Universitat Politècnica de Catalunya • BarcelonaTech

IMTECH 5

Newsletter

Issue 5, January-August 2023

Interviews

- ◇ Xavier Ros-Oton
- ◇ Olli Saari
- ◇ Jaume Franch
- ◇ Xavier Fernández-Real
- ◇ Juanjo Rué
- ◇ Adrián Ponce Álvarez

Research focus

- ◇ Marta Mazzocco
- ◇ Marina Vegué

PhD highlights

- ◇ Alberto Larrauri
- ◇ Franco Coltraro
- ◇ Anastasia Matveeva
- ◇ Patricio Almirón

Outreach

- ◇ Luis À. Caffarelli
- ◇ Andreu Masdeu & José L. Muñoz
- ◇ Ramon Eixarch

Chronicles

- ◇ Juan L. Vázquez
- ◇ Albert Atserias
- ◇ Josep À. Montaner

Reviews by

- ◇ S. Xambó
- ◇ John Urbas
- ◇ S. Xambó
- ◇ Joaquín Pérez

Events

- ◇ Ingrid Daubechies
- ◇ Albert Atserias
- ◇ Jordi Guàrdia
- ◇ Eva Miranda's Hardy Tours & Toni Pou's interview for ARA

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IMTech Newsletter

Issue 5, July 31, 2023

нет войне – No War!

Contents

- Editorial, 1
- Research, 11
- Outreach, 20
- Reviews, 29
- Contacts, 39
- Interviews, 2
- PhDs, 16
- Chronicles, 25
- Events, 34
- Idx NL05, 40

Editorial

In this issue we collect the advances published on March 6, April 14, May 22 and June 22, together with materials received and composed during July. In each section, We have respected the order of publication, up to June 22, and the order of the final editorial composition for the July items (namely, the PhD highlights of Anastasia Matveeva and Patricio Almirón; the Chronicles of Albert Atserias and Josep Àlvarez Montaner, and, in Events, the most recent news about Eva Miranda's Hardy Tours). To start with, we include six Interviews:

- Xavier Ros-Oton, on the occasion of receiving the diploma of correspondent member of Real Academia de Ciencias.
- Olli Saari, on the occasion of his Ramón y Cajal contract at the UPC started in January 2023.
- Jaume Franch, on the occasion of ending his second mandate as dean of the **FME**.
- Xavier Fernández-Real Girona, on the occasion of the Ferran Sunyer i Balaguer Prize awarded to the memoir Integro-Differential Elliptic Equations co-authored with Xavier Ros-Oton.
- Juanjo Rué, on the occasion of having won the Albert Dou Prize for Mathematical Dissemination awarded by the SCM.
- Adrián Ponce Álvarez, who has a Ramón y Cajal contract at the UPC since September 2022.

There are two Research focus. One by Marta Mazzocco, which is a write-up of her **IMTech** Colloquium lecture about *Quantum character varieties* delivered on 8 February 2023, and another by Marina Vegué, with the title *Reducing the dynamics of large interacting system*, an outline of a paper published this year in the *PNAS Nexus* co-authored with V. Thibeault, P. Desrosiers and A. Allard.

PhD highlights:

- Alberto Larrauri: *First Order Logic of Random Sparse Structures*, advised by Marc Noy.
- Franco Coltraro: *Robotic manipulation of cloth: mechanical modeling and perception*, advised by Jaume Amorós and Maria Alberich-Carramiñana.
- Anastasia Matveeva: *Poisson Structures on Moduli Spaces and Group Actions*, advised by Eva Miranda.
- Patricio Almirón: *Analytic invariants of isolated hypersurface singularities and combinatorial invariants of numerical semigroups*, advised by Maria Alberich-Carramiñana and Alejandro Melle.

One of the great news disclosed on March 25 was that Luis Á. Caffarelli had been awarded the Abel Prize 2023. We took the opportunity to include his masterful opening lecture of the term 2003-04 at the FME, translated into English, as the first Outreach piece of this issue: *The heat equation*. We are also grateful to Juan L. Vázquez for the first piece of the Chronicles section: *A personal glimpse on Professor* Luis Caffarelli *on the occasion of the Abel Prize 2023*, which is an English version of the portrayal he published in the Boletín of the RSME on March 31. The Abel Prize award ceremony 2023 was held on May 23.

There are two more Outreach notes. One by Andeu Masdeu and José L. Muñoz on *Research and development at **Basetis**: The strength of blending Mathematics and Artificial Intelligence*; and the other by Ramon Eixarch, the **Wiris** CEO, on *Enhancing STEM Learning: online solutions for Education*. To note that both **Basetis** and **Wiris** were launched by former students of the FME. There are also two additional Chronicles. One by Albert Atserias, on the winner of the IMU Abacus Medal 2022: *Mark Braverman's Work on Information Complexity*; and another by Josep À. Montaner on the international conference *Algebraic and topological interplay of algebraic varieties* in honor of Enrique Artal and Alejandro Melle (12-16 June, Jaca, Spain).

