Universitat Politècnica de Catalunya ▪ BarcelonaTech

# IMTECH 5*d*

# Newsletter

Issue 5, Release 5*d*, June 26, 2023

## Interviews

◇ ADRIAN PONCE

## Research focus

◇ MARINA VEGUÉ

***Reducing the dynamics of a large interacting system.***

## PhD highlight

◇ FRANCO COLTRARO

## Outreach

◇ ANDREU MASDEU AND JOSÉ L. MUÑOZ (**Basetis**)

◇ RAMON EIXARCH (**Wiris**)

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# Issue 5, Advances 5*d*, June 2023

нет войне – No War!

## Contents



## Editorial

This issue, NL05d, contains June advances of the NL05 (January-August 2023).

It features the following pieces:

- An Interview with Adrián Ponce Álvarez, who has a Ramón y Cajal contract at the UPC since September 2022.
- A research focus by Marina Vegué (DMAT) on Reducing the dynamics of a large interacting system.
- A PhD highlight about Franco Coltraro's thesis *Robotic manipulation of cloth: mechanical modeling and perception*.
- An Outreach by Andreu Masdeu and José Luis Muñoz (both from **Basetis**) with the title Research and Development at **Basetis**: The strength of blending Mathematics and Artificial Intelligence.
- An Outreach by Ramon Eixarch (**Wiris** CEO): Enhancing STEM Learning: online solutions for Education.

The NL is most thankful to the authors of these contributions.

The full NL05 will be published in July. It will assemble the four advances published so far (February, April, May and June) together with other materials that are expected to arrive within the first three weeks of July.

Adrian Ponce Alvárez started with a Ramón y Cajal contract at the UPC in September 2022. His research is focused on the study of neural networks, at different scales and in different states. He uses a combination of data analysis and theory, by means of stochastic dynamical systems, information theory, statistical mechanics, machine learning, and network theory.

He completed his master's degree in Physics at University of Paris-Sud 11 in 2006 and did his PhD in Neurosciences in Aix-Marseille University, France. After finishing his PhD in 2010, he was a postdoctoral fellow at Gustavo Deco's lab, after which he moved to the Departament de Matemàtiques at UPC to start his current position.

NL. ***Welcome to the UPC, and congratulations for the Ramón y Cajal contract. Could you comment on what features of UPC contributed to your decision to come here with your RyC?***

Thank you, I am happy to be at the UPC and to be able to continue my research in the framework of the Ramón y Cajal. I highly value the commitment that the UPC has with scientific research and, before coming here, I already knew the theoretical neuroscience research groups that work in the Dynamical Systems Group of the Department of Mathematics at the UPC, specifically Gemma Huguet and Antoni Guillamon.

***Your master's degree in Physics of Biological Systems at the University of Paris-Sud suggests a strong interdisciplinary mix. Was it decisive in your research orientation? What was the topic of your master thesis?***

During my master's in Physics of Biological Systems I learned tools to study biological systems as interacting systems with emergent properties. That master focused mainly on dynamical systems and statistical physics. During those years, I devoured the book Mathematical Biology by James Murray. This background has allowed me to ask neuroscientific questions from a system perspective. My Master thesis was about resonance phenomena in a single-cell model, i.e., a leaky integrate-and-fire model with hyperpolarization-activated currents, supervised by Claude Meunier (Paris Descartes University).

***What was the topic of your PhD thesis? What are your recollections of that research period?***

During my PhD at Aix-Marseille University (funded by a personal PhD grant from the Mexican Government), I studied the neuronal activity during delayed motor preparation and decision-making, in Alexa Riehle's laboratory (CNRS). For this, I trained monkeys to perform cognitive tasks and recorded the neuronal activity in several cortical areas, as well as the animal's behavior. Due to this experience, I learned to design and conduct behavioral experiments, perform extracellular recordings in vivo, and analyze and interpret neuropsychological data. I used tools from information theory and machine learning to analyze the recorded data. Acquiring some experience on experimental neuroscience has been extremely useful to develop my research on theoretical neuroscience and to stablish collaborations with experimentalists.

***Let us move forward to your postdoctoral position in Deco's lab. What aspects of those stays would you like to highlight?***

My interest in the mechanisms underlying brain function led me to transit towards theoretical neuroscience. In 2011, I joined the group of Gustavo Deco (UPF), as a postdoctoral fellow, to study neural networks at the microcircuit and whole-brain levels. To study these models, I acquired expertise in stochastic dynamical systems, graph theory, and information theory. I used this knowledge to investigate how network statistics emerge and affect information processing. In a series of works, we studied the emergence of state-dependent neural interactions (among neurons or among brain regions) in different brain states, task conditions, diseases, and neuromodulatory processes, and their functional consequences in terms of information capacity and communication. Some of these models were included in The Virtual Brain, an open-source platform for whole-brain simulations constraint by biological connectivity data (*connectomes*) to test both scientific and clinical questions.

Along with the study of dynamical systems to understand neural systems, I studied these systems with a statistical mechanics approach, which has proven to be useful to link collective behavior in high-dimensional systems, model inference, and information. By inferring models from neuronal activity, we derived macroscopic properties that quantify the system's capabilities to process and transmit information. By studying these inferred models, together with simulations of neural networks, we have showed that cortical/brain states relate to phase transitions. Of particular interest for information processing, we found evidence of critical behavior in neural systems.

