians awarded with an ERC starting grant. What did this represent in your career?

From the personal viewpoint, it was extremely important to me. I got the ERC grant in 2014, I was a Ramón y Cajal researcher at the ICMAT, so I did not have a permanent position yet. At that time the only way to get a permanent position at the ICMAT-CSIC was to obtain an ERC grant (something very sad, in my view, which fortunately changed drastically a few years later). So for me (and my friend Alberto, who also got in 2015 another ERC grant) this was the way to continue at the CSIC. From the scientific viewpoint, I could hire a few postdocs and students, and this was a big boost to my career, concretely because it was how Francisco joined the ICMAT as a PhD student, as well as Alejandro Luque, as a postdoctoral researcher. With Alejandro we produced excellent works, culminating in an article that will be published in JEMS (also with Alberto) where we produced the first weak counterexamples to Grad's conjecture that are far from symmetry. Alejandro is an excellent mathematician, one of the best I met from the dynamical systems school in Barcelona, so this is why I was very sad when he left academia, although I understood the very unfair reasons that led him to take such a decision.

You are one of the very few Spanish mathematicians delivering a plenary talk in one of the ECMs. How did this bolster your career? Did such a pinnacle distinction open new paths for international collaboration?

I gave a plenary lecture at the 7th ECM in Berlin, in 2016. I was the second spanish mathematician doing so (the first one was Prof. Carles Simó at the ECM in Barcelona in 2000). Now it sounds like funny that I did not have a permanent position at the ICMAT yet. It was a great experience, giving a talk for an audience with more than 1000 mathematicians, in the big auditorium of the engineering school in Berlin. This said, I am not sure how it bolstered my career. Certainly it did not change my reserch interests nor ideas, probably it gave more visibility to my work, although I did not notice it immediately (but maybe some of the recognitions I got later were influenced by this? I do not know). The main point of a distinction like this is that you feel that (at least some of) your colleagues value your work and consider that it is of high profile. But this is difficult to detect in mathematics, colleagues do not usually say that you are doing an outstanding work, people are too reserved in this regard. This said, I have always believed that the main distinction for a mathematician is the theorems, the theories, the techniques that he/she has proved, and how it influences a line of research. When I turn 80 (hopefully!) I think that my main memories will come from the results I proved, how I enjoyed doing so, and the excellent students or colleagues I had that were able to go further than me with some of my advise.

Recently, you were appointed as an EMS distinguished speaker at

the 29th Nordic Congress of Mathematicians. In what ways does this enhance your profile?

It was excellent news, when you get a recognition you never know if it will be the last one. It was a great opportunity to talk about very recent stuff on complexity in steady fluid flows based on joint works with Pierre Berger, Ana Florio, Robert Cardona and Eva Miranda.

You have also been speaker in the Floer Lectures, so altogether you have undergone a quite intense international exposure. How do you value this dimension in the development as a researcher? Have you ever considered accepting a prestigious position abroad?

I think it is a measure of the quality of your work, its impact in the international community and the high profile of your research lines. Also invitations like this are related to your abilities as a speaker. Of course, apart from the quality of the articles, the international exposure of a researcher is one of the points that I pay more attention when evaluating a candidate for a grant, a position or an award. I never considered seriously to accept a position abroad, although I had some offers in the past (and I believe that if I wanted to move now, I could do it easily to some good maths department in many places outside Spain). There are multiple reasons for this. I like Madrid, my family and friends are here, and there are psicological factors: as an only child I was highly protected by my mother and it has always been difficult for me to take steps that represent a drastic change in my life. This is why I have never stayed abroad for a long time, even with a postdoctoral position.

In a broader perspective, what advice would you offer, based on your own experiences in academia and research, for young researchers aspiring to follow a path similar to yours?

My path has been very strange, rather unusual for most of researchers, not only in mathematics, full of "good luck" and match points. I did not go abroad with a postdoctoral position, I never followed much the ideas and problems of others, but I like to have my own view of the things, and I was never interested in the specific fashion topic of the moment (although sometimes it turned out that the problems I liked were also of interest to many people and involved long standing conjectures). Taking this into account, I am not sure I am the right person to give advice to young researchers. But let me try a bit. I would say that in most cases it is extremely valuable, many times crucial, to spend some time abroad and to learn from the best mathematicians. I also recommend to enjoy what you are doing, to have passion for mathematics, not to take this only as "a job" but also as "a pleasure" (which sometimes can be orgasmic ... can this be published?). When you are a student or a postdoc, of course you follow the ideas and problems suggested by your advisors, but at the same time you have to build your own ideas, your own vision of the subject, to find the way of doing and understanding mathematics that you like, the problems that you want to solve and for which you see yourself stronger. Not everybody that does a PhD or even a postdoc will be able to continue his/her career in the academia, but if you enjoy what you are doing (even taking into account it is easier to say than to do, a lot of pressure for young people, I know), it will be an unforgettable experience.

Collaboration often entails mentoring students and postdocs. Can you share instances where collaborative projects involved the active participation of young researchers? How does such involvement impact the learning experience and research outcomes?

I have been very fortunate with my students and young collaborators. I already talked about Francisco Torres de Lizaur and Robert Cardona, both are extremely strong and have a bright future ahead, I am looking forward to reading their theorems in the future. Of course I have to mention Maria Ángeles García-Ferrero,with whom we (Alberto and myself) developed the theory that allowed us to construct solutions to the heat equation with prescribed paths of hot spots; and Álvaro Romaniega, an outstanding student with whom we developed the theory of random Beltrami fields which allowed us to prove a probabilistic version of Arnold's conjecture in hydrodynamics. I am also very happy with my current students at the ICMAT, Alba Garcia-Ruiz and Javier Peñafiel, and Soren Dyhr, a student at UPC with Eva Miranda and Ángel González. Recently I also worked with a young assistant professor in Paris, Anna Florio, she is a tremendous mathematician, who did a great job in our joint project with Pierre Berger where we established GST universality of Beltrami flows in Euclidean space. I am also looking forward to seeing the progress of Josep Fontana, now a master student in Oxford; he already did an excellent job in our joint articles with Eva and Cédric on *b*-contact dynamics.

Beyond individual collaborations, how would you describe IMTech's role as a hub for mathematical research? In what ways can the institute facilitate interactions and collaborations among researchers working on diverse mathematical topics?

I think the creation of **IMTech** was a great enterprise and provides and excellent opportunity for increasing the mathematical research in Barcelona (which is already of very high quality). Probably the best way to facilitate interactions is to finantially support the organization of synergetic activities, but not anything, those for which there is a substantial basis and interest among the local people, and for which there are interesting ideas to develop. It also helps if people with stronger research profile have a teaching reduction, so that they can focus more on research and possible collaborations, but I know that this (and the finantial support for activities) depends on many factors which are not easy to control.

Finally, do you have some advice from ICMAT to IMTech?

I am not sure, because they are different institutions. **ICMAT** is a joint research center involving three universities in Madrid, and the **CSIC**. **IMTech** is a mathematical institute of the UPC, so in this sense I think it depends only on one university. The most important aspect of a research institute is its excellence, the stronger the members of the institute, the easier will be to get funding in public or private calls. If the members of a research center are active and do high profile research, then they should have the freedom to develop the research lines they want. I have been always very fortunate at the **ICMAT**, where I have been able to work on any topic I have been interested in. Finally, I will mention that a very important contribution to the research in any center comes from the postdocs, so it is crucial to get funding for hiring young people; I know this is not easy in Spain, where there are very few postdoctoral positions.

Finally, I want to thank the **IMTech** for this interview. It has been a wonderful opportunity to express some of my thoughts on my own career and mathematical research in general.

Marcel Guardia Munarriz is Full Professor at Universitat de Barcelona and member of the Centre de Recerca Matemàtica (**CRM**). His research deals with Hamiltonian systems. He analyzes the unstable motions that these systems may possess, focusing on models coming form Celestial Mechanics and also on Hamiltonian PDEs such as the nonlinear Schrödinger equation.

He holds a degree in Mathematics from the **FME** of the **UPC** and a PhD from the same University supervised by Tere. M. Seara. Before the current position he held postdoctoral positions at Pennsylvania State University, University of Maryland, the Fields Institute, the Institute for Advanced Study in Princeton and the Université de Paris VII and was also associate professor at **UPC**. He obtained an **ERC** Starting grant (2017 call) for the project HAMINSTAB: *Instabilities and homoclinic phenomena in Hamiltonian systems* and an ICREA Academia distinction (2018 call). Since 2022 he is the scientific director of the María de Maeztu distinction awarded to the **CRM**.

NL. ***You earned the degree in Mathematics at the end of the term 2004-2005, which the FME dedicated to Albert Einstein, mainly in celebration of his* annus mirabilis *(1905). The previous term had been dedicated to Henri Poincaré —the first of a series of thirty-one names celebrated up till the current academic year. What are your more salient memories about that five-year span at the FME?***

I have very fond memories of my five years at the **FME**. On the one hand, I greatly enjoyed the degree and I learned a lot. On the other, the environement at **FME** was extremely pleasant, I was very involved in the student life at **FME**and I still keep many friends from that time. By the way, I like that you, as dean of the **FME**, started the Mathematician of the year series with Poincaré, he is the founding father of dynamical systems!

During your undergraduate studies, what subjects did wake up your interest as possible research areas? Was there a winning choice? If so, what circumstances favored it?

During the degree I had very broad interests: algebra, geometry, analysis... So, when I decided to pursue a PhD I was very hesitant on which field to choose. I ultimately chose dynamical systems because it is a field which mixes up analysis and geometry and, at the same time, is geared towards applications.

The title of your doctoral thesis, supervised by Professor Tere Martínez-Seara and defended in 2010, was From Non-smooth To Analytic Dynamical Systems: Low Codimension Bifurcations And Exponentially Small Splitting of Separatrices. Would you like to recall the main insights and results concerning that work?

To carry out the PhD in the **UPC** dynamical systems group and under the advice of Tere M. Seara was an absolute pleasure. The environment was very stimulating, with many seminars, advanced courses, etc. Tere was always eager to send me to other Universities to collaborate with other people (I did three short stays during my PhD). I think it is very important for a PhD student to be exposed to other ways of doing research. Concerning my thesis, the work I value most is that dealing with what is called exponentially small splitting of separatrices. These are techniques that allow to prove the existence of chaotic behavior and, in general, of unstable motions, in nearly integrable Hamiltonian systems. The Barcelona school of dynamical systems (and in particular Tere M. Seara) has been very important in developing this field and in my thesis I gave one of the first general results. Some tools I developed in my thesis where crucial later in my career to build unstable motions in Celestial Mechanics models.

Let us now focus on your four-year postdoctoral journey in the period 2011-2017. You visited the Pennsylvania State University (USA), the Fields Institute (Canada), the Institute for Advanced Study (USA), The University of Maryland at College Park (USA), the CNRS and the Université Paris 7 Denis Diderot (France) with a Marie Curie Fellowship, and the UPC, first with a Juan de la Cierva postdoctoral fellowship (2 years) and then with a Ramón y Cajal postoctoral fellowship (1 year). How do you value this research odyssey? What were its most memorable experiences?

My postdoctoral period was rather unusual in the sense that I had several short postdocs. This gave me the oportunity to be exposed to different areas of Mathematics and different styles of research. This deeply influenced my career. Most important for me were the postdocs at the University of Maryland. Maryland had at that moment (and still does) one of the strongest dynamical systems groups in the world and with a large number of PhD students and postdocs. Interacting with them really broadened my vision of the field of dynamical systems. I have also to admit that at the personal level all this journey was rather exhausting (I changed appartment 8 times in 4 years!), so after my stay in Paris I was ready to come back to Barcelona and settle down.

That journey was followed by a tenured position at the UPC (Associate Professor, 09/2017 to 08/2022). How did you face the related responsibilities in teaching and research? What significance has it had for you to be granted an ICREA Academia distinction and endowed with an ERC Starting Grant (2018-2023) for the project Instabilities and homoclinic phenomena in Hamiltonian systems?

These were years of big changes in my career. I quickly went from being a postdoc myself to creating the ERC Starting grant team with several PhD students and postdocs. At the begining, starting up the ERC team seemed daunting but I have to say that I really enjoyed working with the team I created and these collaborations have allowed me to obtain some of the best results in my career. It is also very rewarding to see that the people that were in the team are having now very successful careers. The first two postdocs in the team, Filippo Giuliani and Andrew Clarke, have obtained tenure-track positions (one in Milano and the other one at UPC) and the first two PhD studends, Mar Giralt and Jaime Paradela, have obtained postdocs in Paris and Maryland, exactly the places were I did postdocs myself. In the period 2007-2022 I did most of my teaching at the **FME**. For several years I taught the Master course in Hamiltonian systems, which I oriented towards the dynamics of nearly integrable systems and with a particular emphasis towards Celestial Mechanics.

Since September 2022 you are Full Professor at the UB and, for the period 2022-2025, Scientific director of the María de Maeztu Distinction awarded to the CRM. How do you envisage these new responsibilities in your career? How do you foresee mathematical research in the coming years from the point of view of institutional organization?

I understand the Mathematicians in **UB**, **UAB**, **UPC** and the **CRM** as a pieces of the same research system. I think that going all together strengthens our community, boosts the research we carry out and helps make Barcelona attractive for researchers in Mathematics. It is crucial, now and in the near future, to deal with a huge generational shift. In this sense, I think that the transformation that the **CRM** has undergone by affiliating people from the Universities is momentous. This strengthens the bonds between us and allows us to apply for projects and funds collectively. I have tried to contribute to this project by assuming the scientific direction of the María de Maeztu Distinction but this is and must be a collective endeavour.

In relation to your research, in this issue you and* ANDREW CLARKE *contribute with the piece* Why are inner planets not inclined? *in the* Research focus *section. Could you describe the key stages and insights that culminated in the remarkable breakthrough reported there?

The origin of this work goes back to my postdoc in Paris in 2013. At that time, together with JACQUES FEJOZ, we analyzed the existence of chaotic motions in the secular 3 body problem (which is a simplified model of the three body problem). This model was too simple to lead to global instabilities but we already realized that adding more bodies to the problem could lead to stronger results. However other projects kept us busy and it wasn't until ANDREW CLARKE joined the ERC Research Team that we decided to continue with this project. All three have spent more than two years working on it. It mixes up several different techniques in geometry and analysis and it can be split in three steps. We first constructed orbits transferring angular momentum between planets for a simplified model, the secular 4 body problem. We then generalized it to the "true" 4 body problem. This lead to drastic changes of the eccentricity and inclinations of the planets ellipses. However, in these first results the semimajor axes of the ellipses were very stable. Then, we realized that slightly changing the approach we could also create drastic changes in semimajor axes. This culminated the project. Let me say that this third step came somehow as a surprise since folklore says that the semimajor axes are much more stable. However, we are able to show that this is not the case in certain regimes.

What would be the next steps in that program (instability of solar systems) that you would like to achieve? What difficulties do you expect to meet along the way?

So far we have constructed unstable motions of Solar system models provided one planet is very inclined. The next natural step would be to construct unstable motions in configurations where initially all planets perform close to circular close to coplanar motions, as happens for our Solar System. This regime is known to be more stable.

In a broad sense, your research interests are in Hamiltonian systems and Hamiltonian PDE's. Aside from the just commented celestial mechanics work, could you describe the main results you have obtained since your doctoral thesis? What problems are in your current research agenda?

Consider a Hamiltonian PDEs such as the nonlinear Schrödinger or wave equations with periodic boundary conditions. A fundamental question is to assert whether there are solutions of the PDE that, as time evolves, transfer energy from low to high modes. Such behavior is related to turbulence and is believed to be rather typical in many Hamiltonian PDEs, however results proving it exists are very scarce. I have done contributions in this field by applying dynamical systems tools to PDEs. More recently, I have also started new projects in the analysis of transfer of energy on Hamiltonian systems on infinite lattices. So far I have been considering toy models but the ultimate goal would be to analyze these behaviors in (simple) models coming from solid state physics.

Thank you so much for attending the NL for this interview!

Research focus

Multiview varieties: a bridge between Algebraic Geometry and Computer vision,

by Angélica Torres (Maria de Maeztu fellow at CRM)
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Computer Vision is the area of Artificial Inteligence that focuses on computer perception and processing of images. One of the main problems that arise in this area is the so called *Structure-from-motion* (SfM) problem or *3D-image reconstruction* problem, where the main goal is to create a 3D model of an object appearing in multiple two-dimensional images.

The input of the problem is a data set of images and the output is a 3D model of the scene consisting of an estimation of the relative position of the cameras and a relative position of the objects in the pictures with respect to the cameras.

Solving this problem as accurately and quickly as possible is fundamental for autonomous driving, videogames, and animation, just to name a few examples.

The SfM pipeline is depicted in Figure 1 and it has four key steps:

1. **Data collection**.
2. **Matching**. In this step we match points or lines in one image that are identifiable as the same point or line in another. These pairings are called *correspondences*.
3. **Camera pose**. In this step, some point and line correspondences are used to estimate the relative position of the cameras.
4. **Triangulation**. The camera positions are used to estimate the position of the world and line points whose images are the point and line correspondences obtained in the matching step.

The final 3D model is given by the camera positions and the world points and lines obtained from steps 3 and 4.

Although the input of the SfM pipeline is a set of images, at the end of Step 2 the images are forgotten and we are left with purely geometric information: correspondences of points or lines that are believed to come from the same world feature. To analyse this geometric information, the use of Algebraic Geometry is a natural choice.

Algebraic models for computer vision

In this note we will focus on algebraic varieties appearing in the Triangulation step. To understand them we start by modeling the process of taking a picture.

A *camera* is a function $C : \mathbb{P}^3 \to \mathbb{P}^2$. Depending on the camera that we want to model, the definition of this function is going to change. A *pinhole camera* C is a linear projection from $\mathbb{P}^3$ to $\mathbb{P}^2$ defined by a 3×4 matrix of full rank. For this model we assume that it is possible to take pictures of every point in space except for $c = \ker(C)$, which is called the *camera center* and, intuitively, represents the position of the camera in the world.

Given m pinhole cameras $C_1, \ldots, C_m$, the *joint camera map* is defined as

$$\begin{aligned} \varphi_{\mathcal{C}} : \mathbb{P}^3 &\longrightarrow (\mathbb{P}^2)^m \\ X &\longmapsto (C_1X, \ldots, C_mX). \end{aligned} \tag{0.1}$$

This map models the process of taking the picture of a world point with m cameras, that is, for each $X \in \mathbb{P}^3$ the tuple $\varphi_{\mathcal{C}}(X) \in (\mathbb{P}^2)^m$ is a point correspondence.