In the Reviews section we have reproduced, in homage to the authors, the John Urbas review in the BAMS of L. Caffarelli and X. Cabré landmark book on Fully Nonlinear Elliptic Equations (AMS Colloquium Publications, vol. 43, AMS, 1993). It is a welcome coincidence that we have been able to review the memoir Integro-Differential Elliptic Equations, by Xavier Fernandez-Real and Xavier Ros-Oton, which was the winner of the Ferran Sunyer i Balaguer Prize 2023. The other two reviews deal with Joaquim Bruna's book on Analysis in Euclidean Spaces (World Scientific 2023) and with the acclaimed paper *The Gaussian Double-Bubble and Multi-Bubble Conjectures*, by E. Milman and J. Neeman (we are grateful to Joaquín Pérez, the Director of the Institute for Research in Mathematics of the University of Granada, for his expert review).

Finally, in the Events section, we pay homage to Ingrid Daubechies (winner of the Wolf Prize in Mathematics); to Albert Atserias, for having been awarded the ICREA Academia distinction; to Jordi Guàrdia, for having been elected Dean of the FME (he took office of March 28), and to the members of his team; and last, but definitely not least, an account of Eva Miranda's first two Hardy Tours (the third and last will be in September, for a total of nine Hardy lectures).

Xavier Ros-Oton delivered a lecture on 11 January 2023, on the occasion of receiving the diploma of correspondent member of RAC Real Academia de Ciencias (see the picture at the end), with the title *Teoría de regularidad para Ecuaciones en Derivadas Parciales elípticas* (Regularity Theory for Elliptic Partial Differential Equations). This is currently the last distinction in recognition of his research contributions on a wide variety of topics in the field of partial differential equations (PDE), which are "the equations that move the world", as he himself asserted in one of his lectures. These equations play a central role in all areas of physics, as well as engineering, biology and finance, and also in some of the most famous problems in pure mathematics.

Born in Barcelona in April 1988, he completed his Bachelor's Degree in Mathematics at the Polytechnic University of Catalonia (UPC) in 2010, the master thesis in 2011, and his doctoral thesis in June 2014, both under the supervision of Xavier Cabré. Consequence of his thesis was the remarkable article [1], co-authored with Joaquim Serra (interviewed in NL02, pp. 5-6), which is the most cited article of the year 2014 in MathSciNet (among all mathematics articles) and as of today has over 730 citations (according to Google Scholar).

Ros-Oton pursued his postdoctoral studies at the University of Texas at Austin) from 08/2014 to 08/2017, where he collaborated with the great teacher Luis Caffarelli, world leader in problems of free boundaries, and with Alessio Figalli, young star of the Italian PDE. In 2017 he was called to the University of Zurich as Assistant Professor, and there he continued his close collaboration with Alessio Figalli, who in 2018 was awarded the Fields Medal at the ICM-2018. In 2020 he returned to Spain after being appointed as Full Professor at the UB and as ICREA Professor.

The results of his research have been published in the best mathematical journals worldwide and it is said that he is currently the most cited mathematician of his age in the world. In the few years after his PhD, he has been awarded various distinctions: the Vicent Caselles Prize in 2015 (awarded by the RSME and the FBBVA); the Rubio de Francia Prize in 2017 (awarded by the RSME); the Antonio Valle Prize to Young Researchers in 2017 (awarded by SeMA); the Princess of Girona Foundation Scientific Research Award in 2019; and the Stampacchia Gold Medal in 2021. The mention of the Princess of Girona Foundation Award says: *For being one of the most brilliant mathematicians with the greatest impact worldwide in his age group. His research on partial differential equations, a central field in many scientific disciplines, has had a very prominent impact on the scientific community.*

In 2018, he received an ERC Starting Grant ERC Starting Grant. In 2019 he was one of the plenary speakers at the RSME biennial congress. A person who loves communication and an excellent speaker, he has increasingly participated in these years in congresses, seminars and mini-courses in various countries, has written popular articles and has collaborated with mathematical societies (Spanish and Catalan). The book [2] (with Xavier Fernández-Real) has just appeared in the Zurich Lectures in Advanced Mathematics collection of the EMS.

NL. ***Let us begin at the beginning: When did you decide to become a mathematician? Did you consider other options? What were the main factors that tipped your resolution?***

I decided to study mathematics during the last two years of high school. A key factor that fostered my interest in mathematics were the mathematical competitions organized by the SCM (such as the Cangur and math olympiads), and especially the preparation courses of Prof. Grané at the FME. After participating in these activities during one or two years, it became very clear to me that I wanted to become a mathematician, and pursue a research career in mathematics.