Gustavo Deco has supported me a lot, both scientifically and personally, allowing me to develop my own ideas.

***You have collaborated with researchers from different disciplines, particularly biology and medicine. How has been the experience?***

I truly admire experimental scientists. I'm always impressed by their intuitions about the mechanisms behind the biological phenomena they study. Nowadays experimental neuroscientists have a good understanding of mathematical models and, thus, in an authentic collaborative work, where time is invested, a fruitful dialogue between experiments and theory can be established. In recent years, I had the opportunity to participate in the design of experiments to answer theoretical questions.

***Which are in your view the main open questions in theoretical neuroscience? How mathematics can contribute?***

There are several open questions, but in my opinion, an interesting one is how neural systems self-organize during development and during interaction with the environment. We need analytic tools and models to understand how such a high-dimensional system, as the brain, self-organizes in interaction with a changing environment.

***What vision do you have for your research during the RyC contract? What would you like to achieve?***

In the following years, I would like to lead a research group to pursue my research on statistical mechanics and dynamical systems applied to study empirical and theoretical neural networks to answer fundamental neuroscience questions, in collaboration with national and international research institutions.

***In the mathematical ecosystem of the UPC, there is a wide room for teaching subjects in several studies offered by the FME at various levels. Can you share how do you see your role in that respect for the next few years?***

I'm really enjoying teaching within the Mathematical Models in Biology course of the Master's in Advanced Mathematics and Mathematical Engineering (MAMME). For me, mathematical biology has been always fascinating. I think it conveys a mix of esthetic experience, intellectual challenge, and social relevance. I would like to transmit this drive to the students.

***You are strongly committed in favour of gender, race, and class equity in Academia. How do you see the current situation? How can we have more women and minorities involved in mathematics?***

I consider it necessary to conduct research and teach from an inclusive-conscious perspective that seeks to include social minority groups in academia and to consider the mechanisms that could exclude these groups. I have co-organized meetings to discuss scientific papers on gender, race, sexual identity, and class equity in academia (Gender and Science Journal Club). Through this group, we have successfully convinced research centers to implement gender balanced policies and to confront unconscious biases. We have organized several PhD seminars on gender issues at the UPF-DTIC PhD program and we have been invited to round tables on gender-equity and equality plans at several research institutions in Barcelona, but also outside the academia, which is very important. Furthermore, I was member of the Center for Gender Studies (CEdGE) at the UPF since its foundation in 2015 until 2022. The CEdGE is a think tank that creates multidisciplinary knowledge and interdisciplinary research on gender by exploring the complex interactions between gender and class, identity, ethnicity, sexuality or functional diversity.

I think it is very important that discussions on discrimination within the academia also happens outside the academia and not only in the closed environment of the University.

***We hope that you will enjoy your contract and that it will turn out to be a most positive time for your academic career. We also wish you good luck!***

Thank you!

## Research focus

**Reducing the dynamics of a large interacting system**,
by Marina Vegué (DMAT)
*Received June 20, 2023*

The natural world is full of large systems of interacting units. A human brain, for example, contains around 86 billion ($86 \cdot 10^9$) neurons whose activities evolve in time as a function of the activities of the neurons they are connected to. An ecosystem can also be regarded as an interacting network in which the units are the different species and their abundances are variables that depend on the abundances of the species they interact with. A social network is another environment through which information, opinion or diseases can spread or disappear depending on the structure of its interactions.

Systems as the ones defined by these examples are usually dubbed "complex", a broad term that is used to emphasize the fact that their behavior is often difficult to study and predict. A natural question to ask is whether a generic complex system composed of $N$ interacting units or nodes (e.g., neurons, species, individuals) can be reduced to a simpler system composed of $n \ll N$ units. In other words: if one can construct a smaller system that is easier to treat and understand while preserving some of the characteristic properties of the original system. Imagine an ecosystem in which the species' abundances are relatively stable in time. These abundances may change smoothly as some perturbation is introduced, for example a change in the average annual temperature, which alters the capacity of some species to survive or reproduce. If the perturbation is too strong, however, this equilibrium could disappear and the system might abruptly drift to a state characterized by a partial or total species extinction. It is thus important to know what is the critical temperature at which such a catastrophe occurs, and we would like a reduced version of the system to have an analogous transition at roughly the same temperature. Being able to construct such a reduced system is not only useful for practical reasons (i.e., simulating it and predicting its behavior using less resources) but also from a purely theoretical point of view, because in doing so we can learn about the inner workings of the original system and the reduced dimension $n$ is a measure of the "effective complexity" that characterizes it.

In 2016, Gao, Barzel and Barabási [1] proposed a way to reduce a system on $N$ nodes whose activity evolves in time according to

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} w_{ij}\, g(x_i, x_j), \tag{1}$$

where $x_i = x_i(t)$ is the activity of node $i$ at time $t$ and $w_{ij} > 0$ stands for the weight of the positive interaction from node $j$ to node $i$. Functions $f$ and $g$ define the nodes' self-dynamics and the dynamical coupling between pairs of nodes, respectively.