Using the camera map, we can define similar functions to model line correspondences. Given a camera C, and two points $u, v \in \mathbb{P}^3$, denote by $L_{u,v}$ the line spanned by u and v. The map

$$\begin{aligned} \upsilon_C : \mathrm{Gr}(1, \mathbb{P}^3) &\longrightarrow \mathrm{Gr}(1, \mathbb{P}^2) \\ L_{u,v} &\longmapsto L_{Cu,Cv}, \end{aligned} \tag{0.2}$$

where $\mathrm{Gr}(1, \mathbb{P}^n)$ denotes the grassmannian of lines in $\mathbb{P}^n$, takes a line spanned by u and v and sends it to the line in $\mathrm{Gr}(1, \mathbb{P}^2)$ spanned by Cu and Cv. This map models the process of taking the picture of a line with a camera C. It is straightforward to prove that the definition of the map does not depend on the choice of u and v, so from now on we can omit these subindices from the notation. Since C is not defined in the camera center, the map υ_C is not defined in the lines in $\mathrm{Gr}(1, \mathbb{P}^3)$ that go through the camera center. The readers with a background in Algebraic Geometry will see that this forms a Schubert cell.

The joint camera map for lines is defined as

$$\begin{aligned} \phi_{\mathcal{C}} : &\mathrm{Gr}(1, \mathbb{P}^3) \longrightarrow (\mathrm{Gr}(1, \mathbb{P}^2))^m \\ &L \longmapsto (\upsilon_{C_1}(L), \ldots, \upsilon_{C_m}(L)). \end{aligned} \tag{0.3}$$

In the Computer Vision setting, the lines in $\mathrm{Gr}(1, \mathbb{P}^3)$ are considered world lines and the lines in $\mathrm{Gr}(1, \mathbb{P}^2)$ are called image lines. For each world line L, the tuple $\phi_{\mathcal{C}}(L)$ is a line correspondence.

For a fixed camera arrangement $\mathcal{C} = (C_1, \ldots, C_m)$ the triangulation problem consists precisely in finding elements in the fibers of the joint camera maps, that is, given a point correspondence $(x_1, \ldots, x_m)$, its triangulation is a point $X \in \mathbb{P}^3$ such that $\varphi_{\mathcal{C}}(X) = (x_1, \ldots, x_m)$, and, similarly, given a line correspondence $(\ell_1, \ldots, \ell_m)$ its triangulation is a line $L \in \mathrm{Gr}(1, \mathbb{P}^3)$ such that $\phi_{\mathcal{C}}(L) = (\ell_1, \ldots, \ell_m)$.

Data collection — Matching features for pairs of images — 3D model

Figure 1: Structure from Motion pipeline. In the final 3D model the cameras are in red and the triangulated points are in black. The reconstruction was done with COLMAP [7] using their data set *person-hall*.

Multiview varieties

In practice the triangulation is done using noisy data. The noise comes from lens distortion in the cameras, different light effects, or different quality images. This implies that the point or line correspondences obtained in the matching process might not be in the image of the joint camera maps, but are close enough. This is why understanding the images of $\varphi_{\mathcal{C}}$ and $\phi_{\mathcal{C}}$ is necessary. Here is where the multiview varieties come into play.

The *Point multiview variety*, denoted $\mathcal{M}_{\mathcal{C}}$, is defined as the Zariski closure of $\mathrm{Im}(\varphi_{\mathcal{C}})$. Similarly, the *Line multiview variety* is the Zariski closure of $\mathrm{Im}(\phi_{\mathcal{C}})$.

The point and line multiview varieties are, respectively, the smallest algebraic sets containing all the perfect point and line correspondences. They model perfect data. If we understand these varieties, then we can correct the error of a noisy correspondence by triangulating its closest point in the corresponding multiview variety.

The point multiview variety has been thoroughly studied (see for example [3] or [8] for the basic properties). We highlight the fact that a Groebner basis for its vanishing ideal is known [1] and its Euclidean distance degree is known [5]. For this note we just mention the following theorem from [8] that gives a set of polynomials that cut out the point multiview variety.

Theorem 1. Let $\mathcal{C} = (C_1, \ldots, C_m)$ be an arrangement of pinhole cameras, and define the $3m \times (m+4)$ matrix

$$A(\mathbf{u}) = \begin{pmatrix} C_1 & \mathbf{u_1} & 0 & \cdots & 0 \\ C_2 & 0 & \mathbf{u_2} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ C_m & 0 & 0 & \cdots & \mathbf{u_m} \end{pmatrix},$$

where $\mathbf{u_i} = (u_{i1}, u_{i2}, u_{i3})^T$ for $i = 1, \ldots, m$ are the variables associated with the image of each camera. The multiview variety is cut out by the maximal minors of the matrix $A(\mathbf{u})$ above.

More recently in [2] the authors start the study of the line multiview variety. They introduce the formal definition that we saw above, provide a set of polynomials that cut out the variety, find its singular locus, and compute its multidegree. The results are given for camera arrangements such that at most four of them are collinear, this means that the they are valid for random camera arrangements or, in the language of Algebraic Geometry, the results are valid generically. We highlight the following theorem from [2]

Theorem 2. Given an arrangement of pinhole cameras $\mathcal{C} = (C_1, \ldots, C_m)$, denote by ℓ_i the point in the dual of $\mathbb{P}^2$ defining the line $\upsilon_{C_i}(L)$ in $\mathrm{Gr}(1, \mathbb{P}^2)$. The line multiview variety $\mathcal{L}_{\mathcal{C}}$ is equal to the set

$$\left\{(\ell_1, \ldots, \ell_m) \in \left(\mathbb{P}^2\right)^m \mid \mathrm{rank}\left(\left[C_1^T \ell_1, \ldots, C_m^T \ell_m\right]\right) \leqslant 2\right\}$$

if and only if no four cameras are collinear.

From the geometric point of view, the point multiview variety contains the tuples of points such that their back-projected lines intersect in a point (see Figure 2). Similarly, the line multiview variety contains all the line tuples whose back-projected planes intersect either in a plane or a line (see Figure 3).

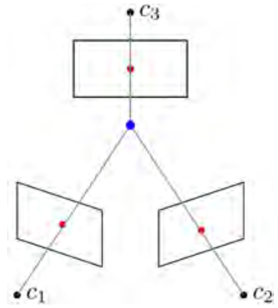

Figure 2: In red a point correspondence for 3 cameras. The back projected lines of each point are depicted in gray. Since they intersect in the blue point, the point correspondence is in the point multiview variety $\mathcal{M}_{\mathcal{C}}$.

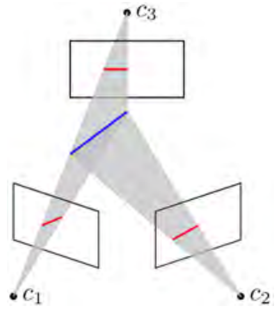

Figure 3: In red a line correspondence for 3 cameras. The back projected planes of the image lines are depicted in gray. Since they intersect in the blue world line, the line correspondence is in the line multiview variety $\mathcal{L}_{\mathcal{C}}$.

Error correction for real data

In practice, the equations obtained from Theorem 1 and Theorem 2 can be used to develop solvers for triangulation in SfM pipeline. Indeed, triangulating a point cloud requires finding solutions of the same parametric system of polynomial equations as many times as data points. Computer vision engineers have developed implementations that allow for a very fast solution of specific systems coming from theoretical results as the ones introduced above (see for example [4, 6, 7]).

As a final remark we highlight that in [2] the authors conduct numerical experiments to estimate the number of critical points of the distance funcion to the line multiview variety. This is a first approach to the Euclidean Distance degree of the Line multiview variety and it measures the complexity of the triangulation problem using algebraic methods. Although the numerical experiments suggest that this degree is polynomial in the number of cameras, this is still an open question.

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Undecidable trajectories in Euclidean ideal fluids, after [2],

by Robert Cardona (UB), Eva Miranda (DMAT, IMTech, CRM), and Daniel Peralta-Salas (CSIC, ICMAT)*

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1. Introduction

Fundamental to the understanding of physical phenomena and dynamical systems in general is the study of the computational complexity that may arise in a given class of systems. This complexity can include undecidable phenomena and computational intractability, which is relevant not only from a purely theoretical point of view but also in terms of applications to developing algorithms to determine the long-term behavior of a given physical system.

Several dynamical systems have been shown to exhibit undecidable trajectories: there exist explicitly computable initial conditions and open sets of phase space for which determining if the trajectory will intersect that open set can be undecidable from an algorithmic point of view. These include ray tracing problems in 3D optical systems [6], neural networks [7], and more recently ideal fluid dynamics [2, 3], a problem asked in the 90s by Moore [5]. In [3], it was shown that in compact 3D domains, one can find examples of stationary ideal fluids that possess undecidable trajectories. The caveat of the proof is that the Riemannian ambient metric is not canonical in any sense, for instance, the proof does not work to construct such examples in the standard flat three-torus, the standard round sphere, or the standard Euclidean space.

In this note we give a short introduction to the ideas developed in [2], where we construct stationary ideal fluids in the standard Euclidean space $\mathbb{R}^3$, i.e., equipped with the flat metric, that possess undecidable trajectories. The price to pay to obtain solutions in a space with a fixed Riemannian metric like the Euclidean one is working on a non-compact space and obtaining solutions that do not have finite energy.

2. Undecidability and Turing machines

The most used technique to prove that a class of dynamical systems might exhibit undecidable trajectories is by constructing an example of that system that is "Turing complete". This means roughly that the system encodes the evolution of any Turing machine, which is a symbolic system encoding a certain algorithm. Let us recall what a Turing machine is.

Turing machines

A Turing machine is defined as $T = (Q, q_0, q_{halt}, \Sigma, \delta)$, where Q is a finite set (called "states"), including an initial state q_0 and a halting state q_{halt}, another finite set Σ called the alphabet of symbols and that contains a blank symbol that we denote by a zero, and a transition function

$$\delta : (Q \times \Sigma) \longrightarrow (Q \times \Sigma \times \{-1, 0, 1\})$$

that will encode the dynamics (or "algorithm"). A configuration of the machine at a certain step of the algorithm is given by a pair in $Q \times \mathcal{A}$, where $\mathcal{A}$ denotes the (countable) set of infinite sequences in $\Sigma^{\mathbb{Z}}$ that have all but finitely many symbols equal to 0. An "input" of the algorithm, or starting configuration of the machine, is given by a pair of the form (q_0, t), where q_0 is the initial state and $t = (t_i)_{i \in \mathbb{Z}}$ is an arbitrary sequence in $\mathcal{A}$, which is commonly referred to as the tape of the machine.

The algorithm works as follows. Let $(q, t) \in Q \times \mathcal{A}$ be the configuration at a given step of the algorithm.

1. If the current state is q_{halt} then *halt the algorithm* and return t as output.
2. Otherwise, compute $\delta(q, t_0) = (q', t_0', \varepsilon)$, where t_0 denotes the symbol in position zero of t. Let t' be the tape obtained by replacing t_0 with t_0' in t, and shifting by ε (by convention $+1$ is a left shift and -1 is a right shift). The new configuration is (q', t'), and we can go back to step 1.

We emphasize that there is no loss of generality in the restriction to those sequences $\mathcal{A} \subset \Sigma^{\mathbb{Z}}$ that have "compact support", meaning that all but finitely many symbols are the blank symbol 0. The set $\mathcal{P} := Q \times \mathcal{A}$ is the set of configurations, and the algorithm determines a global transition function

$$\Delta : Q \setminus \{q_{halt}\} \times \mathcal{A} \to \mathcal{P},$$

that sends a configuration to the configuration obtained after applying one step of the algorithm.

Let us finish this section with two more facts about Turing machines. When reproducing an algorithm, one would like to know whether a given Turing machine with a given initial configuration will eventually reach a configuration whose state is the halting state, or if the algorithm will keep running forever. This is known as the halting problem and is known to be (computationally) undecidable as shown by Alan Turing in 1936. This means that there is no algorithm that, given any Turing machine T and any of its initial configurations c, will answer in finite time whether T halts with c or not. A particular consequence of computational undecidability in this context is that for some pairs (T, c), the statement "T halts with c" can be true/false but unprovable, i.e., undecidable in the sense of Gödel.

The second fact that we will need is that there exist "universal Turing machines". Those are Turing machines that can simulate in some sense any other Turing machine, and thus that are capable of reproducing any possible algorithm. One can think of those as a "compiler" in modern computer science. More formally there are different definitions of universal Turing machine, we will use one that is easier to state and that is sufficient for our purposes.

Definition 1. A Turing machine T_U is universal if the following property holds. Given any other Turing machine T and an initial configuration c of T there exists an initial configuration $c_U(T, c)$ of T_U, that depends on T and its initial configuration, such that T halts with c if and only if T_U halts with c_U.

In particular, determining whether a universal Turing machine with a given initial condition will ever halt is a (computationally) undecidable problem, and there exist initial configurations of T_U for which it is (logically) undecidable to determine if T_U will halt with that initial configuration.

Turing complete systems

Having understood the relation between Turing machines and undecidability, we relate them to dynamical systems in the following way. Let X be a dynamical system on a topological phase space M, where X can be either discrete or continuous

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on a finite or infinite-dimensional phase space. For concreteness, we can keep in mind the example of an autonomous flow on a smooth manifold.

Definition 2. A dynamical system X on M is Turing complete if there exists a universal Turing machine T_U such that for each initial configuration c of T_U, there exists a (computable) point $p_c \in M$ and a (computable) open set $U_c \subset M$ such that T_U halts with input c if and only if the positive trajectory of X through p intersects U_c.

In this case, the halting of a given configuration can be deduced from the evolution of an orbit of X. It is essential to require that p and U_c are in some sense explicit, namely computable, since otherwise, one could run into trivial systems being Turing complete. If we are working on a manifold, a point p is computable (in terms of c) if in some chart the coordinates of p can be exactly computed in finite time (in terms of c), for instance having explicit rational coordinates. Computability of an open set U_c can be loosely defined as saying that one can explicitly approximate U_c with any given precision. This notion is formalized in a subject called computable analysis. A Turing complete system has undecidable trajectories, meaning that there exist an explicit point p and open set U for which determining if the trajectory of p reaches U is an undecidable statement. This is different from being chaotic, where the sensitivity to initial conditions yields a practical unpredictability of trajectories since we are saying that even if we know exactly the initial point p, the long-term behavior can be completely unpredictable.

In practice, most Turing complete systems are constructed in the following way. We first encode, in a computable way, each configuration (q, t) of a universal Turing machine T_U as a point or an open set $U_{(q,t)}$ of the phase space M. For our purposes, assume we encode the initial configuration (q_0, t^{in}) as points $p_{(q_0,t^{in})}$, and every other configuration as an open set. We then require that for each initial configuration (q_0, t^{in}), the trajectory of X through $p_{(q_0,t^{in})}$ sequentially intersects the sets corresponding to the configurations obtained by iterating the Turing machine starting with (q_0, t^{in}). Namely, the trajectory through (q_0, t^{in}) will first intersect the set that encodes $\Delta(q_0, t^{in})$, then the set that encodes $\Delta^2(q_0, t^{in})$ and so on, without intersecting any other coding set in between. With this property, we can consider the open set U obtained as the union of all the open sets $U_{(q_{halt},t)}$ for each $t \in \mathcal{A}$, and the trajectory through $p_{(q_0,t^{in})}$ will intersect U if and only if the machine T_U halts with initial configuration (q_0, t^{in}).

3. Constructing stationary ideal fluids that are Turing complete

Having defined Turing complete systems, let us now describe the equations for which we would like to construct a solution that is Turing complete.

The Euler equations and sketch of the main theorem

The motion of an incompressible fluid flow without viscosity is modeled by the Euler equations. In $\mathbb{R}^3$, the equations can be written as

$$\begin{cases} \frac{\partial}{\partial t} X + \nabla_X X &= -\nabla p \,, \\ \operatorname{div} X = 0 \,, \end{cases}$$

where p stands for the hydrodynamic pressure and X is the velocity field of the fluid (a non-autonomous vector field). Here $\nabla_X X$ denotes the covariant derivative of X along X. If X is a stationary solution, i.e., time independent, then the first equation is equivalent to $X \times \operatorname{curl}(X) = \nabla B$, with $B := p + \frac{1}{2}\|X\|^2$ and curl denotes the standard curl operator induced by the Euclidean metric. A vector field that satisfies $\operatorname{curl}(X) = \lambda X$ for some constant $\lambda \neq 0$ is called a Beltrami field. It is a particular case of a stationary Euler flow with constant Bernoulli function. The main theorem we proved in [2, Theorem 1] is:

Theorem. *There exists a Turing complete Beltrami field u in Euclidean space $\mathbb{R}^3$.*

The strategy of the proof can be sketched as follows.

(1) We show that there exists a Turing complete vector field X in the plane $\mathbb{R}^2$ that is of the form $X = \nabla f$ where f is a smooth function.

(2) Furthermore, we require that if we perturb X by an error function $\varepsilon : \mathbb{R}^2 \to \mathbb{R}$ that decays rapidly enough at infinity, then we obtain a vector field that is Turing complete as well.

(3) We show that a vector field in $\mathbb{R}^2$ of the form $X = \nabla g$, where g is an entire function, can be extended to a Beltrami field v in $\mathbb{R}^3$ such that $v|_{z=0} = X$. That is v leaves the plane $\{z = 0\}$ invariant and coincides with X there.

(4) We approximate f by an entire function F with an error that decays rapidly enough. Hence $\tilde{X} = \nabla F$ is Turing complete and extends as a Beltrami field u on $\mathbb{R}^3$. It easily follows that u is Turing complete as well.

In this note, we will sketch the arguments of steps (1) and (2). The third step is done via a global Cauchy-Kovalevskaya theorem adapted to the curl operator. The fourth step is a general result about approximation of smooth functions by entire functions with errors with arbitrary decay [4].

Weakly robust Turing complete gradient flow in the plane

The goal of this section is to construct a Turing complete gradient flow on $\mathbb{R}^2$ and sketch how to make sure that its Turing completeness is robust under perturbations that decay fast enough at infinity. Following the recipe explained in Section 2, we will first show how to encode the configurations and initial configurations of a given universal Turing machine T_U into $\mathbb{R}^2$. Without loss of generality, we assume that $T_U = (Q, \Sigma, q_0, q_{halt}, \delta)$ with $Q = \{1, ..., m\}$ for some $m \in \mathbb{N}$ and $\Sigma = \{0, 1\}$.

The encoding. We first construct an injective map from $\mathcal{P} = Q \times \mathcal{A}$ to $I = [0, 1]$, where we recall that $\mathcal{A}$ is the set of sequences in $\Sigma^{\mathbb{Z}}$ with finitely many non-zero symbols. Given $(q, t) \in \mathcal{P}$, write the tape as

$$\cdots 000 t_{-a} \cdots t_b 00 \cdots \,,$$

where t_{-a} is the first negative position such that $t_{-a} = 1$ and t_b is the last positive position such that $t_b = 1$. If every symbol in a negative (or positive) position is zero, we choose $a = 0$ (or $b = 0$ respectively). Set the non-negative integers given by concatenating the digits $s := t_{-a} \cdots t_{-1}$, $r := t_b \cdots t_0$, and introduce the map

$$\varphi(q, t) := \tfrac{1}{2^q 3^r 5^s} \in (0, 1),$$

which is injective and its image accumulates at 0. There exist pairwise disjoint intervals $I_{(q,t)}$ centered at $\varphi(q, t)$, for instance of size $\frac{1}{4}\varphi(q, t)^2$. To introduce an encoding into $\mathbb{R}^2$ we proceed as follows. Fix $\epsilon > 0$ small, we encode (q, t) as

$$U_{(q,t)} := \bigcup_{j,k=0}^{\infty} I^j_{(q,t)} \times (k - \epsilon/2, k + \epsilon/2)$$

where $I^j_{(q,t)} := I_{(q,t)} + (2j, 2j)$. In other words, we are looking at any interval of the form $I^{j,k} := [2j, 2j+1] \times \{k\} \subset \mathbb{R}^2$, with $j, k \in \mathbb{N}$, and considering an ε-thickening of $I_{(q,t)}$ understood as a subset of $I^{j,k} \cong [0,1]$. Figure 1 gives a visualization of part of one of the open sets $U_{(q,t)}$ in a region of the plane.

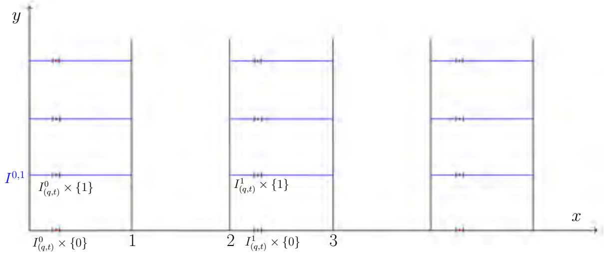

Figure 1. The open set $U_{(q,t)}$ is the ε-thickening of the intervals $I^j_{(q,t)} \times \{k\}$

The countable set of initial configurations $\mathcal{P}_0 = \{(q_0, t) \mid t \in \mathcal{A}\}$ admits a (computable) ordering which we will not specify, so that we can write it as $\mathcal{P}_0 = \{c_i = (q_0, t^i) \mid i \in \mathbb{N}\}$. Given c_i, the initial condition associated to the vector field that we will construct will be $p_{c_i} = (\varphi(c_i) + i, 0) \in \mathbb{R}^2$. This corresponds to the point $\varphi(q_0, t^i)$ of the copy $I^{i,0}$ of the several intervals we considered.

Integral curves capturing the steps of the algorithm. Iterating the global transition function from an initial configuration $c_i = (q_0, t^i)$ gives a countable sequence of configurations $c_i^k = (q_k, t_k^i) = \Delta^k(c_i)$ for each k an integer greater than 1. On each band $[2i, 2i+1] \times [0, \infty)$, we construct a smooth curve γ_i such that $\gamma_i \cap \{[2i, 2i+1] \times \{k\}\}$ is the point $(2i + \varphi(q_k, t_k^i), k)$, which lies in $U_{(q_k, t_k^i)}$, see Figure 2.

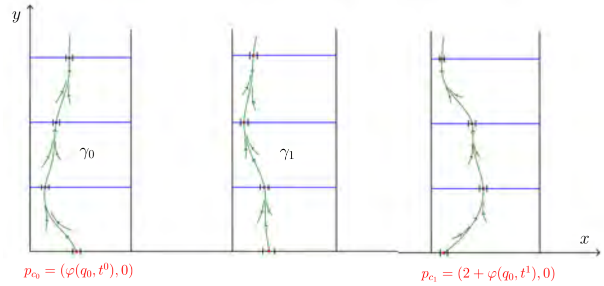

Figure 2. Integral curves following the computations of the Turing machine

We conclude by constructing a gradient field $X = \nabla f$ such that each γ_i is an integral curve of X. Observe now that given an initial condition c_i, the integral curve through p_{c_i} will intersect sequentially the open sets $U_{(q_k, t_k^i)}$, thus keeping track of the computations of the machine with initial configuration c_i. One easily shows that X is Turing complete, where to each c_i we assign the initial condition p_{c_i}, and the open set U for which the trajectory through c_i intersects U if and only if the machine halts with initial configuration c_i is simply $U = \bigcup_{t \in \mathcal{A}} U_{(q_{halt}, t)}$, that is, every open set encoding a halting configuration.

Weak robustness and conclusion. Recall that in order to apply the Cauchy-Kovalevskaya theorem, we need X to be the gradient of an entire function. To achieve this, we construct X in a way that the flow normally contracts towards each curve γ_i at a strong enough rate. This can be used to show that if we perturb X by an error function $\varepsilon(x, y)$ with fast decay at infinity, we obtain a vector field that is again Turing complete. This is because even if the curve γ_i will no longer be an integral curve, the integral curve through any of the points p_{c_i} of the perturbed vector field will still intersect sequentially the open sets $U_{(q_k, t_k^i)}$, hence capturing the computations of the Turing machine. The fast decay of the error is necessary since the open sets $U_{(q_k, t_k^i)}$ have no uniform lower bound on their size. This is because the intervals $I_{(q,t)}$ accumulate at zero, and hence their size tends to zero. One can estimate the decay rate of the size of the open sets that need to be intersected by the curves in terms of the distance to the origin, and hence robustness can be achieved under fast decay errors. The construction concludes by approximating f by an entire function $\tilde{f}$ (using [4]), and applying the Cauchy-Kovalevskaya theorem for the curl to the Cauchy datum $\nabla \tilde{f}$ on the plane $\{z = 0\}$.

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Why are inner planets not inclined? (after [2,3]),

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Consider the N-body problem, namely the motion of N bodies in 3-dimensional space subject to the Newtonian universal attraction:

$$\ddot{x}_j = \sum_{\substack{0 \leqslant i \leqslant N-1 \\ i \neq j}} m_i \frac{x_i - x_j}{\|x_i - x_j\|^3}, \tag{1}$$

where $x_j \in \mathbb{R}^3$ is the position and $m_j > 0$ the mass of body j. Of particular interest is the planetary problem, where the masses of bodies $1, \dots, N-1$ (the planets) are small with respect to the mass of body 0 (the sun). Since the masses of the planets are so small, the gravitational planet-planet interaction is much weaker than the attraction of the planets to the sun.

If one neglects the planet-planet interactions, the N-body problem becomes $N-1$ decoupled sun-planet 2-body problems, and Kepler's classical laws of planetary motion assert that the planets move on fixed ellipses. In this Keplerian approximation, all of the elliptical parameters (i.e. semimajor axes and eccentricities), as well as the mutual inclinations between the planes of the ellipses, are constants of motion. Of course when we consider the full N-body problem, by taking into account the gravitational attractions of the planets on one another, the Keplerian ellipses now vary slowly. A fundamental problem in Celestial Mechanics is to understand the effect of planet-planet interactions in the N-body problem: do these weak forces average out over time so that the planets perform near-elliptical motions, or do the small variations accumulate over time making the planets' orbits deviate strongly from ellipses? This question can be phrased colloquially as *Is the Solar system stable?* and it was called by M. Herman the *oldest problem in* *dynamical systems* [4]. Nowadays it is known that the answer to this question is rather nuanced, and that, generally speaking, stable and unstable motions coexist.

This problem has attracted a tremendous amount of attention over the centuries. In the direction of stability, a series of increasingly strong arguments of Laplace, Lagrange and others throughout the XIX century indicated stability of the semimajor axes of planets' orbits. This culminated in what has come to be known as the *first stability theorem of Laplace and Lagrange*: variations in the semimajor axes have zero average over certain time scales. Further to this, the classical Kolmogorov-Arnold-Moser Theory ensures that there exists a positive measure set of initial conditions whose orbits perform approximately for all time near-circular near-coplanar Keplerian ellipses.

Results concerning unstable motions in the N-body problem, however, are scarce. Instability, in this context, is represented by the existence of orbits along which nearly constant quantities drift a large amount over long time periods. In 1964, Arnold published a note, in which he proved that a specific near-integrable Hamiltonian system possesses orbits along which a constant of motion of the integrable approximation drifts a distance independent of the size of the perturbation [1]. In a footnote of that paper he wrote "*I believe that the mechanism of 'transition chains' which guarantees that nonstability in our example is also applicable to the general case (for example, to the problem of three bodies).*" This statement is now referred to as *Arnold's conjecture*, and the phenomenon is called *Arnold diffusion*.

In the setting of the planetary problem, Arnold diffusion can be understood as the existence of orbits along which the interactions between planets cause the elliptical parameters to drift significant distances. Prior to the work described in this article, there have been no complete analytical proofs of Arnold diffusion in a planetary problem, and the existing picture has been one of stability.

In [2, 3], we construct the first example of unstable motion in the planetary problem. We consider the planetary 4-body problem and we assume that the semimajor axes of the Keplerian ellipses are of different orders, meaning that planet 2 is much farther from planet 1 than planet 1 is from the sun, whereas planet 3 is revolving even further away. Furthermore, we make the crucial assumption that the mutual inclination between the ellipses of planets 1 and 2 is large: we need it to be more than 55°. Under these assumptions, we prove the following theorem.

Theorem. *There exist orbits of the planetary 4-body problem along which:*

1. *The eccentricity e_2 of the orbital ellipse of planet 2 follows any finite predetermined itinerary with arbitrary precision.*
2. *The mutual inclination i_{23} between the orbital planes of planets 2 and 3 follows any finite predetermined itinerary with arbitrary precision.*
3. *The semimajor axis a_3 of planet 3 follows any finite predetermined itinerary with arbitrary precision.*

Observe that all of these quantities, e_2, i_{23}, a_3 are constants of motion in the Keplerian approximation, and so this phenomenon is an instance of Arnold diffusion.

The motion in e_2 described by part 1 of the theorem implies that, if we choose any $\eta > 0$ and any finite sequence $\{e_2^1, \dots, e_2^N\} \subset (0,1)$, there exists an orbit of the 4-body problem and times $0 < t_1 < \cdots < t_N$, such that at time t_j the eccentricity $e_2(t_j)$ of planet 2 satisfies $|e_2(t_j) - e_2^j| < \eta$. In particular, we can make the orbit start close to circular (i.e. $e_2 \sim 0$), and at some point in the future, it can become highly eccentric ($e_2 \sim 1$).

Moreover, if we choose any finite sequence $\{i_{23}^1, \dots, i_{23}^N\} \subset \mathbb{T}$, we can find orbits of the 4-body problem such that, at time t_j, the mutual inclination $i_{23}(t_j)$ between the orbital planes of planets 2 and 3 satisfies $|i_{23}(t_j) - i_{23}^j| < \eta$. For example, there are orbits where planets 2 and 3 start on almost coplanar Keplerian ellipses, going in the same direction (i.e. *prograde* motion), but then at some point in the future, the orbital plane of planet 2 can flip over so that again, planets 2 and 3 are almost coplanar but are now revolving in opposite directions (*retrograde* motion). Indeed, there are orbits where planet 2 flips over and back between prograde and retrograde motions arbitrarily many times.

Finally, and perhaps most curiously, if we choose any finite sequence $\{a_3^1, \dots, a_3^N\} \subset [1, \infty)$, we can find orbits of the 4-body problem such that the semimajor axis $a_3(t_j)$ of the outermost planet at time t_j satisfies $|a_3(t_j) - a_3^j| < \eta$. Along such orbits, the semimajor axis of planet 3 can grow to any predetermined size from its starting point, and indeed can oscillate. This is contrary to the findings of Laplace and Lagrange.

With regards to the title of this piece (as well as the title of [2]), we conjecture that this diffusion mechanism will likely lead to collisions in any solar system in which the inner planets are inclined. This is not proved in our work, as our analysis is valid away from collisions. Note, moreover, that our results do not apply to our own solar system, as the planets' orbital ellipses are almost coplanar.

We point out that these phenomena described in parts 1-3 of the Theorem occur simultaneously along the same orbits. This constitutes the first analytical proof of Arnold's conjecture in a planetary problem.

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PhD highlights

Iñigo Urtiaga Erneta defended his PhD thesis *Elliptic problems: regularity of stable solutions and a nonlocal Weierstrass extremal field theory* on July 4, 2023.

The thesis was produced within the **UPC** doctoral program on Applied Mathematics and his thesis advisor was Xavier Cabré

Currently, he is a Hill Assistant Professor at Rutgers University under the supervision of Yanyan Li

Thesis summary

Partial Differential Equations (PDEs) are used to model almost every phenomenon affecting our daily lives, and they arise in areas as complex and diverse as physics, engineering, biology, or economics. Among these equations, *elliptic PDEs* describe stationary situations such as the equilibrium configurations of an evolution process. In applications, the interest lies in nonlinear equations which may admit too many different solutions. However, the only "physical" solutions one sees are the *stable* ones, namely, those that do not disappear under small perturbation of the data. Our thesis investigates qualitative properties of this natural class of solutions to elliptic problems.

The first part of the dissertation is devoted to the regularity of stable solutions to semilinear equations. This question is motivated by problems in combustion, where the temperature of a combustible mixture solves a reaction-diffusion equation and is expected to be near a stable solution to the associated elliptic problem. It has been known for a long time that these solutions may be singular when the dimension n (i.e., the number of variables in the problem) is sufficiently large. Namely, when $n \geqslant 10$, there are explicit examples of singular stable solutions where the "reaction term" (i.e., the nonlinearity of the problem) is an exponential function. Recently, in the breakthrough paper [3], X. Cabré, A. Figalli, X. Ros-Oton, and J. Serra showed that if $n \leqslant 9$, then stable solutions are smooth for *any* nonlinearity. Their proof applies to semilinear equations involving the Laplacian (an operator with constant coefficients) in a sufficiently regular domain. In [4–6] we extended their techniques to operators with variable coefficients, establishing the regularity of stable solutions in the same optimal range of dimensions. As a consequence of our analysis, we have even improved the known results for the Laplacian by significantly weakening the regularity requirements of the domain.

In the second part of the thesis, we develop an *extremal field theory* for nonlocal elliptic problems. Nonlocal equations (such as integro-differential equations) have gained much interest in recent years, as they are more suited than PDEs to model phenomena driven by long-range interactions. Many such equations arise from variational problems, where solutions can be interpreted as critical points (also known as "extremals") of some energy functional. A fundamental question in the Calculus of Variations is to determine whether an extremal actually minimizes this energy. For classical, local problems, sufficient conditions for minimality have been known since the XIXth century. Most notably, we have the following remarkable result of Weierstrass: if a critical point is embedded in a family of extremals whose graphs produce a foliation (an "extremal field"), then it is a minimizer with respect to competitors taking values in the foliated region. To prove this theorem, one constructs a *calibration* for the energy, i.e., an auxiliary functional (satisfying appropriate technical conditions) which yields the minimality of the critical point as a direct consequence. Together with X. Cabré and J.C. Felipe-Navarro, in [2] we have extended the theory of extremal fields to the nonlocal setting for the first time. Our main achievement has been to construct a calibration for general nonlocal energy functionals. Before our work, calibrations had only been obtained for nonlocal minimal surfaces in [1]. To find a calibration for the Gagliardo seminorm (the most basic fractional functional) was an important open problem that we have solved.

Selected Publication: [2].

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Armando Gutiérrez Terradillos defended his PhD thesis *Theta correspondences and arithmetic intersections* on May 26, 2023.

The thesis was produced within the **UPC** doctoral program on Applied Mathematics and his advisors were Víctor Rotger and Gerard Freixas.

Starting January 2024, he will be a postdoctoral researcher at the Morningside Center of Mathematics of the Chinese Academy of Sciences working in the group of Yichao Tian.

Thesis summary

Automorphic representations are an evolution of the classical notion of modular forms going back to Hecke in the 1900's. These objects are central in number theory and arithmetic grometry, providing the theoretical framework of deep conjectures due to Langlands. The theory of automorphic representations is formulated in terms of representations of Hecke algebras in spaces of L^2-functions defined over the adelic points of a group. Fundamental for this framework is the theory of representation of Lie algebras and algebraic groups.

The questions adressed in this thesis are motivated by the special values of L-functions, which are fundamental invariants of automorphic representations. Following general conjectural principles one expects deep connections with the geometry of relevant spaces in arithmetic geometry, known as Shimura varieties. A key tool to deal with this kind of problems is the so called theta correspondence. It allows us to relate automorphic representations for different groups, transferring certain properties from one representation to another.

The thesis is mainly divided into two parts. In essence, the first one is an extension of the paper [5]. The integrals of the logarithm of the Borcherds forms have been related to zeta and L-values in a wide variety of papers. In [2], the author studies the integral of the logarithm of the Borcherds forms for certain quasi-projective Shimura varieties associated to the group GSpin, obtaining an expression involving certain Fourier coefficients of Eisenstein series. One of the main tools in [2] is the Siegel-Weil formula in the convergent range of Weil and for anisotropic quadratic spaces. On account of the eventual divergence of the integral of the theta function over the modular curve, the integral of the logarithm of the Borcherds forms over the modular curve was not addressed in [2]. Along this chapter, using the regularized Siege-Weil formula of [3], we obtain an explicit expression for the truncated integral of the Siegel theta function. The main application of this result is an explicit formula for the integral of the logarithm of the Borcherds forms. The final result involves different zeta values and coefficients of Eisenstein series, completing the work of [2].

In chapter two, the analytic properties of L-functions are analyzed from a representation theoretic perspective. It is an extension of the work with Antonio Cauchi in [4]. First, we consider a zeta integral of $\mathrm{GU}_{2,2}$ which unfolds to a unitary Shalika functional. In order to compute this function we proceed from local to global, leading us to perform a detailed analysis of local Shalika models for unramified representations of $\mathrm{GU}_{2,2}$. On the one hand, under some local conditions, we show that the multiplicity of the Shalika model of unramified representations for the group $\mathrm{GU}_{2,2}$ is one. Using this result and following the ideas of [1], we are able to find an expression of the Shalika functional in terms of the Satake parameter of a representation in GSp_4. Similarly to the classic Casselman-Shalika formula for Whittaker functionals, the above result can be used to explicitly calculate Z-integrals. In fact, it allows us to establish a relationship between the zeta integral for the group $\mathrm{GU}(2,2)$ and a twisted standard L-function of GSp_4, where the relation between the involved automorphic representations is given by the theta correspondence.

Selected Pre-publication: [5]

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Jaime Paradela Díaz defended his PhD thesis *Unstable motions in the Three Body Problem* on June 30, 2023.

The thesis was produced within the **UPC** doctoral program on Applied Mathematics and his advisors were Marcel Guàrdia and María Teresa Martínez-Seara.

At present he is a Novikov Postdoc at the University of Maryland. His supervisor is Bassam Fayad.