They proposed to reduce the system to a system of dimension $n = 1$ whose only *effective* (or *macroscopic*) activity $\mathcal{X}_{\text{eff}}$ is a linear combination of the activities in the original system according to

$$\mathcal{X}_{\text{eff}} = \sum_{i=1}^{N} a_i\, x_i, \qquad a_i = \frac{w_i^{\text{out}}}{\sum_{j=1}^{N} w_j^{\text{out}}} = \frac{\sum_{j=1}^{N} w_{ji}}{\sum_{j,k=1}^{N} w_{kj}}. \tag{2}$$

The quantity $w_i^{\text{out}}$ is the (weighted) out-degree of node $i$. Eq. (2) thus states that the effective activity is a weighted average of the microscopic activities according to a positive vector $\boldsymbol{a} = (a_i)_{i=1}^N$ with sum 1 whose entries are proportional to the nodes' out-degrees. The authors showed that $\mathcal{X}_{\text{eff}}$ approximately follows an ODE which has the same structure as the original dynamics:

$$\dot{\mathcal{X}}_{\text{eff}} \approx f(\mathcal{X}_{\text{eff}}) + \mathcal{W}_{\text{eff}}\, g(\mathcal{X}_{\text{eff}}, \mathcal{X}_{\text{eff}}), \tag{3}$$

the effective interaction weight $\mathcal{W}_{\text{eff}}$ being the weighted average, according to the same vector $\boldsymbol{a}$, of the in-degrees:

$$\mathcal{W}_{\text{eff}} = \sum_{i=1}^{N} a_i\, w_i^{\text{in}} = \sum_{i,j=1}^{N} a_i\, w_{ij}. \tag{4}$$

They argued that such a one-dimensional reduction can capture the tipping points of several complex systems as different parameters are perturbed.

Despite some systems may be effectively reduced this way, it is likely that other systems will not be properly represented by a system of dimension just one. So can we generalize this result in some way? Can we construct a set of $n \ll N$ linear effective variables —called *observables* from now on— and a reduced matrix of interactions $\boldsymbol{\mathcal{W}} = (\mathcal{W}_{\nu\rho})_{\nu,\rho=1}^n$ whose dynamics, at least approximately, has the same form as the original one? In what follows I will outline a possible way to that end (exposed in detail in Ref. [2]), assuming that the original system's evolution is given by Eq. (1). Fig. 1 provides a visual sketch of the process.

Imagine that we start by partitioning the units in our network according to their connectivity properties so that nodes within a group connect similarly to nodes in other groups. A neuronal network, for instance, might contain functional modules composed of neurons that are densely connected among them but sparsely connected to neurons in other modules. In an ecological network, species that exploit similar niches (and thus interact with other species in a similar way) could be grouped together. If we do not have any information about the presence of modules in the network, we could use a community-detection algorithm on the matrix of interactions $\boldsymbol{W} = (w_{ij})_{i,j=1}^N$ and try to identify them.

Our reduced system will have as many variables as the number of groups in the partition, $n$. We denote by $G_1, \cdots, G_n$ these groups. We require the $\nu$-th observable to be a weighted average of the activities within group $G_\nu$, that is, denoting by $m_\nu = |G_\nu|$ the size of $G_\nu$, we want to find a non-negative vector $\boldsymbol{a}_\nu = (a_{\nu i})_{i=1}^{m_\nu} \in \mathbb{R}^{m_\nu}$ with sum 1 and construct from it the observable associated to $G_\nu$,

$$\mathcal{X}_\nu = \sum_{i=1}^{m_\nu} a_{\nu i}\, \widehat{x}_{\nu i}, \tag{5}$$

where $\widehat{\boldsymbol{x}}_\nu = (\widehat{x}_{\nu i})_{i=1}^{m_\nu} = (x_j)_{j \in G_\nu}$ is the vector of activities of nodes in group $G_\nu$. We call the vectors $\boldsymbol{a}_1, \cdots, \boldsymbol{a}_n$ the *reduction vectors*.

The next step is to write down the exact ODE for the evolution of the $\nu$-th observable, which is a function of $f(\widehat{x}_{\nu i})$ and $g(\widehat{x}_{\nu i}, \widehat{x}_{\rho j})$ for $i \in \{1, \cdots, m_\nu\}, \rho \in \{1, \cdots, n\}$ and $j \in \{1, \cdots, m_\rho\}$, and, generically, cannot be expressed as a function of the observables only. To approximately close the observables' dynamics, we hypothesize that, if the node partition is defined properly, nodes within the same group will

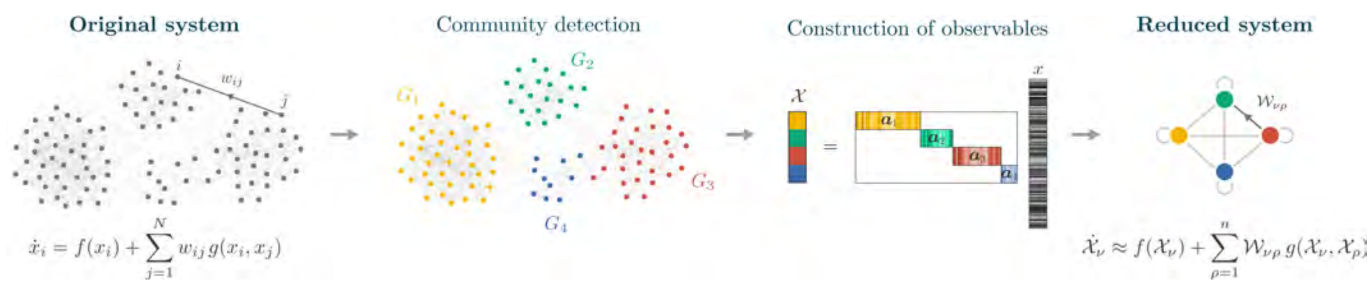


Figure 1: Schematics of the dimension-reduction strategy.