Thesis summary

Broadly speaking, Dynamics aims at understanding the long term behavior of systems for which an infinitesimal evolution rule is known. It was already realized by the french mathematician Henri Poincaré, by the end of the 19th century, that it is in general hopeless trying to give a precise, quantitative description, of all the orbits of a given dynamical system. Instead of trying to solve the differential equations, in his studies of the 3 Body Problem, which models the motion of three bodies interacting via Newtonian gravitation, Poincaré drew attention on a more *qualitative* picture of the dynamics. One of the main actors in this qualitative description are the *hyperbolic periodic orbits* of the system. These are periodic orbits for which the linearization of the vector field posseses contracting and expanding directions. Close to the hyperbolic periodic orbits of the system, Poincaré identified mechanisms causing the exponential divergence of nearby orbits. Namely, two arbitrarily close initial conditions can lead, after a sufficiently long time, to quite different behaviors.

Somehow paradoxically, hyperbolicity, although creating local instability, can lead to global stability, in the sense that the whole orbit structure does not change when the system is slightly modified. This was the idea that lead Stephen Smale to introduce in the 1960's the concept of *uniformly hyperbolic systems*, in which, at each point of the phase space the local picture of the dynamics resembles that of the dynamics close to a hyperbolic periodic orbit. By now, we have a satisfactory description of the dynamics of uniformly hyperbolic systems. On the other extreme of the spectrum, another class of systems which are well understood are the so called *integrable systems*. For these systems, the phase space is foliated by invariant submanifolds on which the dynamics resembles that of a linear translation of the torus. This foliation is usually called the *Arnold-Liouville foliation*. Understanding what happens between these two distant regimes is the goal of the modern theory of dynamical systems. In the last decades, succesful programs have emerged to study what happens at the boundaries of both uniformly hyperbolic and integrable systems.

In this thesis, we present some modest contributions to the understanding of the dynamics of systems close to integrable ones. More concretely, we study the dynamics of the 3 Body Problem (3BP) in a regime where it can be studied as a perturbation of the 2 Body Problem (2BP), which is integrable. In this setting, it is natural to ask what new behavior, not present in 2BP, can appear in the 3 Body Problem.

One of our main results [2] is the existence of *topological instability* in the *restricted* 3BP.[*] Namely, the leaves of the Arnold Liouville foliation of the 2BP are not invariant for the flow of the restricted 3BP, and, we show, there exist orbits of the restricted 3BP connecting arbitrarily far leaves of this foliation.

A second set of results [1,3] deals with the existence of *non*

[*] The region of the parameter space for the 3BP in which one mass is negligible compared to the other two is usually referred to as the restricted 3BP.

trivial hyperbolic sets both in the 3BP and restricted 3BP. Non trivial hyperbolic sets are Cantor like subsets of the phase space where, at each point, the tangent space splits into two complementary uniformly contracting and uniformly expanding subspaces. The dynamics restricted to this set is extremely rich as it displays strong mixing and transitivity.

Although very different in nature, a key ingredient in the proof of both results is the identification of (partially) hyperbolic invariant objects of the phase space of the 3BP. Indeed, we are able to show that the local instability created by these objects can accumulate to induce global changes in the orbit structure when compared to the dynamics of the integrable 2BP, leading to the aforementioned phenomena.

Selected Pre-publication: [1]

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Outreach

My role in the development of LIGO and the detection of gravitational waves,[†]

by Kip Thorne (Caltech).

Thank you, Enrique,[‡] for your much too generous description of me and my contributions to science. This honorary doctorate from the Universitat Politècnica de Catalunya is of great significance to me. It honors, especially, my contributions to LIGO's discovery of gravitational waves. For this reason, I regard myself as sharing it with the large team of scientists and engineers, whose contributions were essential to our discovery.

There are only two types of waves that bring us information about the universe: electromagnetic waves and gravitational waves. They travel at the same speed, but aside from this, they could not be more different. Electromagnetic waves—which include light—, infrared waves, microwaves, radio waves, ultraviolet waves, X-rays and gamma rays—these are all oscillations of electric and magnetic fields that travel *through* space and time. Gravitational waves are oscillations in the *fabric* of space and time.

Galileo Galilei opened up electromagnetic astronomy 400 years ago, when he built a small optical telescope, turned it on the sky, and discovered the four largest moons of Jupiter. We LIGO scientists opened up gravitational astronomy in 2015 when our complex detectors discovered gravitational waves from two colliding black holes a billion light years from Earth.

The efforts that produced these two discoveries could not have been more different: Galileo made his discovery alone, though he built on ideas and technology of others. We LIGO scientists made our discovery through a tight collaboration of more than 1000 scientists and engineers.

Professor Garcia-Berro refers to me as one of the *leaders* of the LIGO Project. However, I was a leader in a formal sense for only three years, from 1984 to 1987, when I chaired LIGO's steering committee, consisting of LIGO's three founders: Rai Weiss, Ronald Drever and me. Our steering committee was a miserable failure. We frequently disagreed and so could not make decisions fast enough to move the project forward efficiently. Richard Isaacson, our funding officer at the US National Science Foundation (NSF) told us, unequivocally, that success would require LIGO to have a single director, a director with the authority and power to make all major technical and managerial decisions. The director did not need prior experience in the field of gravitational waves. What he or she *did* need was great skill in designing a collaboration structure and management structure, skill in learning the essential physics quickly, skill in seeking the relevant advice from the best experts, skill in making wise decisions based on that advice, and skill in convincing the members of the collaboration to accept his or her decisions.

LIGO's success is largely due to a sequence of directors who had those skills: Robbie Vogt, then Barry Barish, then Jay Marx, and now David Reitze. Of these four, only David Reitze had experience in gravitational wave science before becoming LIGO's director.

Many other big science projects have floundered or even collapsed because they failed to give sufficient power to a single director with the necessary skills.

Since I was not a successful organizational leader or decision-making leader of LIGO, what did I actually contribute? As Professor Garcia-Berro indicated, I formulated a vision for the science that LIGO would do. And I kept my eyes on the 'end game'. I continually asked myself, "what will be required for full success in the end? What is missing, that must come together with the mainstream LIGO experimental effort in order to open up gravitational astronomy and maximize the information it brings us?"

For me, developing the vision, and keeping my eyes on the end game, were a half century quest:

I was inspired to focus on gravitational waves as a student, in 1963—inspired by my mentor, John Wheeler, and by the first experimenter to attempt to detect gravitational waves, Joseph Weber. Especially important were my conversations with Weber during long hikes in the French Alps.

In 1966, when I joined the faculty at Caltech, I began formulating my vision for sources of gravitational waves and the insights that gravitational waves might bring us—a vision based largely on information and ideas gleaned from my own students, and from physics and astronomy colleagues around the world. I continued to update that vision over the decades from then until the 2000s. I incorporated my vision into the funding proposals that we wrote for LIGO, and I offered it to my LIGO colleagues as guidance for their data analysis efforts.

In 1972, Rainer Weiss created a detailed design for a new type of gravitational-wave detector: one based on laser interferometry that would ultimately become LIGO. In the most powerful experimental paper I have ever read [1], Rai identified also the major noise sources that such a detector would confront, he described ways to deal with each noise, and he estimated the sensitivity of the resulting detector. By comparing them with the wave strengths that my colleagues and I were estimating, he concluded that such a detector had a good chance of succeeding.

At first I was skeptical. In the book *Gravitation* [2] that I published the next year with Charles Misner and John Wheeler, I labeled Rai's detector "not promising". It required using light to monitor motions of mirrors that were a trillion times smaller than the light's wavelength. That seemed ridiculously impossible.

But after studying Rai's paper in depth and after long discussions with him, I changed my opinion; and then I spent most of the rest of my career trying to help Rai succeed; trying to help in any way that a theorist could.

Ronald Drever, in Glasgow, Scotland, had invented several major improvements on Rai's ideas, so in the late 1970s, I spear-headed bringing Drever to Caltech to create an experimental effort there, in collaboration with Rai's at MIT. And together, in 1983, the three of us created the LIGO project.

In parallel with the project's experimental effort, I was keeping my eyes on the end game. What *else* would be required for full success?

One obvious additional ingredient was data analysis: How do you extract very weak signals from LIGO's noisy data? Here, again, I was initially naive: I thought the data analysis would be easy. But I was wrong. Bernard Schutz in Cardiff began thinking deeply about LIGO data analysis in the late 1980s, in collaboration with Alberto Lobo and others. Bernard quickly

[†]Acceptance Speech of the Doctor Honoris Causa degree conferred by **UPC** on 25 May 2017.

[‡]Enrique García-Berro (1959-2017), sponsor of the nomination. The whole ceremony was documented in the booklet accessible at this link.

convinced me that the data analysis was difficult. So in the early 1990s I started a data analysis effort at Caltech, in parallel with Bernard's and Alberto's efforts in Europe. Soon a talented cadre of young people, inspired by Bernard, Alberto and me, took up the challenge and formulated data analysis plans that looked promising. So by 2000, I quit worrying about data analysis. But I had another, deeper worry:

In 1980, it had become clear to me that the strongest sources—the first things LIGO would detect—would likely be colliding black holes. To detect the collisions' waves and extract their information, we would need a catalog of all the wave *shapes* that the collisions could produce: their gravitational 'waveforms'. Computing those waveforms was so difficult that it could *not* be done with just pencil and paper. It would require computer simulations: solving Einstein's relativity equations on a computer, an enterprise called *numerical relativity*.

So I regarded numerical relativity as a crucial effort that would have to feed into LIGO's data analysis, in order for LIGO to succeed. In the early 1990s there were a dozen small research groups, around the world, trying to perfect computer codes for numerical relativity. But progress was very slow. Under pressure from Richard Isaacson—the NSF funding officer who had forced us to appoint a single LIGO director—these research groups banded together into a worldwide collaboration called the *Grand Challenge Alliance*. As chair of the Alliance's advisory committee, I got a very clear picture of their progress—or, more accurately, their *lack* of progress, through the 1990s and into the 2000s.

By the early 2000s I began to panic. It appeared likely that the simulations of colliding black holes would *not* be ready in time for LIGO's first discovery of gravitational waves. To speed up the research, we would need a much larger and more focused effort. Fortunately, by then I had trained a set of young scientists who could take over the roles I was playing inside the LIGO project; so I left the day-to-day involvement with LIGO and turned to creating a large, focused effort on numerical relativity, in collaboration with the intellectual giant of that field, Saul Teukolsky at Cornell University. We called our effort the SXS project: the project to Simulate eXtreme Spacetimes.

In 2004 a postdoc in our SXS project, Frans Pretorius, succeeded in creating a computer code that simulated two black holes orbiting each other, and gradually spiraling inward as they lost energy to gravitational waves; and simulated the holes' collision, and their final, spectacular burst of gravitational waves. Building on Pretorius's breakthrough, by 2015 our SXS collaboration had constructed a sufficiently complete catalog of gravitational waveforms to underpin LIGO's data analysis and its gravitational wave discoveries.

I hasten to add that, just as I did not contribute significantly to the actual construction of LIGO's detectors, I did also not contribute to the construction of the SXS computer codes. My primary role was to identify what was needed and inspire others to develop it.

LIGO's advanced detectors today are at one-third of their design sensitivity. When fully perfected, they will see three times farther into the universe, encompassing a volume 27 times larger, so instead of discovering roughly one black hole collision per month, when searching for waves, they will discover roughly one per day. With further planned improvements, they will discover a few per hour by the late 2020s, and also waves from many other kinds of sources. This, however, will require a major change in the detectors' designs: a change predicted half a century ago by Vladimir Braginsky, a superb Russian experimental physicist.

In 1968 Braginsky told us that, for ultimate success, our gravitational wave detectors would have to monitor the motions of very heavy objects, such as LIGO's mirrors, with such high precision that we would see their motions fluctuate unpredictably: fluctuations controlled by the laws of quantum mechanics. For the first time, humans would see human-sized objects behave quantum mechanically—a behavior only seen, previously, in atoms, molecules and fundamental particles. It took me ten years, but by about 1978 understood Braginsky's worry qualitatively, and began urging my students and postdocs to probe it quantitatively.

In 1983, when Weiss, Drever and I were co-funding LIGO, my student Carlton Caves developed a detailed understanding of these quantum fluctuations of LIGO's mirrors. And from then until now, my Caltech research group and Braginsky's in Moscow have collaborated to develop what Braginsky called *quantum nondemolition* techniques for LIGO: experimental and data analysis techniques to ensure that gravitational wave signals passing through LIGO's 40 kilogram mirrors are not demolished by the mirrors' quantum fluctuations. The first of these quantum nondemolition techniques will be implemented in LIGO later this year; and over the next several years, they will become crucial.

Here, again, I must confess: these quantum nondemolition techniques were devised largely by my students and postdocs, and by Braginsky's students and postdocs. My role here, as most everywhere else, has been one of identifying what needed to be done, and exhorting others to do it.

In that sense I have been crucial to LIGO. But the real credit for LIGO's success belongs to others: to the scientists and engineers who actually built LIGO's detectors and made them work; and those who actually developed LIGO's data analysis algorithms and made them work; and to those who actually perfected the numerical relativity computer codes and used them to simulate colliding black holes; and to those who actually formulated and perfected LIGO's quantum nondemolition techniques, and will begin to implementing them in LIGO later this year.

To these colleagues, I give thanks; enormous thanks. LIGO's successes are theirs; they deserve the credit; and I regard them as sharing this wonderful honorary doctorate with me.

Laudatio of Prof. Kip S. Thorne by Prof. Enrique García-Berro.

A moment of the investiture ceremony: Rector Enric Fossas handing over the gloves to Prof. Kip S. Thorne in the presence of Prof. Enrique García-Berro.

Prof. Kip S. Thorne delivering the *doctor honoris causa* lecture.

NL postface. On September 23, 2017, Enrique García-Berro died after a dreadful accident while hiking in the Huesca Pyrenees (Spain). On October 3, 2017, the Royal Swedish Academy announced that the 2017 Nobel Prize in Physics was awarded to Rainer Weiss, Barry C. Barish and Kip S. thorne *for decisive contributions to the LIGO detector and the observation of gravitational waves*. The Nobel awarding ceremony was held on December 8, 2017, and the three honorees agreed on the same title for their Noble lectures (*LIGO and the Discovery of Gravitational Waves*) but focused on three aspects: I, by R. Weiss, [3, 4]; II, by B. C. Barish, [5, 6]; and III, by K. S. Thorne, [7, 8].

The **NL** dedicates this edition of Kip Thorne's wonderful lecture to honor the memory of Enrique, dear colleague and friend.

Addenda (by Santiago Torres from GAA/UPC). Since the historic first detection in 2015 by the LIGO-Virgo collaboration of gravitational waves produced by the merger of two black holes, a new window in exploring our Universe has opened. Two years later, in 2017, the detection of gravitational waves along with electromagnetic signals from the merger of a binary neutron star marked the beginning of a new era in multi-messenger astronomy [9]. Since then, the catalog of gravitational waves has continued to expand, extending the gravitational-wave spectrum from attohertz to kilohertz frequencies. This ongoing expansion provides data that tests fundamental physics, confirming Einstein's general theory of relativity and unveiling mysteries about the universe's evolution (see [10], and references therein).

In particular, in this year 2023, a fascinating discovery was revealed, found by the North American Nanohertz Observatory for Gravitational Waves project (NANOGrav) and confirmed by several groups [11]. By using a few tens of pulsars within our Galaxy, whose periodicity is affected by the passage of gravitational waves, acting as buoys in a cosmic sea, scientists were able to observe the gravitational wave background, also known as the stochastic background, for the first time in history. These new discoveries bring forth new questions, such as the origin of this gravitational background attributed to supermassive black holes, primordial black holes, or other hypotheses that are currently under discussion.

In the following years, we anticipate a captivating race in search of new discoveries, involving projects like the space interferometer LISA, the already launched Euclid mission, or the Einstein Telescope [10]. These initiatives will likely yield new discoveries and open up possibilities to raise further questions that expand our understanding of the Universe.

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Mathematical principles of fluid mechanics,

by Joan de Solà-Morales i Rubió (**DMAT**, **IMTech**).

Received on October 12, 2023.§

The title of this lecture is a topic of the bachelor's degree in Mathematics offered by the **FME**. There is a subject called *Mathematical Models of Physics*, which I have never taught, and which I am sure covers it much better than I will. But I will give my point of view. And my point of view is inspired, or wants to be inspired, by Newton's *Philosophiæ Naturalis Principia Mathematica*. I need not explain to you that this is a very important book. In it Newton created mechanics, the three laws of mechanics; he created gravitation with the law of universal gravitation; and he created infinitesimal calculus, an extraordinary advance. And how did Newton proceed? He studied some problems and created the mathematics he needed. The mathematics he needed was the infinitesimal calculus and he created it. He was able to do it. I would like to look at the most typical fluid mechanics' equations from this point of view, the point of view of the mathematics involved in those equations.

Linear algebra. Let me start with a central equation from Linear algebra:

$$\frac{d}{d\varepsilon}\det(\mathrm{I}+\varepsilon A)\Big|_{\varepsilon=0} = \mathrm{Tr}(A). \qquad (*)$$

Since I have never taught Linear Algebra, it is quite likely that Eq. $(*)$ is taught on the first day in that subject, I don't know, but I do teach it on the first day. My proof of the formula uses the definition of the determinant by permutations. Indeed, all permutations except the identity produce terms in ε of order $\geqslant 2$, while the identity produces $(1+\varepsilon a_{11})\cdots(1+\varepsilon_{nn}) = 1+\varepsilon(a_{11}+\cdots+a_{nn})+\cdots$, where the last $\cdots$ denote terms of order $\geqslant 2$ in ε. And why do I say it is central? Because in the determinant on the left A may a Jacobian matrix and then second term, the trace, is a divergence.

This reasoning seems to go back to L. Euler (see [7]), but if we have to credit a name behind $(*)$, it has to be Jacobi, as he discovered a formula that is more general than $(*)$, but more complicated. Jacobi was an Ashkenazi Jew and I cannot refrain

§This text is a write up in English of the the opening lecture for the **FME** academic year 2023-24 delivered by the author on October 11, 2023, with the title *Principis Matemàtics de la Mecànica de Fluids* (Catalan), and dubbed by him *Lectio Brevis* (Latin).

to quote Bell's assessment found on borrowing his photograph below from Wikipedia: "Carl Gustav Jacob Jacobi was not only a great German mathematician but also considered by many as the most inspiring teacher of his time". I do not use the term 'inspiring' in Catalan or Spanish, although I know who my 'inspiring' teachers have been and I am very grateful to them. If I wouldn't know what 'inspiring' is, I surely know what it ends up being.


Carl Gustav Jacob Jacobi (1804-1851) and Joseph Liouville (1809-1882).