have similar activities. Thus, these activities will be close to the corresponding observable and it makes sense to use first-order Taylor approximations of $f(\widehat{x}_{\nu i})$ and $g(\widehat{x}_{\nu i}, \widehat{x}_{\rho j})$ around $\mathcal{X}_\nu$ and $(\mathcal{X}_\nu, \mathcal{X}_\rho)$, respectively. In doing so, we obtain an approximate system of ODEs that will not be closed unless the reduction vectors fulfill the following conditions:

$$\boldsymbol{K}_{\nu\rho}\, \boldsymbol{a}_\nu = \mu_{\nu\rho}\, \boldsymbol{a}_\nu \tag{6a}$$

$$\boldsymbol{W}^T_{\nu\rho}\, \boldsymbol{a}_\nu = \lambda_{\nu\rho}\, \boldsymbol{a}_\rho, \tag{6b}$$

where $\nu, \rho \in \{1, \cdots, n\}$, $\boldsymbol{K}_{\nu\rho} = \operatorname{diag}(w_1^{\text{in},\rho}, \cdots, w_{m_\nu}^{\text{in},\rho})$ is the $m_\nu \times m_\nu$ diagonal matrix whose diagonal is the vector of in-degrees of nodes in $G_\nu$ taking into account connections coming from $G_\rho$ only, and $\boldsymbol{W}_{\nu\rho}$ is the $m_\nu \times m_\rho$ matrix of interactions from nodes in $G_\rho$ to nodes in $G_\nu$. The scalars $\mu_{\nu\rho}$ and $\lambda_{\nu\rho}$ are additional parameters to be found. Eqs. (6a),(6b) are called the *compatibility equations*. They impose constraints on the reduction vectors.

Once the compatibility equations are fulfilled, the approximate reduced dynamics has the form

$$\dot{\mathcal{X}}_\nu \approx f(\mathcal{X}_\nu) + \sum_{\rho=1}^{n} \mathcal{W}_{\nu\rho}\, g(\mathcal{X}_\nu, \mathcal{X}_\rho), \quad \mathcal{W}_{\nu\rho} = \sum_{i=1}^{m_\nu} a_{\nu i}\, w_i^{\text{in},\rho}. \tag{7}$$

The matrix $\boldsymbol{\mathcal{W}} = (\mathcal{W}_{\nu\rho})_{\nu,\rho=1}^{n}$ is thus the *effective interaction matrix* in the reduced system. Notice that both the approximate reduced dynamics and $\boldsymbol{\mathcal{W}}$ have the same form as Eqs. (3) and (4) proposed by Gao et al., for an arbitrary $n$. The difference with their approach is the construction of the observable vectors, as we will show now.

In our approach, we must solve the compatibility equations to find the reduction vectors. If the nodes in $G_\nu$ have similar connectivity properties, their in-degrees from $G_\rho$ will be close to one another and $\boldsymbol{K}_{\nu\rho}$ will be close to a multiple of the identity matrix so Eq. (6a) can be assumed to be automatically fulfilled. We thus concentrate in solving Eq. (6b) for all $\nu, \rho$, that is,

$$\boldsymbol{W}^T_{\nu\rho}\, \boldsymbol{a}_\nu = \lambda_{\nu\rho}\, \boldsymbol{a}_\rho, \qquad \nu, \rho \in \{1, \cdots, n\}. \tag{8}$$

For $n = 1$, this is a single equation and it states that the vector $\boldsymbol{a}_1$ must be an eigenvector of the transposed connectivity matrix $\boldsymbol{W}^T$. If this matrix is positive, since we ask the reduction vector to be non-negative, then it has to be the dominant, or Perron-Frobenius, eigenvector of $\boldsymbol{W}^T$, normalized to have sum 1. Despite in some particular situations the normalized dominant eigenvector and the vector of normalized out-degrees proposed by Gao et al. coincide, in general they are not the same.

For $n > 1$, Eqs. (8) are coupled, but we can decouple them using, again, the Perron-Frobenius theorem when $\boldsymbol{W}$ is a positive matrix. In this case, Eqs. (8) are equivalent to the following decoupled equations:

$$\boldsymbol{W}'_{\nu\rho}\boldsymbol{a}_\nu = \lambda'_{\nu\rho}\boldsymbol{a}_\nu \qquad \nu, \rho \in \{1, \cdots, n\},$$
$$\boldsymbol{W}'_{\nu\rho},\, \lambda'_{\nu\rho} := \begin{cases} \boldsymbol{W}^T_{\nu\nu}, & \lambda_{\nu\nu} \quad \text{if } \nu = \rho, \\ \boldsymbol{W}^T_{\rho\nu}\boldsymbol{W}^T_{\nu\rho}, & \lambda_{\rho\nu}\lambda_{\nu\rho} \quad \text{if } \nu \neq \rho. \end{cases} \tag{9}$$

For a fixed $\nu$, Eqs. (9) state that $\boldsymbol{a}_\nu$ has to be, simultaneously, the (normalized) dominant eigenvector of a collection of $n$ positive matrices. Of course, such a vector will not exist in general because these matrices will not share their dominant eigenspace. To approximately solve the problem, we propose to set $\lambda'_{\nu\rho}$ to the dominant eigenvalue of $\boldsymbol{W}'_{\nu\rho}$ (this is what it would be if there were an exact solution) and then find the vector $\boldsymbol{a}_\nu$ that minimizes the quadratic error associated to Eqs. (9). We call this dimension-reduction strategy the *spectral reduction*.

Fig. 2 shows an application of the spectral reduction to a susceptible-infected-susceptible (SIS) infectious dynamics on a friendship network obtained from Facebook contacts, where nonexisting interactions were assumed to have an arbitrary small value [2]. In this example every unit's activity represents the probability of the node being infected. We plot the value of the average observable at equilibrium as we increase, along the $x$-axis, a global factor that multiplies all the interactions. There is a point from which the disease cannot be eradicated: the average observable becomes different from zero. This critical point is well predicted by the $n = 1$ spectral reduction, and it is better captured than when the reduction vector is either homogeneous or proportional to the vector of out-degrees. Moreover, the spectral diagram becomes more accurate as $n$ increases.

What is the interpretation of the spectral reduction vectors for $n > 1$? From the decoupled compatibility equations [Eqs. (9)], we know that $\boldsymbol{a}_\nu$ has to be the dominant eigenvector of a collection of matrices. The first matrix is $\boldsymbol{W}^T_{\nu\nu}$, whose entries are the weights of the inverted interactions from nodes in $G_\nu$ to nodes in $G_\nu$. The other matrices are $\boldsymbol{W}^T_{\rho\nu}\boldsymbol{W}^T_{\nu\rho}$ for every $\rho \neq \nu$, whose entries represent the weight sum of all the inverted 2-step paths from nodes in $G_\nu$ to nodes in $G_\nu$ going through nodes in $G_\rho$. The $i$-th component of the (normalized) dominant eigenvector of a non-negative matrix is the so-called "eigenvector centralitiy" of node $i$ in the corresponding network: a measure of the relative importance of the node in the network (a variant of which is used by Google to rank web pages upon a search, for example).

In summary, our findings suggest that in order to reduce the dynamics by means of grouping nodes and defining linear observables within the groups, the contribution of each node to its corresponding observable should be the relative eigenvector centrality of the node within that group, with this centrality taking into account both direct (1-step) and indirect (2-step) interactions via nodes in other groups.

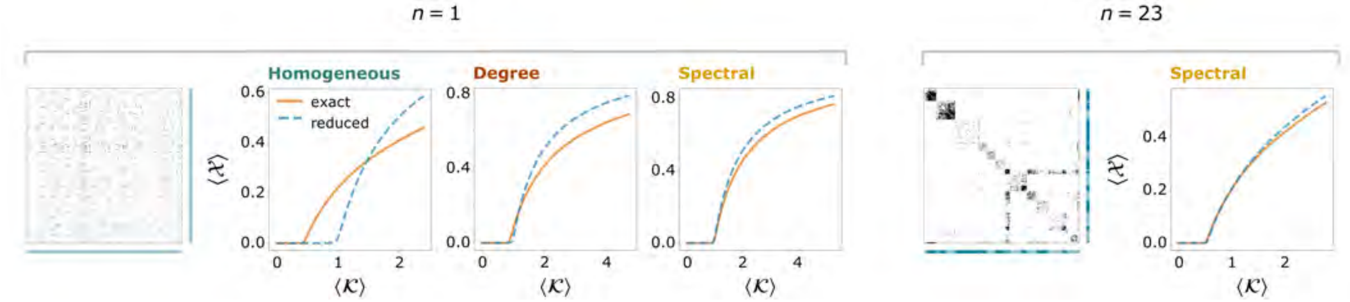


Figure 2: Dimension reduction results for a susceptible-infected-susceptible (SIS) infectious dynamics given by Eq. (1) with $f(x) = -x$ and $g(x, y) = (1 - x)y$ on a network based on Facebook contacts with $N = 362$ nodes [2]. We plot the average observable at equilibrium, $\langle \mathcal{X} \rangle$, as a function of the average weighted in-degree of the reduced system, $\langle \mathcal{K} \rangle$, (both of them weighted by the relative sizes of the different groups) for the exact and the reduced dynamics. The parameter $\langle \mathcal{K} \rangle$ increases as a consequence of multiplying all the interaction strengths by a global factor. For $n = 1$ (i.e., when considering the whole network as a unique group), we compare three reductions: a homogeneous reduction in which all the nodes contribute equally to the observable (left), the degree-based reduction defined by Gao et al. [1] (middle), and our spectral reduction (right). The last diagram shows the spectral reduction results when the nodes have been partitioned into $n = 23$ groups.

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# PhD highlights

Franco Coltraro defended his PhD thesis *Robotic manipulation of cloth: mechanical modeling and perception*, supervised by Professors Jaume Amorós and Maria Alberich-Carramiñana, on March 30th, 2023 within the UPC doctoral program in Applied Mathematics. Currently, he is a postdoctoral researcher at the Institut de Robòtica i Informàtica Industrial (IRI), CSIC-UPC in Barcelona.