Differential equations. Liouville's theorem for matrix solutions $X(t)$ of ordinary linear homogeneous differential equations, $X'(t) = M(t) \circ X(t)$, can be stated as follows:

$$\frac{d}{dt}\det(X(t)) = \mathrm{Tr}\big(M(t)\big) \cdot \det(X(t)).$$

This is an easy consequence of the following relation and $(*)$ for $\varepsilon = \Delta t$:

$$\begin{aligned} X(t+\Delta t) &\backsimeq X(t) + \Delta t\ X'(t) \\ &= X(t) + \Delta t\ M(t) \circ X(t) \\ &= (\mathrm{I} + \Delta t\ M(t)) \circ X(t). \end{aligned}$$

Indeed, these relations imply that

$$\det\big(X(t+\Delta t)\big) \backsimeq (1 + \Delta t\,\mathrm{Tr}(M(t)) \cdot \det(X(t));$$

consequently

$$\det\big(X(t+\Delta t)\big) - \det(X(t)) \backsimeq \Delta t\,\mathrm{Tr}(M(t)) \cdot \det(X(t)),$$

and the claim follows readily from this.

Liouville was (like Cauchy and like Navier) an engineer of "Ponts et Chaussées", and also an engineer of the École Polytechnique. Apparently at that time mathematical analysis was taught at the École Polytechnique, and the teacher was Cauchy, while at the school of Ponts et Chaussées they made, well, bridges and roads. Let us keep this in mind for what we will say later.

A little more complicated, but still in the subject of Differential Equations, is the problem of finding the derivative of an ordinary system of differential equations with respect to the initial conditions. This can phrased by means of the function Φ, called the *flow* of the equations, which it sends the initial condition at time t_0 to the final condition at time t. The first formula below is the differential equation, and the second, the initial conditions:

$$\begin{cases} \dfrac{d}{dt}\Phi_{t,t_0}(\boldsymbol{x}) = \boldsymbol{v}(\Phi_{t,t_0}(\boldsymbol{x}), t) \\ \Phi_{t_0,t_0}(\boldsymbol{x}) = \boldsymbol{x}. \end{cases}$$

Taking the derivative D with respect to $\boldsymbol{x}$, we get

$$\frac{d}{dt}\underbrace{\mathrm{D}\Phi_{t,t_0}(\boldsymbol{x})}_{X(t)} = \underbrace{\mathrm{D}\boldsymbol{v}(\Phi_{t,t_0}(\boldsymbol{x}),t)}_{M(t)} \circ \underbrace{\mathrm{D}\Phi_{t,t_0}(\boldsymbol{x})}_{X(t)}$$

and by Louville's theorem,

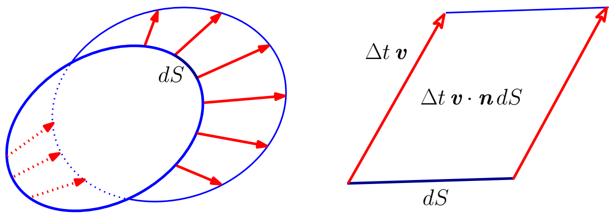
$$\frac{d}{dt}\mathrm{J}\Phi = (\nabla \cdot \boldsymbol{v})\,\mathrm{J}\Phi,$$

where $\mathrm{J}\Phi = \det(\mathrm{D}\Phi_{t,t_0}(\boldsymbol{x}))$ is the Jacobian of the flux and $\mathrm{Tr}(\mathrm{D}\boldsymbol{v}(\Phi_{t,t_0}(\boldsymbol{x}_0),t))$ is the divergence of $\boldsymbol{v}$. If the divergence is positive, the flow expands, and when it is negative, it contracts. Actually, this is the reason why $\nabla \cdot \boldsymbol{v}$ is called 'divergence' of $\boldsymbol{v}$.

Integral calculus. The divergence theorem asserts the second equality in the following formulas:

$$\frac{d}{dt}\int_{\Omega(t)} d\mathcal{V} = \int_{\Omega(t)} \nabla \cdot \boldsymbol{v}\, d\mathcal{V} = \int_{\partial\Omega(t)} \boldsymbol{v} \cdot \boldsymbol{n}\, dS.$$

Let me look at those formulas in another way (for this view I recognize the influence of a former brilliant student I had, Joaquim Serra). For the first equality we use the change of variables formula $\int_{\Omega(t)} d\mathcal{V} = \int_{\Omega(s)} \mathrm{J}\Phi_{t,t_0}\, d\mathcal{V}$, for a fixed t_0. Note that then the derivative with respect to t can be moved inside the integral and we have seen that the derivative of the Jacobian is the divergence. We can also consider what volume the border of Ω sweeps in its motion, which leads to the third formula: only the normal component of the velocity accounts for the change in volume (see next figure). In sum, we have two ways of calculating $\frac{d}{dt}\int_{\Omega(t)} d\mathcal{V}$ and therefore they yield the same result, which amounts to a proof of the divergence theorem.



Reynolds' transport theorem. Although it is possible to present fluid mechanics without recourse to this theorem, I am much in favor of its use. It asserts the following:

$$\frac{d}{dt}\int_{\Omega(t)} f(\boldsymbol{x},t)\ d\mathcal{V} = \int_{\Omega(t)} \left[\frac{\partial}{\partial t} f + \nabla \cdot (f\boldsymbol{v})\right] d\mathcal{V}.$$

The left-hand side is again the derivative with respect to t of an integral over a domain moving with the flow, but now the integrand is not 1, as in the divergence theorem, but a general function $f = f(\boldsymbol{x},t)$. In the integrand of the right-hand side we have the term $\partial_t f$ (the variation of f with respect to t) and the divergence $\nabla \cdot (f\boldsymbol{v})$, which is equal to $\nabla f \cdot \boldsymbol{v} + f\nabla \cdot \boldsymbol{v}$.

Conservation of mass and the continuity equation. Now we begin to look at fluid mechanics. The principle of conservation of mass is that the mass within any domain $\Omega(t)$ moving with the flow does not change. This can be expressed by asserting the vanishing of the derivative with respect to time of the integral of $\rho(\boldsymbol{x},t)$ over $\Omega(t)$:

$$0 = \frac{d}{dt}\int_{\Omega(t)} \rho(\boldsymbol{x},t)\, d\mathcal{V}.$$

If we evaluate this derivative by means of Reynold's transport theorem, we conclude that

$$\int_{\Omega(t)} \left[\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho\boldsymbol{v})\right] d\mathcal{V} = 0.$$

Since this happens for any domain, this relation is equivalent to

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

which is known as the *continuity equation* (for mass). In the case of an incompressible and homogeneous fluid, $\rho = \rho_0$ (a constant) the continuity equation reduces to

$$\nabla \cdot \boldsymbol{v} = 0.$$

Balance of the linear momentum. The linear momentum of the mass $\rho\, d\mathcal{V}$ contained in the infinitesimal volume $d\mathcal{V}$ is $\rho \boldsymbol{v}\, d\mathcal{V}$. Therefore the integral $\int_{\Omega(t)} \rho \boldsymbol{v}\, d\mathcal{V}$ is the momentum of the mass contained in $\Omega(t)$. Its derivative with respect to t is the total force acting on $\Omega(t)$ (Newton's second law). This total force is the sum of *stress* and *body* forces. The stress forces on the matter contained in $\Omega(t)$ are exerted by the matter outside the region through the boundary. They are represented by the vector $\boldsymbol{S}$ (force per unit boundary area) and hence the total stress force is given by the integral $\int_{\partial\Omega(t)} \boldsymbol{S}\, d\mathcal{S}$. The body forces are represented by the vector $\boldsymbol{F}$ that encodes the force per unit mass, so that the total of such forces on $\Omega(t)$ is $\int_{\Omega(t)} \rho \boldsymbol{F}\, d\mathcal{V}$. Gravity and Coriolis forces are of this kind. In sum, the balance of linear momentum for a fluid is expressed by the equation

$$\frac{d}{dt} \int_{\Omega(t)} \rho \boldsymbol{v}\, d\mathcal{V} = \int_{\partial\Omega(t)} \boldsymbol{S}\, d\mathcal{S} + \int_{\Omega(t)} \rho \boldsymbol{F}\, d\mathcal{V}. \qquad (**)$$

Leonhard Euler (1707 – 1783)

Euler's equations for incompressible non-viscous fluids. For non-viscous fluids, the stress forces are necessarily normal to the boundary: $\boldsymbol{S} = -p\boldsymbol{n}$ (this is analogous to the fact that a perfectly slippery surface can only react with normal forces to any object touching them). The scalar p is called *pressure*. Using the divergence theorem, it can be seen, as in [2], that the total stress force can be expressed as a volume integral: $\int_{\Omega(t)} -\nabla p\, d\mathcal{V}$. In this case the balance of momentum gives, using the transport theorem for each component $\rho \boldsymbol{v}^i$ of $\rho \boldsymbol{v}$ ($i = 1, 2, 3$),

$$\rho_t \boldsymbol{v}^i + \rho \boldsymbol{v}^i_t + \boldsymbol{v}^i \nabla \cdot (\rho \boldsymbol{v}) + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v}^i = -p_{x_i} + \rho \boldsymbol{F}^i,$$

which, using the continuity equation, yields Euler's equations (1757) for an incompressible non-viscous fluid:

$$\begin{cases} \boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho} \nabla p + \boldsymbol{F} \\ \nabla \cdot \boldsymbol{v} = 0 \end{cases}$$

In the PDEs course taught in our curriculum these equations are only studied in a much simpler version, namely Burger's equation: $u_t + uu_x = 0$. Here the unknown u is a scalar, not a vector. The term $uu_x = (u\partial_x)u$ corresponds to $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$, and there are no terms corresponding to the gradient of the pressure nor the equation analogous of $\nabla \cdot \boldsymbol{v} = 0$.

PDEs. Euler's equations have a remarkable connection with Laplace's equation $\Delta\phi = 0$. If we assume that $\boldsymbol{v}(\boldsymbol{x}, t) = \boldsymbol{v}(\boldsymbol{x}) = \nabla\phi$ (in this case we say that $\boldsymbol{v}$ is a *potential flow*), the condition $\nabla \cdot \boldsymbol{v} = 0$ becomes $\nabla^2\phi = \Delta\phi = 0$ and Euler's equations are satisfied with $p = -\frac{1}{2}\rho\|\boldsymbol{v}\|^2 + C$ (this is Bernoulli's equation, which is also valid for irrotational flows). Two significant propertis of potential flows (see [2]) are that, on one hand, their circulation around any closed curve is zero, and, on the other, that they minimize the kinetic energy among all flows that satisfy the same (Neumann) boundary conditions (i.e., imposing the value of the normal derivative at a boundary).

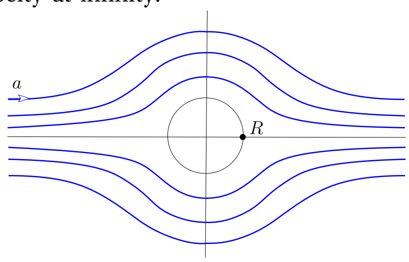
If a field is the gradient of a function, its rotational is zero. The converse is true only when the domain of the field is simply-connected (a topological condition).

Another remark is that the wave equation

$$\partial_t^2 \phi = c^2 \Delta\phi$$

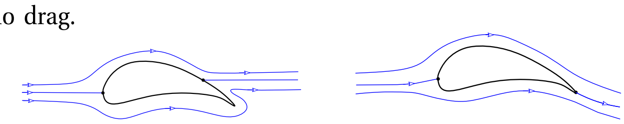
can also be regarded as a fluid mechanics equation, but this time for compressible flows ($p = p(\rho)$). Indeed, in this case it turns out to be a consequence of Euler's equation for potential flows (see [1]).

Complex analysis. In Complex analysis (one of the subjects of our Mathematics curriculum), it is proved that the harmonic functions are the real part of holomorphic functions in simply-connected domains. If this is the case, a *complex potential* $\Omega(z) = \phi(x, y) + i\psi(x, y)$ is introduced, where $\phi(x, y)$ is the velocity potential. Now the Cauchy-Riemann conditions imply that the integral curves of $\boldsymbol{v}$ are the level curves of $\psi(x, y)$ (i.e., the curves $\psi(x, y) =$ constant). For example, level curves corresponding to the imaginary part of the function $\Omega(z) = a(z + R^2/z)$, which is holomophic outside the disc of radius R, are depicted in the following picture, where a is the velocity at infinity:



However, potential flows around an obstacle are cursed by the D'Alembert's paradox (in dimensions 2 and 3): Such flows, with given velocity at infinity, *do not produce any net force on the obstacle*.

An important breakthrough for curing that paradox was discovered by Nikolai Zhukovsky (1847-1921). He was a Russian scientist who is considered a founding father of hydrodynamics and aeronautical engineering. In his research of how a flow could produce a lifting force on a plane wing profile, he took recourse in his knowledge of complex analysis, including the technique of computing integrals by means or residues, and thereby he modified the complex flow considered before by adding a term $\frac{\Gamma}{2\pi i} \ln z$ (this term is irrotational, but not potential, inasmuch as the exterior of the profile is not simply-connected). The net result is that it produces a lift force with no drag.



Flows around and airfoil profile. The right image depicts Jukowsky's flow.

Viscous flows: Navier–Stokes equations. Drag is produced by viscosity, which is reflected in the form of the stress vector $\boldsymbol{S}$. Cauchy showed that $\boldsymbol{S} = \sigma \cdot \boldsymbol{n}$, where σ is a symmetric matrix (it is called Cauchy's stress tensor). It follows that we can write $\sigma = -pI + \sigma_{\boldsymbol{v}}$, where $\sigma_{\boldsymbol{v}}$ is also a symmetric tensor. This tensor is called the *viscosity tensor* as $\sigma_{\boldsymbol{v}} = 0$ precisely when the flow is non-viscous (that is, $\sigma = -pI$).

The fundamental fact about the viscosity tensor is that $\sigma_{\boldsymbol{v}}$ *depends linearly on the differences of velocity between nearby particles* (Navier, Stokes). Therefore it is reasonable to write $\sigma_{\boldsymbol{v}} = \sigma_{\boldsymbol{v}}(D\boldsymbol{v})$, where $D\boldsymbol{v}$ stands for the differential of $\boldsymbol{v}$, because $D\boldsymbol{v}$ is a good encoding of the 'velocity differences between nearby particles'. An additional remark is that the differences in velocity among nearby particles produced by rigid rotations do not produce viscous effect, and so we can replace $D\boldsymbol{v}$ by $D\boldsymbol{v} + D\boldsymbol{v}^T$. By using $(**)$, it can be seen, as in [1, 2], that the result is the Navier-Stokes equations (NS), where μ is the fluid *viscosity*:

$$\underbrace{\frac{d}{dt}\int_{\Omega(t)} \rho \boldsymbol{v} d\mathcal{V}}_{\text{Transport and cont.}} = \underbrace{\int_{\partial\Omega(t)} \mu(D\boldsymbol{v} + D\boldsymbol{v}^T)\boldsymbol{n}\, d\mathcal{S}}_{\text{Divergence Th.}} + \int_{\Omega(t)} \rho \mathbf{F}\, d\mathcal{V}.$$

These equations lead to the standard form, with $\nu = \mu/\rho$ (*kinematic viscosity*):

$$\begin{cases} \boldsymbol{v}_t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\dfrac{1}{\rho}\nabla p + \nu\Delta\boldsymbol{v} + \mathbf{F} \\ \nabla\cdot\boldsymbol{v} = 0. \end{cases}$$

Claude-Louis Navier (1785 – 1836) and Sir George Stokes (1819 –1903)

The history of the NS equation is fascinating. As stated in [3], "Navier's original proof of 1822 was not influential, and the equation was rediscovered or re-derived at least four times, by Cauchy in 1823, by Poisson in 1829, by Saint-Venant in 1837, and by Stokes in 1845. Each new discoverer either ignored or denigrated his predecessors' contribution. Each had his own way to justify the equation. Each judged differently the kind of motion and the nature of the system to which it applied". Cauchy and Poisson espoused the more mathematical approached, but otherwise their methodologies were quite different: Cauchy introduced tensor techniques and spatial symmetry arguments and Poisson, among other things, emphasized the need of molecular reasoning (discrete sums instead of integrals), as Navier had already pioneered. Saint-Venant and Stokes acknowledged Navier's work, but differed from him, and among themselves, on the procedures to get the equations.

So, why did the equations end up being called after Navier and Stokes? This question has been discussed in the history texts (see [3, 4], [5], [6], for example). One fact that is recognized is that Navier and Stokes were the most practical, the ones who cared more about contrasting their results with experiments, whereas the other names referred to above cared more about geometry. Navier, who died in 1836, remained a little skeptical about his equations because the experiments carried out by Girard, a professor at the École Polytechnique, did not quite turn out the way he expected. After his death, Hagen and Poiseulle designed more careful versions of Girard's experiments and they found a better accord between theory predictions and experiments. As expressed in [3], it was unfortunate for Navier to trust Girard's findings about flows in capillary tubes.

Stokes was also concerned with experiments. One important problem he studied was the drag F of a sphere moving inside a fluid. He came up with a formula named after him: $F = 6\pi\mu Rv$, where R is the radius of the sphere and v the flow speed. I wish I had time to spend explaining where this formula comes from, and how many assumptions have to be made to get there. This formula is only valid in dimension 3. For the problem in dimension 2, that is, the movement of a disc in a plane fluid, or of a cylinder in a three-dimensional fluid, Stokes did not find a solution, and this is why it has remained as *Stokes' paradox*. Now we know, as it was found out later, that there is no solution in dimension 2 (this would be a topic for another occasion). Stokes also introduced singular limits: his formula holds for a small sphere in very slow motion in a very viscous fluid.

A new derivation of the viscous term. The derivation I propose of the term $\mu\Delta\boldsymbol{v}$ is based on the following formula:

$$\lim_{r\to 0^+} \frac{\left(\frac{1}{r^{n-1}|S_{n-1}|}\int_{\|\boldsymbol{h}\|=r} f(\boldsymbol{x}_0+\boldsymbol{h})\, d\mathcal{S}_{\boldsymbol{h}}\right) - f(\boldsymbol{x}_0)}{r^2} = \tfrac{1}{2n}\Delta f(\boldsymbol{x}_0).$$

I learned this formula, which is not very well known, from Xavier Cabré. The term within parenthesis in the numerator is the average $\bar{f}_{\boldsymbol{x}_0,r}$ of the function f over the sphere of radius r centered at $\boldsymbol{x}_0$, i.e. the integral of the function over the sphere divided by the sphere's area. The formula thus says that the value of the Laplacian of f at a point $\boldsymbol{x}_0$, $\Delta f(\boldsymbol{x}_0)$, can be obtained (note the factor $1/2n$) from the limit when $r \to 0^+$ of the difference $\bar{f}_{\boldsymbol{x}_0,r} - f(\boldsymbol{x}_0)$ divided by r^2.

In the formula the function f can be vector valued and hence it can be applied to a flow $\boldsymbol{v}$. In this case, the left hand side takes into account the difference $\bar{\boldsymbol{v}}_{\boldsymbol{x}_0,r} - \boldsymbol{v}(\boldsymbol{x}_0)$, which is a measure of the difference in a speed of the flow at a distance r from $\boldsymbol{x}_0$ and at $\boldsymbol{x}_0$. This view of the viscosity term goes to the root of the viscosity phenomenon: viscosity is produced by differences in speed between nearby particles, where 'nearby' means for 'small' r, that is, for $r \to 0^+$. It sheds a different light on the nature of the viscosity term included in the presentations by Navier, Cauchy, Poisson, Saint-Venant and Sokes. Whereas these authors derive the viscosity term by means of differential calculus, the proposed presentation relies on integral calculus.

On the fractional Laplacian and Stokes' paradox. To conclude, I am going to propose a similar approach to justify the use of the fractional Laplacian in the NS equations, as in [8, 9], a point that is hard to find, if at all, in the literature.

The following formula is the same as the preceding one, but written by moving $f(\boldsymbol{x}_0)$ within the integral and $|S_{n-1}|$ to the right-hand side:

$$\lim_{r\to 0^+}\int_{\|\boldsymbol{h}\|=r} \frac{f(\boldsymbol{x}_0+\boldsymbol{h}) - f(\boldsymbol{x}_0)}{\|\boldsymbol{h}\|^{n+1}} d\mathcal{S}_{\boldsymbol{h}} = \frac{|S_{n-1}|}{2n}\Delta f(\boldsymbol{x}_0).$$

Next formula has a similar look, and it is also the limit of an integral average, but it is different, as the integration runs over the exterior of the sphere of radius r.

$$\lim_{r\to 0^+}\int_{\mathbb{R}^n\setminus\{\|\boldsymbol{h}\|\leqslant r\}} \frac{f(\boldsymbol{x}_0+\boldsymbol{h}) - f(\boldsymbol{x}_0)}{\|\boldsymbol{h}\|^{n+2s}} d\mathcal{V}_{\boldsymbol{h}} = \frac{-1}{c(n,s)}(-\Delta)^s f(\boldsymbol{x}_0).$$

It is another way of construing the notion of velocity differences between nearby particles, and on the right hand side we get the *fractional laplacian* $(-\Delta)^s$. With this interpretation we obtain the NS equation with viscosity term expressed as a fractional Laplacian:

$$\boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p - \nu'(-\Delta)^s \boldsymbol{v} + \boldsymbol{F}.$$

None of the central problems of the NS equations has been solved with this new equation, but one that I think has not been studied is this little nugget for which I have some sympathy: the Stokes paradox. Such study might produce, in dimension 2, an explicit solution to the flow problem around a disk. In other words, the question (perhaps open) is whether for some values of s in the fractional Laplacian there is no Stokes paradox in dimension 2, and thereby spawning an explicit solution for the flow around a disc.

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Remembering G.E.P. Box: Life, Contributions, and Some Personal Experiences,

by Daniel Peña (UC3M), Víctor Peña (**EIO**) and Josep Ginebra (**EIO**)

Received on November 28, 2023.

This academic year, the Facultat de Matemàtiques i Estadística (**FME**) is honoring the legacy of one of the most influential statisticians of all time, George E.P. Box (1919-2013). The keynote lecture of this dedication was delivered on September 14 by Professor G. Geoffrey Vining (see the Chronicle of that event authored by Professor Marta Pézez Casany in this issue).

On November 22, 2023, professor Daniel Peña, who was a friend and collaborator of Box, shared his reflections on the life and contributions of the eminent statistician in a lecture delivered at the **FME**. This piece is based on the slides he presented on that occasion.

Box was born into a working-class family in England. At age 16, he started working as an assistant in a chemical company. At age 20, he was studying chemistry at the University of London when he was called up for military service during World War II. In the British Army, he was involved in the design and analysis of experiments with poison gas, and turned to Ronald A. Fisher for advice. After the war, Box earned an undergraduate degree in mathematics and statistics and in 1953 he obtained a Ph.D. in statistics from the University of London under the mentorship of Egon Pearson.

After obtaining his Ph.D., Box worked at Imperial Chemical Industries as a statistician. During that time, he developed the foundations of evolutionary operation, which is based on sequential experimentation and continuous process improvement [1]. He took a one year leave to visit North Carolina State, where he met Professor Gertrude Cox and other prominent American statisticians. He and his family enjoyed his year in the United States and moved to Princeton in 1956. At Princeton, Box served as the Director of the Statistical Research Group. He further developed industrial statistics and worked on revolutionary work in the field of time-series analysis with Gwilym Jenkins (see [2, 3]).

In 1960, Box moved to the University of Wisconsin-Madison, where he wrote his most celebrated works in Bayesian data analysis, design of experiments, and quality control [4–6]. He built a strong Department of Statistics serving as its Chair, attracting some of the most talented statisticians at the time. Under his leadership, Wisconsin-Madison became the best applied statistics department in the US. Additionally, he was successful in conducting and promoting interdisciplinary research between statisticians, engineers, and scientists in a wide array of fields. Towards the end of his career, he founded the Center for Quality and Productivity Improvement with William G. Hunter.

When Box officially retired as a professor in 1992, he continued his involvement in the Center for Quality and Productivity Improvement. He kept on working on interdisciplinary problems and took a renewed interest in statistical process control and monitoring. He also wrote the memoir [7], a book that intertwines personal and professional experiences, providing insights into his life and influence as a statistician.

Professor Peña first saw Box at a conference on time-series analysis in Cambridge in 1976. At the time, Box's work was controversial: theoreticians thought that the work lacked mathematical rigor, and applied statisticians thought it was too complicated. In 1979, Peña interacted with Box at the first Valencia Bayesian meetings (organized by J. M. Bernardo) and later invited him and George Tiao to give a course in Madrid. Peña and Box became friends and collaborators; they visited each other several times and they co-authored groundbreaking work on dynamic factor models [8]. In 1994, Box became doctor honoris causa by the Universidad Carlos III.

In the mid 1980s, Box visited the Statistics department at **UPC** invited by Albert Prat, where he taught a course on time series analysis together with Tiao, and he visited the department several times after that. In the late 1990s, Box spent a year at Universidad de Cantabria to work on his book on statistical process control [9] in collaboration with Professor Alberto Luceño.

Professor Peña sees George Box as a pioneering data scientist: his work on sequential experimentation, model combination, dimensionality reduction, and exploration of non-linear surfaces lies at the core of the field. Box's work is alive in our everyday life, from the recommendations we get from streaming services (which are based on sequential design of experiments) to the answers we get from ChatGPT (which are based on neural networks that handle non-linearities in ways that are related to Box's work on transformations).

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How to construct experiments: The quest for random combinatorial designs, by Patrick Morris (GAPCOMB) and Guillem Perarnau (DMAT/UPC, **IMTech**)

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From Biology to Economics, experiments play a crucial role across the sciences. In many cases one has a lot of freedom in how to design an experiment. How do you design it well? How do you make sure it is fair and efficient? For answers to these questions, we can turn to Mathematics and in particular, a corner called *Combinatorics*, where such questions have been formalised and studied for centuries. Two key concepts arise: *randomness* and *combinatorial designs*. Despite huge progress in the mathematical understanding of these notions, one key challenge remains, namely how to marry these two ideas and *generate random designs*. This is a notoriously hard problem, but recent breakthrough results open up entirely new vistas and the future of the mathematical art of experimental design looks very bright.

Figure 1: A row by row distribution of fertilisers.

Randomness: Making experiments fair. Suppose you are given the following task. Design an experiment that compares the effect of different fertilisers on the yield of a certain crop. Let us say you have n fertilisers (with n batches of each) and an n units by n units square field split into individual lots, each lot having size 1 by 1 and taking exactly one fertiliser. How do you arrange the fertilisers on the field in order to test and compare them? One way to do this would be to split your big square field into rows of lots and assign each row a fertiliser. See for example the distribution indicated in Figure 1. Whilst this is certainly a neat arrangement, it leads to an experiment that is not very *fair*. Indeed, it is susceptible to bias that will skew our results. Imagine, for example, that a pest infestation appears from the South. The last row, corresponding to a single fertiliser will then have very poor results, even though it could actually be the best one!

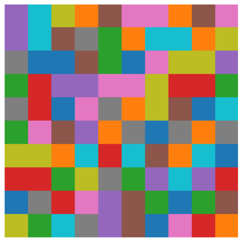

Figure 2: A random distribution of fertilisers.

In order to avoid such unwanted biases, we want a distribution that is somehow far from neat, as any ordered rule for distribution will inevitably lead to a potential bias. How do we achieve such a distribution? A beautifully simple and yet extremely effective idea is to simply use a *random* distribution. That is, for each lot, we roll an n-sided die and assign the fertiliser corresponding to the outcome to that lot (of course, at some point you may run out of a given fertiliser, but then you can simply roll again until you get a choice where you have a free batch to use). See Figure 2 for an example of a random distribution. The great thing about this is that it is extremely fair. Indeed, it gives no (dis-)advantage to any particular fertiliser and will most likely be completely free of unwanted symmetry. Whilst it is impossible to know when the idea of using random objects first originated, it was pioneered by Paul Erdős in the second half of the 20th century, who realised the power of using randomness to get combinatorial objects with desired properties. Often such properties are difficult to obtain by deterministic means, but flourish naturally with the use of randomness. Among the earliest results of the so-called "Probabilistic method", we find the lower bound on diagonal Ramsey numbers and the existence of graphs with large girth and large chromatic number. The use of random objects to answer certain questions also spurred the study of random discrete structures in their own right, and the study of properties of random combinatorial objects is to this day a thriving and fascinating area at the intersection of Combinatorics, Probability Theory, Computer Science and Statistics.

Using randomness is highly effective in being robust to unwanted factors that could effect the outcome of our experiment, but what if there are critical factors that we actually *want* to test? Indeed, let us imagine a more complicated task for an experiment. Again you have different fertilisers which you want to place in a square of lots, but this time, you want to test the performance of each fertiliser in relation to other factors that correspond to the rows and columns of your square. For example, it could be that each row of your square is at a different height and each column receives a different amount of water. Our distribution should still be fair but, crucially, we should have data for each fertiliser and different height/water level, so that we can identify, for example, the optimal combination. Thus we see a problem with the random distribution as in Figure 2; there are several columns and rows that completely miss some fertilisers. There is a fix, but it comes at a price. Indeed, if we use a larger field, say our field is m units by m units, then placing our n fertilisers randomly, will most likely result in a distribution where each row and each column sees each fertiliser when (and only when) m is at least $n \log n$. This example is an instance of the famous *Coupon Collector Problem* from Discrete Probability. Actually in this case each row/column will also see each fertiliser around m/n times. Whilst this looks like a nice experiment distribution, it is very wasteful in resources as the extra $\log n$ factor means that we need lots more of each fertiliser, not to mention a larger field.

Combinatorial Designs: Making experiments efficient. In many cases one can imagine that costs dictate that the design of the experiment should be optimally efficient. Thus, returning to our example, is it possible to use an n units by n units square and just use n copies of each fertiliser but still have each row and each column using each fertiliser once? Given that the random approach does not work for this, maybe it is time to revisit the ordered approach, as we did for Figure 1. Indeed, suddenly our initial arrangement doesn't seem so bad anymore. At least every fertiliser is tested with each water level (in each column) and so we have comprehensive results for half of our task. In fact, it is not too hard to get an ordered arrangement that works, simply consider the diagonal arrangement depicted in Figure 3.

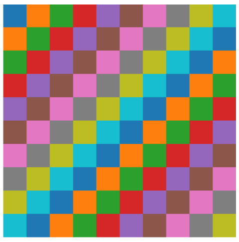

Figure 3: A *diagonal* LS.

Mathematically speaking, what we are talking about is known as an *n-Latin Square (LS)*; an $n \times n$ array with entries in $[n] = \{1, 2, \ldots, n\}$ such that each number appears exactly once in each row and in each column. These objects were introduced by Leonhard Euler in the 18th century and are central in Design Theory. In fact, you have probably also seen a large number of LSs in your life time as they form popular games in the form of (completed) *Sudoku squares*, which are 9 by 9 LSs in which we additionally impose each number to appear exactly once in each of the 3×3 main subsquares.

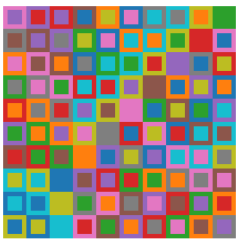

Figure 4: A pair of MOLS.

Euler realised that he could generate many different Latin squares and was interested in increasing the symmetrical requirements, looking for example, for the so called *Mutually Orthogonal Latin Squares (MOLS)* where two LSs overlap in a way that no pair is repeated in a square. See for example Figure 4. In the experimental design analogy, one can think that you are not only testing fertilisers but also pesticides in the same experiment and want to arrange them so that each fertiliser and each pesticide is tested at each height/water level and *additionally* every pair of fertiliser/pesticide is also tested together.

LSs and MOLSs are just one example of a wide family of mathematical objects known as *Combinatorial Designs*. Each is a collection of subsets of a finite set that have strong symmetric and balanced properties. Since Euler, these objects have been studied on a mathematical basis and a deep theory has been developed. They have also found many applications in different scientific disciplines. Indeed, their use in Experimental Design, as in the example above, was pioneered by Ronald Fisher in the 1920s-30s [3] as he was concerned with the agricultural applications of statistical methods. Another application is the construction of error-correcting and error-detecting codes, codes that are used to transmit information robustly and, preferably, efficiently. Indeed, to reduce the amount of additional bits transmitted, we would like to find optimal codes. An important family of optimal codes is *Perfect Codes*, rare objects that decompose the space of binary strings in a highly symmetrical way. A canonical example is constructed from the *Fano Plane*, an example of a combinatorial design known as a *Steiner Triple System* (STS) that can also be interpreted as a finite projective geometry. Take a collection $\{S_1, \ldots, S_7\}$ of 3-subsets of $[7]$ such that each pair appears in exactly one triplet (see Figure 5). Now we construct a code as follows: for each $i \in [7]$ create a binary word v_i of length 7 by writing a 1 at entry j if and only if $i \in S_j$ (see Figure 5). Add the zero word of length 7, 0000000, and also include all the complementary words (words obtained by replacing zeroes by ones, and vice versa). This collection of 16 strings is known as the $(7, 4)$-*Hamming code*. It transmits 4 bits of information by sending 7 bits, and it does so with some level of robustness: it can correct up to 1 error or detect up to 2. That is, if our goal is to correct errors and only 1 bit of the codeword is changed in the transmission (flipped from a 1 to a 0 or vice versa), the codewords are different enough that we can retrieve the original codeword. Similarly, if our goal is to detect errors and there are at most 2 bits changed, the word received will not match up with any codeword and so we will know that some error has occurred along the transmission.

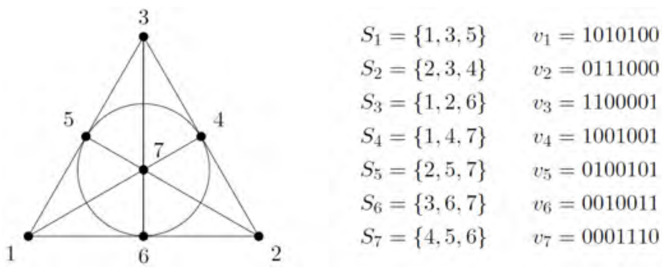

Figure 5: Geometric realisation of Fano plane, collection of 3-subsets and their corresponding elements in the (7,4)-Hamming code.

Let us give one last example of a design, which is in fact also an STS and, like Sudoku squares, came from the world of recreational mathematics. This is known as *Kirkman's school girl problem*, a mathematical puzzle published in 1850 [7]: "Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk twice abreast". This puzzle was the earliest explicit example of an STS (like the one in Figure 5), where one asks for a collection of subsets of some n-vertex set of size 3 such that every *pair* of vertices features exactly once. Thus Kirkman's problem seeks to find an n-vertex STS that, additionally, can be split into 7 partitions of $[n]$. Note that Kirkman's problem can also be interpreted as asking for an experimental design; if we are interested in a social experiment, we may want to ask each girl to rate the other girl's conversation and so it will be necessary for every pair of girls to walk together at least once. As with LSs, an STS represents the most efficient way of achieving this.

An interesting feature of Kirkman's problem is that it is not easy. Indeed, unlike the diagonal LS that we saw in Figure 3, there is no simple rule that can give the configuration necessary and it can take quite some time to come up with a solution, as a good recreational mathematics problem should! This is in fact very indicative of the development of Combinatorial Design Theory since its birth. We have seen some simple examples, but one can easily generalise definitions and require more from our designs, as Euler did already by asking for MOLS. A natural generalisation of STSs is the following: for integers $1 \leqslant t \leqslant k \leqslant n$ we can ask for a collection of k-subsets on n vertices such that every t-subset is covered exactly once, known as an (n, k, t)*-Steiner system (SS)*.

Most of the research in Combinatorial Designs in the 20th century revolved around the question of whether the designs that we ask for actually exist? For some parameters, one can easily prove no such design exists via a combinatorial trick called *double counting*. For instance, if there exists an (n, k, t)-SS of size m, then the total number of t-subsets in $[n]$ is $\binom{n}{t}$, while the number of t-subsets covered by the collection of m k-subsets of the design is $\binom{k}{t}m$. Since m is integer, $\binom{k}{t}$ must divide $\binom{n}{t}$. These sort of conditions are known as *divisibility conditions* and are clearly necessary for the existence of designs. For other sets of parameters, more complicated proofs can show non-existence despite the divisibility conditions being satisfied. On the other side, much effort has gone into constructing designs with various parameters and properties. Although this is sometimes easy, for most designs this very quickly gets hard, like with Kirkman's problem. Solutions often involved using Geometry, as in Figure 5, and Algebra, as in Figure 3, where one can view the construction as placing, for each row i and column j, the fertiliser F_{k+1} where $k = i + j \mod n$.

Random designs: The best of both worlds. We have seen how randomness leads to fair experiments whilst designs can be used to construct efficient experiments testing multiple different factors. Neither approach is perfect. Indeed we saw that randomness can lead to redundancy when we require multiple factors to be tested whilst it is clear that the unwanted symmetry of an arbitrary combinatorial design does not necessarily lead to a fair experiment. For instance, consider the example in Figure 3: if the central South-West to North-East diagonal gets the most light, then the fertilisers represented by (light-)blue will have a pretty sizeable advantage.

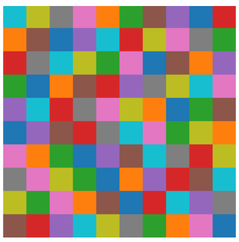

Figure 6: A random LS.

In order to fix this, one idea is to construct the design in a random fashion. Yates already envisioned this in 1933. Discussing about experiment designing, he wrote "...it would seem theoretically preferable to choose a square at random from all the possible squares of given size". For example, Figure 6 depicts a random LS of size 10. It certainly seems well-distributed and so robust against bias. In general though, Yates' request poses a considerable challenge. One could hope to just have a list of all possible designs and then pick one at random but this quickly becomes intractable. There are already 7,580,721,483,160,132,811,489,280 distinct* LSs of size 10 and this number grows superexponentially with n. Therefore some new idea is needed to construct a random (or close to random) design.