### Thesis summary

We introduce a new cloth model for the dynamics of textiles as inextensible surfaces. This assumption challenges most models in literature where elasticity is allowed, sometimes by necessity (Textile Engineering) or in the pursuit of spectacularity (Computer Graphics). Inextensibility is modeled as follows: we assume that our cloth $S$ is a surface (with boundary) moving through space whose metric (first fundamental form) is preserved. In order to implement these conditions (which are in fact PDE's) on a computer, we assume that $S$ has been triangulated (or quadrangulated) and then apply a novel and non-trivial Finite Element discretization to the inextensibility constraints [1]. If we denote by $\varphi(t)$ the position of the nodes of the polyhedron, this gives raise to a smooth (actually quadratic) constraint function $\mathbf{C}(\varphi) = 0$ which must be preserved at all times. Making use of Signorini's contact model, the dynamic equations of motion then would be:

$$\begin{cases} \mathbf{M}\ddot{\varphi} = \mathbf{F}(\varphi, \dot{\varphi}) - \nabla\mathbf{C}(\varphi)^\intercal \boldsymbol{\lambda} + \nabla\mathbf{H}(\varphi)^\intercal \gamma, \\ \mathbf{C}(\varphi) = 0, \\ \mathbf{H}(\varphi) \geqslant 0, \quad \gamma \geqslant 0, \quad \gamma^\intercal \cdot \mathbf{H}(\varphi) = 0, \end{cases} \tag{10}$$

where we have grouped in the force term $\mathbf{F}$ damping, gravity, stiffness, aerodynamics, friction, etc. On the other hand, $\mathbf{H}(\varphi) \geqslant 0$ contains the implicit equation of a given obstacle (e.g. a table) and in addition self-collision constraints. Since the collision and inextensibility forces turn out to be very stiff, the system must be integrated implicitly. This is done by solving a sequence of quadratic problems with linear constraints (basically by linearizing all non-linearities in (10)). In order to solve these quadratic programs efficiently, we develop a novel active-set numerical algorithm which takes into account which constraints were active from one iteration to the next [2]. To our knowledge, our method is the first that results in a non-decoupled resolution of contacts, friction and inextensibility for cloth simulation in a single pass.

The inextensibility assumption is shown to be realistic by comparing simulations to experimental data: we record in a laboratory setting —with depth cameras and motion capture systems— the motions of seven types of textiles (including e.g. cotton, denim and polyester) of various sizes and at different speeds and end up with more than 80 recordings. The scenarios considered are all dynamic and involve rapid shaking and twisting of the textiles, collisions with frictional objects and even strong hits with a long stick. Then we compare the recorded textiles with the simulations given by our inextensible model and find that on average the mean error is of the order of 1 cm even for the largest sizes (DIN A2) and the most challenging scenarios [1, 2].

Furthermore, we also tackle other relevant problems to robotic cloth manipulation such as cloth perception and classification of its states. We present a reconstruction algorithm based on Morse theory that proceeds directly from a point-cloud to obtain a cellular decomposition of a surface with or without boundary: the results are a piecewise parametrization of the cloth surface as a union of Morse cells. From the cellular decomposition, the topology of the surface can be then deduced immediately [3]. Finally, we study the configuration space of a piece of cloth: since the original state of a piece of cloth is flat, the set of possible states under the inextensible assumption is the set of developable surfaces isometric to a fixed one. We prove that a generic simple, closed, piecewise regular curve in space can be the boundary of only finitely many developable surfaces with nonvanishing mean curvature [4]. Inspired by this result we introduce the dGLI cloth coordinates, a low-dimensional representation of the state of a piece of cloth based on a directional derivative of the Gauss Linking Integral. These coordinates —computed from the position of the cloth's boundary— allow us to distinguish key qualitative changes in folding sequences [5].

**Selected Publication**: [1].

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# Outreach

**Research and Development at Basetis: The strength of blending Mathematics and Artificial Intelligence**,
by Andreu Masdeu and by José Luis Muñoz
*Received June 1, 2023*

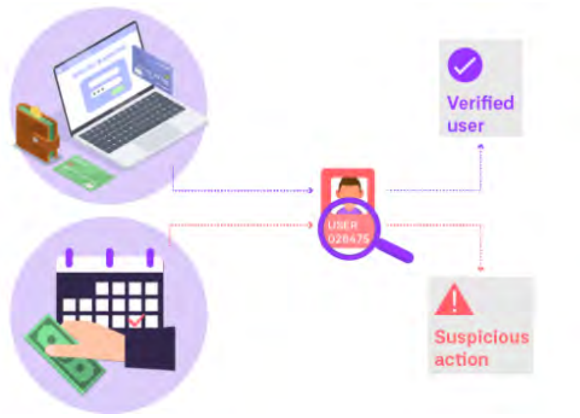
**Basetis** is a Barcelona-based IT consulting firm comprising over 350 employees with a strong focus on social change.* For instance, we have adopted the Teal philosophy, which stands on three pillars: self-management, wholeness, and evolutionary purpose. As a result, decision-making and leadership responsibilities are distributed among **Basetis** personnel rather than following a rigid hierarchical structure.** Our business encompasses a variety of ICT services, including software and mobile application development, graphic design, cloud infrastructure management, as well as data analytics and artificial intelligence (AI) services.