In the 90s there was a big breakthrough by Jacobson and Matthews, who gave a *Markov Chain Monte Carlo (MCMC)* algorithm for generating random LSs [5]. The MCMC method revolutionised the world of random algorithms in the last decades of the 20th century. The basic scheme of MCMC algorithms is the following: Given a class of objects one is interested in sampling from, define a local operation on them which allows us to navigate through the space of such objects. Then, we may follow a random trajectory (commonly referred to as a *random walk*) on our sample space by, at each step, performing a local operation chosen at random from all the possible ones. Under many circumstances, after just a relatively small number of steps (depending on the object size), the trajectory will lead to an object that is close to uniformly random. There is a large and rich theory developed for ways of proving that this is the case, know as *mixing times of Markov chains*. Due to their symmetry constraints, it is not easy to define a local operation on LSs. The pivotal contribution of Jacobson and Matthews was to find a "not so local" operation between them which yielded an MCMC algorithm for sampling an LS. Whilst one can prove that this algorithm will eventually lead to an almost uniform LS, and it seems to work well in practice, to this day we cannot provide mathematical evidence that the algorithm is efficient (that is, polynomial in n). For other designs (for example STSs), the situation is even worse, with a few similar algorithms proposed but very little proven about their uniformity or effectiveness.

Even if we cannot generate random designs, perhaps we can still say something about their properties. Until recently, there has only been sporadic result on this. For example, Babai showed in the 80s that random STSs have a trivial automorphism group [1], providing evidence that random designs do indeed provide bias-free constructions for experimental designs.

Absorption: A new way to construct designs. In 2014, Peter Keevash announced a result that shook the mathematical community. He had found a new way to construct designs through the *absorption method* [6], provided that the divisibility conditions were satisfied. To think about Keevash's method, consider the following random process which aims to build an n-LS. As when we considered the random distribution in Figure 2, we are going to use random dice rolls. In each step we pick a random row and a random column and fill the corresponding entry with a random symbol but this time, in contrast to the simple approach that we discussed at the beginning of the article, we make sure that we do not violate the rules of an LS. So when we use some symbol in a given row and column, we forbid that symbol from being used again in that row and column. This random process, known as the *semi-random method* or the *Rödl Nibble*, introduced by Vojtěch Rödl in 1985, has been used successfully to tackle similar problems. Rödl showed [11] that this process will get quite far and fill almost all of the cells but unfortunately, we will most likely get stuck. That is, before completing the LS, there will be no cells left where we can place symbols in a valid way, without getting two of the same symbol in the same row or column.

*We think of two squares as the same if one can be obtained from the other by permuting rows and columns.

The idea of absorption is to fix this by first putting aside *absorbing structures* made up of some collections of rows, columns and symbols, that have lots of flexibility in how they can contribute to an LS. We then run the random process avoiding these absorbing structures, until we get stuck. At this point, almost all of the remaining entries of the LS have been filled and we reintroduce the absorbing structures using their flexibility to make some room and fill in the remaining empty cells (*absorb* them into the object) thus obtaining a full LS. Of course, this sketch is at a very high level and the real challenge is to define and find absorbing structures in an appropriate way so that they have the power to achieve this. Indeed, before Keevash's result, both absorption and random processes were well-known techniques in the field of Combinatorics but it was not expected that this approach could handle structures so rigid as designs. Keevash managed to do so by using algebraic techniques to construct absorbing structures and shortly after, Glock, Kühn, Lo and Osthus [4] managed to bypass the need for algebra, adopting a new multi-round absorption process coined *iterative absorption*.

These sets of authors were the first who managed to incorporate randomness into the construction of designs, thus providing templates for experiments that, while not being uniformly random, enjoy many of the desired bias-free qualities that are present in them. Whilst this is a fantastic outcome, their main motivation came purely from showing the *existence* of designs. Indeed, as discussed previously, the construction of designs quickly gets complicated and the use of algebraic and geometric techniques is very limited. For example, Wilson [13] in the 1970s notoriously constructed $(n,k,2)$-SSs whenever their existence is not ruled out by the divisibility conditions (the case of STSs was proven already by Kirkman himself!). However in general, (n,k,t)-SSs were not constructed for all possible parameters and a longstanding conjecture of Steiner from 1853 stated they should always exist provided that the divisibility conditions are satisfied[†] and n is sufficiently large (see [12]). The method of construction of Keevash (and likewise the second group of authors) was flexible enough to tackle this and construct designs for all feasible parameters (with n larges enough), which was a huge breakthrough. Indeed before Keevash, not even a single (n,k,t)-SS with $t \geqslant 6$ was known to exist.

These methods opened up completely new vistas for combinatorial designs. Not only could they construct a wide range of different designs but the methods actually construct *many* distinct designs with some fixed set of parameters, thus providing the best known lower bound for the *count* of designs. Recently, these methods have also been used to give designs with special properties, leveraging the control that we can use on the random process to mould our design in a certain desired way. Indeed, Kwan, Sah, Sawhney and Simkin built on the approach of Glöck et al. to construct so-called *high girth* STSs [9], establishing a famous conjecture of Erdős from 1973.

Towards understanding Random Designs. Unfortunately the absorption processes used to construct designs do not give designs that are close to uniform. Indeed, the absorbing structures used are very delicate and so skew the randomness given by the random part of the construction. Although many designs can be constructed this way, it is not even clear that more than a negligible proportion of the set of all designs are given by these methods. Thus Yates' problem of generating designs that are truly random (or close to random) remains elusive.

Nonetheless, somewhat surprisingly, Kwan realised that if certain properties hold with very high probability in the random process, then one can infer that uniformly random designs actually enjoy these properties also. His methods built on those of Keevash and involved providing lower and upper bounds on the number of ways in which a partial design can be completed in order to estimate how far the uniform distribution on designs skews the distribution on partial designs given by the random process. Using this ingenious analysis, he was able to unlock new properties of random designs. He showed that uniformly random STSs contain *perfect matchings* [8]: there is a collection of disjoint 3-sets covering the vertex set of the STS. Further work building on his methods has established more involved properties of STSs as well as LSs [2,10].

Despite these recent developments that were unimaginable before the work of Keevash, it is clear that the study of random combinatorial designs is still very much in its infancy and many beautiful and interesting questions remain wide open, in particular the quest to actually generate random designs and thus have access to the perfect experiment constructions!

The first author of this article will start a project on this topic in April 2024, hosted by the second author of this article and funded by a Marie Curie postdoctoral fellowship awarded by the EU.

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[†]In fact, Steiner conjectured the existence of even more general designs, where one can choose how many times each t-set is covered. These are also given by the methods of Keevash.

Chronicles

Why do we aspire to be second-rate mathematicians when we can be first-rate scientists?

Marta Pézez Casany (EIO/UPC)
Received on 19 September, 2023.

On September 14, 2023, professor G. Geoffrey Vining gave the opening talk of the George Edward Pelham Box school year at the **FME**. The **FME** has dedicated the 2023-24 school year to G. E. P. Box (1919-2013), of whom G. G. Vining was a former student.

The conference opened with an address by the Dean of the **FME**, professor Jordi Guàrdia,[(1)] and continued with the introduction of the speaker by Xavier Tort-Martorell.

The title of professor Vining's lecture was "First-Rate Scientist or Second-Rate Mathematician", which was inspired by the following quote from professor G. P. Box: "Why do we aspire to be second-rate mathematicians when we can be first-rate scientists?"

The lecture began by explaining that G. P. Box, a chemist by training, was a sergeant during the Second World War at a research station, in which the consequences of a chemical war were investigated with animals. As a result of the research he was carrying out, he contacted Sir R. A. Fisher,[(2)] known for being the father of the design of experiments, and later ended up marrying one of his daughters. Almost always, G. P. Box's contributions arose from real problems that appeared as a result of his dedication to improving the quality and productivity of industrial processes. This is how his seminal contributions to the design of experiments and prediction through time series emerged among the most well-known. He was also famous for his quotes, like the one that inspires the lecture we're talking about.

The central part of the lecture versed on the explanation of two NASA projects with which professor Vining has been involved. The first concerned the study of the reliability of carbon-lined vessels through which gas passes at very high pressure, and the second the design of jet turbine engines.

Throughout the lecture the importance of the design of experiments while doing research was highlighted, as well as the need to verify that the assumptions made while defining the models that will be assumed throughout the different analyzes are really acceptable in the environment in which the research is carried out. The speaker ended the lecture by emphasizing the importance of, regardless of our background, considering ourselves scientists in the broadest sense. Research is currently multidisciplinary and requires skills to interact productively with other researchers with very diverse backgrounds.

When asked by a floor participant if he had any suggestions to improve communication between statisticians and researchers, professor Vining mentioned the importance of having scientists who act as a bridge and who have the ability to take decisions.

(1) The NL interviewed him recently: NL04, pp. 6-8.
(2) He was dedicated the **FME** 2012-13 term

Inauguration of the 2023-2024 academic year at the FME

Jaume Franch Bullich (DMAT/UPC, IRI)
Received on 24 October, 2023.

Joan Solà-Morales, Jordi Guàrdia, Gemma Flaquer, and Xavier Cabré.

On October 11th the School of Mathematics and Statistics (**FME**) held the inauguration ceremony of this academic year, 2023-2024, dedicated to George P. Box (see the Chronicle in this issue about the September 14 lecture delivered by professor G. Geoffrey Vining, a former student of G. P. Box).

At the end of the past academic year, three of the five former deans (the first, second and fourth) took retirement. Thus it was a propitious occasion to pay tribute to those former Deans.

The first Dean was Joan Solà-Morales (see also his home page). He and his team launched the brand new **FME** by relying on two important resources: a sizable group of very good professors and a building. He designed a very innovative curriculum for a degree in Mathematics, with a more applied bent than the schools of mathematics existing at that time in the Barcelona area. At the beginning, the curriculum was designed to be completed in 4 years, but a few years later it was found convenient to change it to a 5-year plan. At the start, the school also included a 3-year degree in Statistics. From the very beginning, the **FME** was successful in attracting the best students.

The second Dean, Pere Pascual, consolidated the project, adding a new 2-year degree in Statistics, which could be pursued after completion of either the 3-year degree in Statistics or the first three courses in Mathematics. But he is especially remembered for another initiative: the creation, jointly with the School of Telecommunications, of the first dual degree, a very competitive syllabus allowing selected students to get both degrees (Mathematics and Telecommunications Engineering). This was the seed of what became the Center for Interdisciplinary Studies (CFIS), created in 2003, with Pere Pascual as first director. The creation of **CFIS** allowed the **FME** to increase substantially the attraction of excellent students.

As a consequence of the implementation of the European Higher Education Area, Jordi Quer, the fourth Dean, was in charge of implementing new 4-year bachelor's degrees. This change took place in 2009, when he started his term as a Dean. With the new degrees, and the masters that were created a few years before, the **FME** started an important collaboration with University of Barcelona (**UB**), since the undergraduate and master studies is Statistics are shared between both universities, **UB** and **UPC**.

The ceremony began with an address by the present Dean, Jordi Guardia. He reported on the most important facts from the last academic year and anticipated the more relevant milestones planned for the new academic term. The Dean was assisted in this job by Gemma Flaquer, the Academic Resources' delegate in Guàrdia's team.

The event continued with the introduction by professor Xavier Cabré of the keynote speaker, Joan Solà-Morales (see the Annex at the end of this chronicle). The talk was entitled *Principis Matemàtics de la Mecànica de Fluids* (*Mathematical Principles of Fluid Mechanics*, written up as an Outreach piece in this NL). The lecture was very entertaining and educational, and could be followed by everyone. He emphasized all the mathematics that the students learn during the undergraduate studies in Mathematics that are needed to study Fluid Mechanics (for details, see the Outreach piece in this issue based on that lecture).

After the lecture, the three retired Deans were honored by the Dean and the Rector, professor Daniel Crespo. They received an **FME** pin and a present, and had the opportunity to address a brief speech to the audience. During this homage, a very nice video of selected former and present professors and students was shown. Jordi Guardia also had some words for professor Miguel Muñoz, who was expected to be the fourth Dean, but could not actually run for the election because of health problems.

A tribute to the members of the staff that have been serving FME for more than 25 years concluded this part of the event.

The Rector closed the ceremony emphasizing the excellence of the school and answering the demands expressed by the Dean. He promised some investment in the building in order to accommodate the increasing number of students. He acknowledged and praised the **FME** for having increased the number of students. He also confirmed that new professors will be hired in order to meet the extra demands due to this increment.

After the ceremony, a picture of the six Deans (the four mentioned above plus Sebastià Xambó, the third one, and Jaume Franch, the fifth one) that FME has had through its history was taken. We trust that, in the future, we will be able to take pictures with 7 or more deans!

Right to left: Joan Solà-Morales, Pere Pascual, Sebastià Xambó, Jordi Quer, Jaume Franch, Jordi Guàrdia.

Annex: Introduction of Joan de Solà-Morales by Xavier Cabré (transcription and adapted translation to English by the **NL**)

For me it is a pleasure to present Joan de Solà-Morales, a key figure in Catalan mathematics of the last thirty years. Solà-Morales obtained his doctorate at the **UAB** in 1983, under the direction of Carles Perelló, in the area of *Partial Differential Equations* (PDEs). As early as 1989, he became Full Professor at the **UPC**, and in 1992 he was the main promoter of the creation of this School of Mathematics, the **FME**, and of which he was dean in the first years, from 1992 to 1997. This School, together with the **CFIS**, have caused a very notable improvement, in recent decades, in the quality of mathematical research in Catalonia and Spain. After leaving the job of dean, he was deputy-director of **CRM** from 2007 to 2010 and president of the **SCM** from 2010 to 2014. I will add here that he always had an interest in mathematics outreach and young talent acquisition. As president of the **SCM**, and on a personal level, he was particularly interested in the Kangaroo contest for many years. He has been a full member of the **IEC** since 2012 and is now also an emeritus professor at the **UPC** –luckily for us.

Regarding his research, I would like to give you an idea of his more salient traits. As already said, he obtained his doctorate at the **UAB** in 1983 under the direction of Carles Perelló, in the field of PDEs. Carles Perelló had been the first doctoral student of Jack Kenneth Hale at Brown University (1965: *Periodic Solutions Of Ordinary Differential Equations With And Without Time Lag*). Jack Hale was a very important figure in the field of dynamical systems of infinite dimensions at that time (he supervised 44 doctoral theses). Thus above Joan Solà-Morales we have Carles Perelló and Jack Hale. Below, Solà-Morales has had four PhD students: Marta Valencia, Neus Cónsul, José Antonio Lubary and Marta Pellicer. He has also facilitated the training of other outstanding Catalan mathematicians: Maria Aguareles and Maria Bruna, encouraging them to do their doctorate at Oxford, a place of excellence. To add that Solà-Morales has been the PI of several national projects and that his profile appears in the ArbolMat portal of the **RSME**.

To specify what his research has been, two prominent aspects of his work have to be stressed: the large spectrum of partial differential equations that he has dealt with throughout his career and his unusual vision to raise questions, including conjectures, which have led to outstanding subsequent developments. Indeed, Solà-Morales has important works in the three basic groups of PDEs: fluids, diffusion or heat equations, and wave equations. I know few PDEs specialists of whom I can say this, that is, who have outstanding work in these very different and rather disjoint fields. Let me comment on some of his works.

Fluids: His thesis, which later became a book, *The Navier-Stokes equations in a channel with obstacle*, won the 1983 **IEC**

Prize for PdD thesis. Diffusion or heat equations: His works on boundary reactions stand out; he introduced me, when I became a professor here at the **UPC**, to this topic, and we wrote a joint article, in 2005, which has been influential. He raised two conjectures, also on this subject, which turned out to be related to very hot topics today (the fractional Laplacian) and led to interesting research. One of them remains a conjecture. Neus Cònsul and I disproved the other one, but it has given rise to good research. Wave equations: He has several works in this field, but I will highlight one, because it is recent (from 2019) and already has 44 citations in MathSciNet and 74 in GoogleScholar, which is quite remarkable. It is a work with Marta Pellicer, on the Moore-Gibson-Thompson equation, a dissipative wave equation that appears in acoustics and viscoelasticity.

Apart from these equations, he has been interested in other topics, like equations in networks, geographic maps, financial price formation, or chromatography. Solà-Morales has also a special interest, apart from PDEs per se, in mathematical modeling and in industrial mathematics. In relation to these topics, he visited Oxford to get acquainted with the *Study groups* there and to establish important scientific contacts. Based on that experience, he launched and promoted a form of "Study groups" in Barcelona. The goal was to bring together researchers interested in industrial mathematics and mathematical modeling, on one hand, and companies on the other. The companies proposed the problems and, with their collaboration, the mathematicians tried to solve them. Catalan mathematics has benefited much from this initiative, which still continues today integrated within a larger European framework.

(See also the NL Interview with Joan Solà-Morales in this issue.)

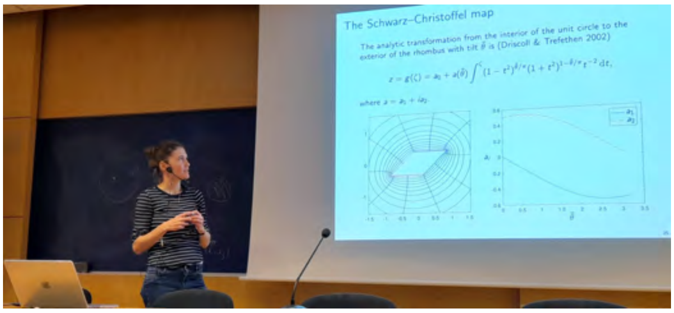
IMTech Fall Colloquium Lecture (29 Novembre 2023)

Speaker: Maria Bruna (University of Cambridge)

Continuum models of strongly interacting Brownian particles.

Report by Gemma Huguet (**DMAT**, **IMTech**)

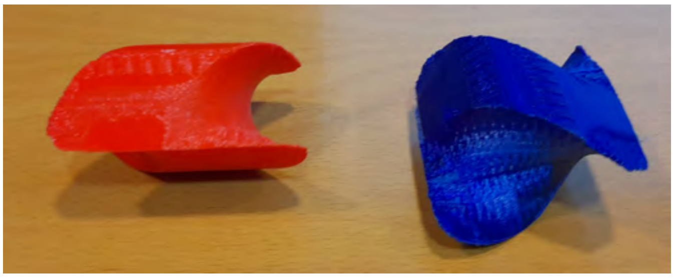
Received on 30 November, 2023.



On November 29, 2023 Professor Maria Bruna (University of Cambridge) delivered the **IMTech** Colloquium Lecture at the Sala d'actes de la Facultat de Matemàtiques i Estadística (FME).

Maria Bruna is Royal Society University Research Fellow at the Department of Applied Mathematics and Theoretical Physics at University of Cambridge, and Fellow of Churchill College, Cambridge. She studied mathematics and industrial engineering as an undergraduate at the CFIS (UPC), completing her studies in 2008. She obtained a PhD in applied Mathematics from the Oxford University in 2012. She has held positions at the University of Oxford, RICAM (Austria), and St John's College (Oxford). Her research interests are stochastic modelling, asymptotic methods and homogenisation techniques in the areas of mathematical biology and industrial mathematics. Her research focuses on methods to capture multiscale phenomena of stochastic systems of interacting particles.

In her talk, Dr Bruna discussed many-particle systems with strong interactions. These models are motivated by the study of many-particle systems in biology or industrial applications, where it is crucial to account for the finite size of particles. She explained how these interactions can be included in the models and different methods to derive continuum PDE descriptions. In the second part of the talk, she showed how these methods can be used to model active matter systems or self-propelled particles such as bacteria or ants, in particular she discussed a system of non-overlapping Brownian needles in $\mathbb{R}^2$ (she presented her most recent results published in PRSA). Starting from the stochastic particle system, she derived a nonlinear nonlocal PDE using matched asymptotics for the evolution of the population density in position and orientation. The method involved the computation of the excluded volume interaction of shaped objects, such as needles, and she showed to the public the printed 3d excluded volume interaction of a needle (see Figure below), which also appeared in the cover of the PRSA.



She considered spatially homogeneous solutions and showed that pure hard core repulsive interactions lead to alignment in angle for large enough density. Finally, in the regime of high rotational diffusion she was able to compare the effective excluded volume of a hard-needles system with that of a hard-spheres system.

The video of the talk is available at Zona video.

Annex: Introduction of Maria Bruna by Joan de Solà-Morales

Maria Bruna Estrach is reputed researcher working in Applied Mathematics, now in a position at the Department of Applied Mathematics and Theoretical Physics of the Cambridge University. Her position is that of a University Research Fellow of the Royal Society, since 2019.

She was born in Sant Cugat del Vallès, and she is well known among us because she studied her two degrees, Mathematics and Industrial Engineering, at the CFIS, in the UPC. She completed these degrees on 2008 and went to the University of Oxford to make a Master on Applied Mathematics and also a PhD, with professor Jon Chapman, at the OCIAM (Oxford Center for Industrial and Applied Mathematics).

After completing her doctorate, Maria Bruna was a postdoctoral researcher in several places: University of Oxford, Johann Radon Institute for Computational and Applied Mathematics in Austria, and again in Oxford, as a junior research fellow in mathematics at St John's College, Oxford, until 2019.

She is spending now a long period in Catalona, working with different people. In particular, she is working now with Gissell Estrada-Rodriguez, from de UPC.

In 2016 Bruna was awarded a L'Oréal-UNESCO Women in Science Fellowship, the first given in mathematics. She is also a 2016 winner of the Aviva Women of the Future Awards. In 2020 the London Mathematical Society gave Bruna a Whitehead Prize "in recognition of her outstanding research in asymptotic homogenization, most prominently in the systematic development of continuum models of interacting particles systems".

Her research, starting with the PhD has been on the random interaction of particles of finite size, and developing continuous models for these phenomena. When the particles are just points, this is the classical approach of diffusion processes, and her new viewpoint also gives rise to Partial Differential Equations, now taking into account the finite size of the particles, and also her interactions with the boundaries, in the confined case, and with different media when this is the case.

A very relevant thing is that she uses asymptotic expansions and matching techniques. These are mathematical methods that have shown to be very useful in many applied problems, and that have not received perhaps enough attention in our mathematical culture.

She has applied her ideas to many applied problems, ranging from biology to industry. Today I think she will told us something about the applications to motion of bacteria or colonies of ants. But she has applied these techniques also to problems in collaboration with industry, like air filtration, designing an optimal porosity profile in order to enhance the lifetime of porosity-graded filters.

Reviews

Review of Andrew Granville's paper [1],
by Marc Noy (**UPC/DMAT**, **IMTech**).

The paper under review addresses a most interesting question: How does the mathematical community accept that a given proof is correct? The author discusses at length Hilbert's program for solving the foundational crisis of mathematics, and how Gödel incompleteness theorems shattered Hilbert's dream. As von Neumann put it in 1930: There can be no rigorous justification for classical mathematics. The author then asks: "how do mathematicians deal with this existential crisis in their subject? The only answer is that they learn to live with it." After that, the author discusses formal computer proofs, and argues that, while certainly useful, they cannot escape the problems posed by Gödel's results. It follows a very enlightening discussion on how proofs are written and reviewed, and on published proofs that contained mistakes and later had to be corrected. This is unavoidable, and the reviewer shares completely the view of robust proofs accepted by the community. Undoubtedly there are published proofs containing mistakes: if the results are relevant and useful for other researchers, the mistakes will most likely be found; if they are not, then there is not much harm in ignoring them.

The author is a renowned number theorist at the University of Montreal. The paper is a pleasure to read and contains many interesting examples and quotations.

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Machine Learning in Pure Mathematics & Theoretical Physics, [1]
edited by Yang-Hui He.
Reviewed by Sebastià Xambó.

Publisher's description: "The juxtaposition of 'machine learning' and 'pure mathematics and theoretical physics' may first appear as contradictory in terms. The rigours of proofs and derivations in the latter seem to reside in a different world from the randomness of data and statistics in the former. Yet, an often under-appreciated component of mathematical discovery, typically not presented in a final draft, is experimentation: both with ideas and with mathematical data. Think of the teenage Gauss, who conjectured the Prime Number Theorem by plotting the prime-counting function, many decades before complex analysis was formalised to offer a proof.

"Can modern technology in part mimic Gauss's intuition? The past five years saw an explosion of activity in using AI to assist the human mind in uncovering new mathematics: finding patterns, accelerating computations, and raising conjectures via the machine learning of pure, noiseless data. The aim of this book, a first of its kind, is to collect research and survey articles from experts in this emerging dialogue between theoretical mathematics and machine learning. It does not dwell on the well-known multitude of mathematical techniques in deep learning, but focuses on the reverse relationship: how machine learning helps with mathematics. Taking a panoramic approach, the topics range from combinatorics to number theory, and from geometry to quantum field theory and string theory. Aimed at PhD students as well as seasoned researchers, each self-contained chapter offers a glimpse of an exciting future of this symbiosis."

From the editor's preface, we quote: "Not only is machine learning used as a tool for speeding up numerical computations that lie at the core of problems from combinatorics to number theory, from geometry to group theory, etc., more importantly, it is helping with the pattern recognition that forms the heart of conjecture formulation.

"With an initial attempt to summarize the progress in, and to advocate the necessity of, machine learning in geometry, especially in the context of string theory [2], I speculated — borrowing terminologies from physics— that mathematics and AI could be in conjunction in two complementary ways [3]. In (1) 'Bottom-up Mathematics', one builds theorems and proofs line by line, using type-theoretic computer languages such as Lean. This is the automated theorem proving program, which has had a distinguished history since the 1960s, and with recent proponents such as K. Buzzard, H. Davenport, et al. (q. v. ICM

2022 addresses by K. Buzzard and by G. Williamson). In (2) 'Top-down Mathematics', one creates pure, noiseless datasets of various mathematical structures, such as representations of finite groups, the arithmetic of algebraic curves and the types of superstring vacuum solutions. Such data are then fed into various machine learning algorithms, supervised and unsupervised, in order to find new patterns, raise new conjectures or find alternative exact formulae.

"It is this second direction of 'Top-down Mathematics' that has been gaining attention and momentum in the last few years and thereto is this present volume devoted. Organized alphabetically by the contributing author, this unique collection of research and survey article is, I believe, the first of its kind in bringing together experts in pure mathematics and theoretical physics, to present their ideas on how machine learning is used to understand the pertinent data and to speed up computation, to formulate new conjectures and to gain intuition on the underlying structure.

"As one could imagine, there is a vast expanse of disciplines which produces the mathematical data, and these will be discussed in detail: explorations in the string landscape, Calabi–Yau manifolds, combinatorics of polytopes, the arithmetic geometry of curves, the number theory of the Birch–Swinnerton-Dyer conjecture, volumes of Einstein manifolds, etc.

"Conversely, one might also wonder how ideas from modern mathematics and physics help with understanding machine learning. Indeed, this is also an emerging field which is destined to prove fruitful. In this volume, we have perspectives from theoretical physicists on inferencing as a dynamical system, as well as correlation functions as a handle on symmetries in neural networks. Finally, since machine learning is also taking over natural language processing, it is natural and expedient to perform meta-studies where one analyzes the language of theoretical physics and mathematics; this is the subject of one of the chapters.

"I sincerely hope that this volume offers the first glimpse onto a fertile land, a cross-disciplinary world of mathematicians, physicists and computer scientists. This nascent collaboration between machine learning and pure mathematics as well as theoretical physics, a taste of the spirit of which we hope to capture here, will undoubtedly continue to flourish."

Contributors: Laura Alessandretti, Andrea Baronchelli, David Berman, Tom Coates, Andrei Constantin, Harold Erbin, Riccardo Finotello, James Halverson, Jonathan Heckman, Johannes Hofscheier, Vishnu Jejjala, Alexander Kasprzyk, Marc Klinger, Anindita Maiti, Brent Nelson, Thomas Oliver, Rak-Kyeong Seong, Keegan Stoner.

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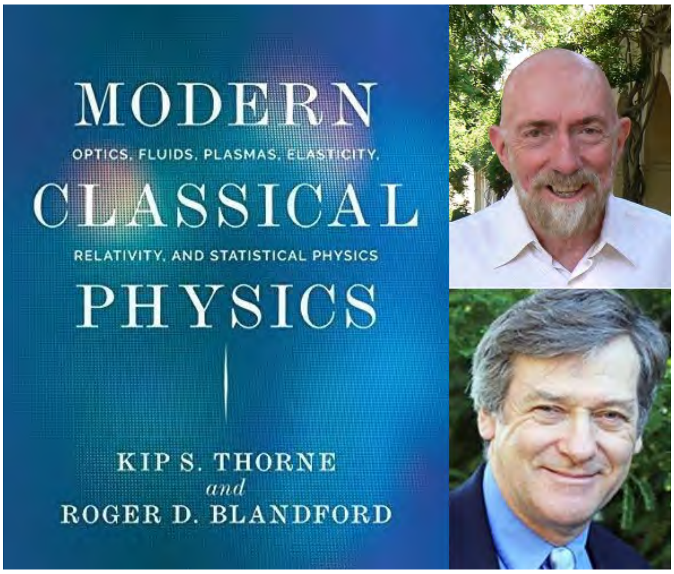
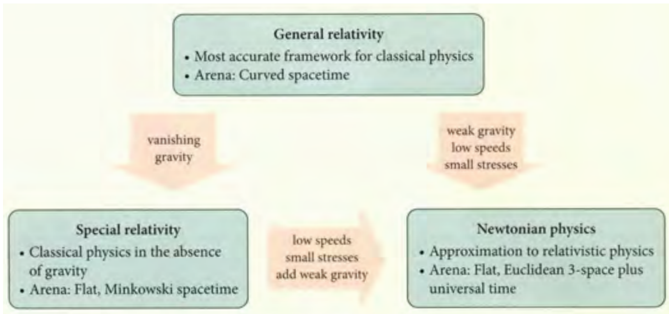
Modern Classical Physics (xl+1511 pages)
by Kip S. Thorne and Roger D. Blandford.
Reviewed by Sebastià Xambó.



The Classical Physics in the title refers, roughly speaking, to macroscopic physics (so there is no chapter devoted specifically to Quantum Physics), while the Modern qualification refers primarily to the extensive presentation of materials devoted to contemporary developments and applications, but also to the systematic use of intrinsic geometric methods. Actually, as stated at the beginning of Part I (Foundations): "In this book, a central theme will be a *Geometric Principle: The laws of physics must all be expressible as geometric (coordinate-independent and reference-frame-independent) relationships between geometric objects (scalars, vectors, tensors,...) that represent physical entities*".

The authors classify the classical laws of physics and their relationships as follows (Fig. 1 in the text):



The book spans 28 chapters collected in seven parts: Foundations (2 chapters: the first on Newtonian Physics and the second on Special Relativity); Statistical Physics (chapters 3-6; chapter 5 covers Statistical Thermodynamics and chapter 6 is devoted to Random Processes); Optics (chapters 7-10); Elasticity (chapters 11-12); Fluid Dynamics (chapters 13-19); Plasma Physics (chapters 20-23); and General relativity (chapters 24-28, the last on Cosmology).

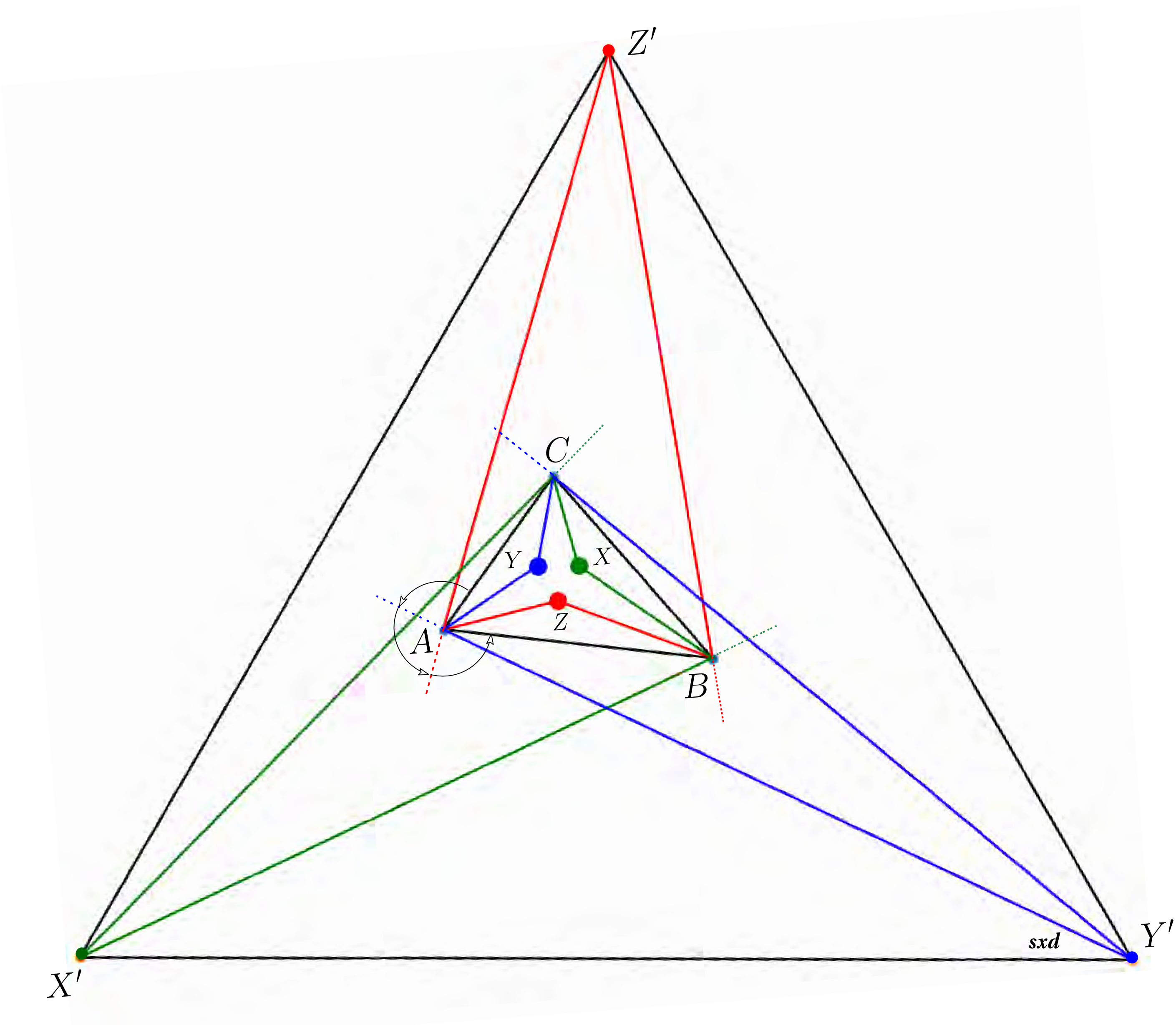
Although Quantum Physics is not treated specifically (it is non-classical), its language is briefly explained in different places (for example: pages 165-166 for Quantum Statistical Mechanics; 174-175, for quantum states of a single particle and of many particles; pages 194-195 for the Bose-Einstein condensate; section 23.3 for 'plasmons') to set up bridges with relevant classical materials: "In our journey, we seek to comprehend the fundamental laws of classical physics in their own terms, and also in relation to quantum physics" (page xxxii); "classical physics should *not* be studied in isolation from quantum mechanics and its modern applications" (page xxxiv); and "Classical physics is sometimes used, pejoratively, to suggest that 'classical' ideas were discarded and replaced by new principles and laws. Nothing could be further from the truth. The majority of applications of physics today are still essentially classical. This does not imply that physicists or others working in these areas are ignorant or dismissive of quantum physics. It is simply that the issues with which they are confronted are mostly addressed classically. Furthermore, classical physics has not stood still while the quantum world was being explored. In scope and in practice, it has exploded on many fronts and would now be quite unrecognizable to a Helmholtz, a Rayleigh, or a Gibbs. In this book, we have tried to emphasize these contemporary developments and applications at the expense of historical choices, and this is the reason for our seemingly oxymoronic title, *Modern Classical Physics*" (pages xxxi, xxxii).

The book evolved from graduate courses taught by the authors over decades in Caltech and, to a lesser extend, in Stanford. As prerequisites, the authors mention an undergraduate-level command of classical mechanics, electromagnetism, thermodynamics, and applied mathematics. As observed by Edward Witten in [1], "The present work is more straightforward in tone and approach than [2], though in spots you'll see an attenuated version of the flair and exuberance for which *Gravitation* is known. [. . .] Given world enough and time, most of us would do well to put everything else aside for a couple of months, study *Modern Classical Physics* systematically, and come back with our knowledge well refreshed. Short of that, we could satisfy our curiosity—or possibly pique it further—on many topics. And certainly, many of us would appreciate this book as a reference. On the whole, Modern Classical Physics is a magnificent achievement".

References

[1] E. Witten, A hefty textbook for an ever-changing discipline, Physics Today **71** (2018), no. 10, 52-53. https://doi.org/10.1063/PT.3.4046.

[2] Charles W. Misner, Kip S. Thorne, and John A. Wheeler, *Gravitation*, Freeman San Francisco, CA, 1973.



Inner and outer Morley triangles (XYZ and $X'Y'Z'$) of a triangle ABC.

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