In these latter domains, Mathematics constitutes a pivotal component in the successful execution of our projects. For this matter, **Basetis** employs numerous mathematicians and physicists. In fact, most of the very first members of **Basetis** were alumni of the **FME**, creating a close connection between the company and the faculty. As a result, many **FME** students come to **Basetis** every year for internships, some of them subsequently joining the company on a permanent basis, constituting a significant portion of the company.

Mathematicians at **Basetis** possess a deep intellectual curiosity which led the company to strategically establish its own AI Team five years ago. This decision capitalized on the intersection between Mathematics and AI, leveraging the strengths of the firm's STEM profiles to develop cutting-edge AI solutions for our clients. **Basetis** already provided data analytics solutions since its foundation, so the incorporation of AI was a natural next step as its relevance increased within the IT industry.

From a mathematical perspective, AI projects usually come with intriguing challenges. There are a wide variety of clients, with different needs and data, hence our spectrum of services is rather broad. In all cases, Mathematics is the backbone of our work, as it provides the theoretical foundation for the algorithms and models that power every aspect of AI technology. These projects can be divided into three distinct categories:

1. **Advanced and Predictive Analytics**
Developing accurate predictive models that can forecast future or real-time outcomes based on historical data is one of the most important challenges faced by the AI Team. Examples of such applications include automated fraud detection in bank transactions, energy consumption forecasting, and stock and demand predictions, among others. The successful execution of these projects requires a profound understanding of statistical principles and techniques. Initially, historical datasets must be subjected to rigorous statistical analysis and interpretation to identify the relevant variables and create new synthetic variables from the data. Also, a comprehensive grasp of the mathematics and complexity underlying the typical models used in machine learning is vital for effective model selection.



Fraud detection in bank transactions

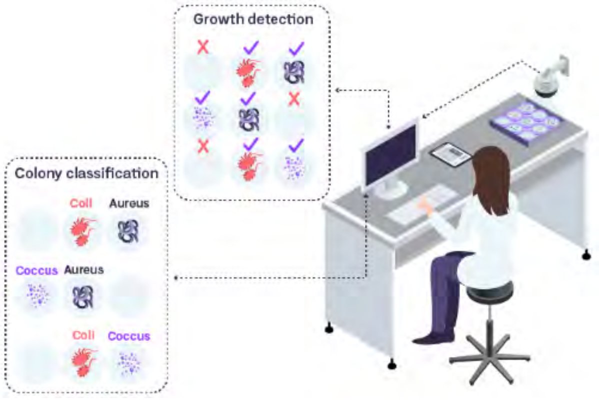
All models inherently involve a procedure for error minimization at their core. However, the specifics of this procedure may vary depending on the model structure, leading to different model behaviors. For instance, decision trees, gradient boosting machines, and logistic regression are examples of machine learning models with a different structure but which can solve the same task. Once the models are fitted to the data using the selected variables, it is crucial to carry out a thorough analysis of their performance. This analysis enables the identification of potential risks and biases associated with the model, enabling appropriate steps to mitigate such issues.

2. **Optimization**
Another key focus of the AI Team is mathematical optimization. Depending on the problem, we can differentiate between continuous optimization and discrete optimization. While continuous optimization problems can be tackled efficiently with machine learning methods, discrete optimization relies on fundamentally different techniques such as graph-based algorithms, mixed-integer programming, Monte Carlo methods, and hybrids of those.


Assigning drivers to services

For instance, the team used graph theory to solve the problem of assigning drivers to services, using one graph to parameterize optimal service concatenation through a minimum path cover problem, and another one to determine the optimal matching between drivers and routes. In such a way, the number of drivers required and the waiting times were minimized and the service efficiency was improved. On a different project, graphs were used to represent a set of academic problems that a student had to face and the dependencies between them. The optimal education itinerary for individual students could be computed using Dijkstra's algorithm and Monte Carlo-based optimization methods.

3. **AI for perception tasks**
In addition to these challenges, we also work on developing natural language processing (NLP) systems, computer vision (CV) algorithms, and other AI applications that require a strong mathematical foundation. Using state-of-the-art techniques, our team has deployed a wide variety of successful AI solutions.




Detecting the growth of bacterial colonies

The AI team has built a chatbot able to function as a drive-through window assistant for a fast-food restaurant chain, using NLP technology to make it capable of understanding the intention of the client's words when listening, and guiding the conversation in order to deliver the order efficiently. On the CV front, the team has developed a system to automatically identify, classify, and quantify olives from images taken of the conveyor belt for real-time quality monitoring. Similarly, an algorithm for detecting the growth of bacterial colonies on a Petri dish was developed, being able to identify typical species and quantify their amount, which can be very helpful for early diagnosis. Moreover, our team has also worked on audio-perception solutions, for instance developing a device able to anticipate malfunctions on industrial machinery by processing sensor data and sound records of the machines.

Overall, the work of the AI team at **Basetis** is highly technical and requires a strong background in mathematics. By leveraging our expertise in math and AI, we are able to provide innovative solutions that help our clients to achieve their business goals.

**Press headlines**

- **El País**/Business Management:
  *Companies witout a boss work (and very well)*.
- Indicador d'Economia/TIC:
  *End hierarchies to empower talent*

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* Marc Castells and Víctor Roquet founded Base Technology and Information Service (**Basetis**) on 6 November 2009, a company in the ICT industry initially focused on providing professional services.

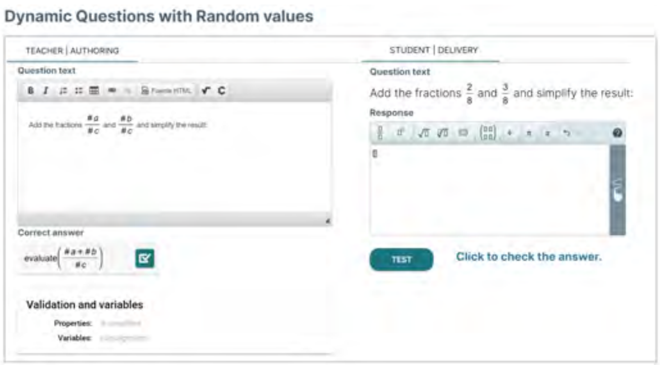
** ***Basetis** is made up of a team of people who are passionate about information and communications technologies. We share an entrepreneurial spirit and base our way of doing things on trust.*

# Outreach

**Enhancing STEM Learning: online solutions for Education**, by Ramon Eixarch (**Wiris** CEO).
*Received June 2, 2023*

The democratization of digital technology has been underway for some time now. In today's world, it would be hard to understand human life without the influence of technology. Moreover, the pandemic has led to the rapid adoption of digital tools to enable various activities to continue remotely, such as virtual meetings, hybrid education, and remote work. Most of these trends continue even after the pandemic has receded. However, with the growing reliance on technology in daily life, not all disciplines were prepared for this context, and some still need to be adapted. STEM* communication and assessment, in particular, have been challenging areas to adapt to the digital environment. This is where **Wiris** has made significant strides, with the aim of making people's STEM work more meaningful.

**Wiris** is a math and science software company that aims to simplify the work of STEM professionals by creating tools that facilitate writing, communicating, and assessing math using digital technology. Our products aim to enable users to have the best experience in STEM, and we accomplish this by integrating our products into all environments to make it as seamless as possible. Currently, **Wiris** offers two primary products: MathType and WirisQuizzes. MathType is a software program that enables the creation of mathematical notation to be inserted into both desktop and web applications. One of its most valuable features is handwriting recognition. On the other hand, WirisQuizzes is an authoring and assessment tool that allows educators to create and deliver math assessments online. Virtual learning environments often pay little attention to tools for mathematical content creation [1], and WirisQuizzes aims to address this need.




WirisQuizzes interface: Example of a dynamic question with random variables.

**Wiris**' beginnings date back over 25 years ago when a group of mathematics students at the Faculty of Mathematics and Statistics (FME) of the Technical University of Catalonia (UPC), myself included, worked out research projects that coalesced into enough know-how to launch a company. A few years later, as the company evolved, its primary goal was to improve and develop digital tools specialized in STEM subjects. **Wiris** released its first product, **Wiris CAS**, in 2002, an online-based computer algebra system that quickly became popular in mathematical environments. Over the years, **Wiris** collaborated with academic institutions and publishing houses, and by 2007, the **Wiris** product family started to flourish, including the first versions of our web-based formula editor (known by then as Wiris editor) and our mathematical assessment tool (WirisQuizzes). In the upcoming years, we continued evolving our products, migrating them to Javascript, and integrating them into popular LMS and HTML editors. The company merged forces with Design Science in 2017, and Wiris editor was rebranded as MathType, becoming a leading solution for mathematical equations. WirisQuizzes also evolved and became compatible with popular LMS systems, and added features such as random learning units [2].

The 2020 pandemic had a significant impact on **Wiris**. Even prior to it, the use of technology in the classroom was a growing trend and a significant concern in educational centers. The lockdown forced the acceleration of this trend as educational institutions globally had to adapt their teaching methods to virtual environments. Most education institutions shifted their learning formats to online or hybrid models, resulting in an unprecedented revolution in education. Both **Wiris**' products played a critical role in this digital transformation regarding STEM subjects. MathType experienced more than 400% growth during the lockdown months, and many educators opted for WirisQuizzes to assess their STEM subjects. During Covid, **Wiris** offered MathType for Google Workspace for free to provide free service to millions of math and science teachers and students who required assistance during a stressful situation. Given the success, we rapidly adapted and integrated with Google Slides by the end of 2020, achieving the perfect solution for educational institutions.

The usage of both products has continued to increase even after the pandemic, demonstrating the sustained demand for digital solutions in STEM education. To put this into perspective, we have experienced incredible growth in MathType users from half a million to double-digit million users, and the figure continues to grow at a fast rate.

Our progress does not end here. As technology continues to advance, **Wiris** recognizes the increasing opportunities to improve STEM education and make it more accessible to people of all ages and backgrounds. The company aims to remain at the forefront of the EdTech landscape, which is evolving at a breakneck pace with the integration of AI into everyday life. We will continue to look ahead and work towards our mission by leveraging all that technology has to offer.

As we expand technologically, Wiris is also rapidly growing in terms of team members. From 2018 onwards, the company has experienced accelerated growth, and today we are about 100 employees with offices in Barcelona and Long Beach, California. We stand out for promoting and harnessing the potential of talent by providing opportunities for individuals and helping them develop their skills to reach their full potential. We believe this is the key to our success and what maintains us at the forefront of the EdTech landscape.

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