Which memories do you have as a student at UPC and how did your training at UPC influence your academic career?

I have very fond memories of my time as a student at the FME: I had a really good time and I learned a lot; I'd definitely make the same choice again. During my last year as an undergraduate student I had to decide the topic and advisor of my PhD, and once I decided that I was mostly interested in Analysis I was very lucky to have Xavier Cabré at UPC.

You did your PhD at UPC under the supervision of Xavier Cabré. Which are your recollections as a graduate student?

Thanks to Xavier Cabré, I learned a lot during my PhD, and worked on a variety of mathematical problems. I not only learned a lot directly from him, but also participated in many conferences and summer schools, which allowed me to interact with several top mathematicians from Europe and US. Furthermore, it was also crucial for me to have Joaquim Serra working on his PhD at the same time: he is still one of my main collaborators and I learned a lot from him, too.

After your PhD you went to Austin and then to Zürich. How was your experience there in personal and professional terms? Which were the main research outcomes from that time?

During my time as a postdoc in Austin I started working on a new topic: free boundary problems. Such a change allowed me to learn a different area of research, while at the same time exporting the knowledge gained during my PhD to a different context. I was lucky enough to collaborate with A. Figalli and L. Caffarelli, and obtained several important results with them. These are the main results for which I received prizes from the RSME, SeMA, and the Fundación Princesa de Girona.

After three years as a postdoc in Austin, I moved to Universität Zürich, where I could continue working with A. Figalli and J. Serra. It was a great time, both from the personal and professional point of view, and it was in Zürich where I obtained my best mathematical results.

Could you please tell us about your own research and the problems that most interest you?

I have worked on quite diverse topics, all related to the broad areas of PDE and Calculus of Variations. The main topic on which I have worked in the last years is one of the most basic and important question in PDE theory: to understand whether all solutions to a given PDE are smooth or if, instead, they may have singularities. Some of my main contributions have been in the context of free boundary problems, that is, PDE problems that involve unknown interfaces. From the mathematical point of view, they give rise to extremely challenging questions, and their study is closely connected to geometric measure theory. In particular, the study of free boundary problems has a strong geometrical flavor.

What problems in areas of your interest would you like to see solved in the coming years?

A type of question I like very much (and for which very little is known) is to understand "generic regularity" for PDE problems. For example, even if some solutions may develop singularities, is it possible to prove that is it an extremely unlikely scenario? In other words, can one prove that in some PDE problems singularities appear with probability zero? I would like to see many more developments in this direction.

How do you value the prizes and distinctions you have been awarded in recognition of the quality of your research?

I feel very honored and lucky to have received these distinctions, and I know there are other mathematicians who would deserve them, too. Moreover, they are awarded for works that I have established with many collaborators, so this is a recognition for them, too.

You have since 2020 a research position in Barcelona. How do you see the research system in Spain and what do you think are its strengths and weaknesses?

Unfortunately, I think that we are still quite far from the research systems and environments of more advanced countries. The main strength is that we have several mathematicians who are working here and doing an excellent job. The main weakness I see is the bureaucracy and lack of administrative support that researchers have, compared to what I saw in the US, Switzerland, and other countries I visited.

References

[1] Xavier Ros-Oton and Joaquim Serra, *The Dirichlet problem for the fractional Laplacian: regularity up to the boundary*, Journal de Mathématiques Pures et Appliquées **101** (2014), no. 3, 275-302.

[2] Xavier Fernández-Real and Xavier Ros-Oton, *Regularity theory for elliptic PDE*, EMS, 2022.

Olli Saari started with a Ramón y Cajal contract at the **UPC** in January 2023. His research is focused on harmonic analysis and analysis of partial differential equations. He completed his master's degree in engineering physics and mathematics at Aalto University in 2013 and continued then as a doctoral student in the same institution under supervision of Juha Kinnunen, funded by the Vilho, Yrjö and Kalle Väisälä Fund. During his doctoral studies, he spent a semester as a visiting researcher at UPV/EHU in Bilbao. After finishing his PhD in 2016, he spent the fall semester 2016 at the University of Bonn and the spring semester 2017 in the Mathematical Sciences Research Institute (California) as a post-doctoral fellow. Starting from the fall 2017 and ending with the year 2022, he was employed by the University of Bonn as a post-doctoral assistant in the working group of Christoph Thiele, after which he moved to Barcelona to his current position.

NL. ***Welcome to Barcelona and the UPC, and congratulations for the Ramón y Cajal contract. Could you comment on what features of Barcelona and its mathematical community contributed to your decision to land here with your RyC?***

A Ramón y Cajal contract offers a lot of freedom so the choice was a difficult one. But Barcelona was my clear favourite in the end. The analysis group at UAB is famous and extremely strong. The PDE group at UB is also very strong and was also an important attractive factor. Our working group at the UPC is slightly smaller, but we share a seminar with the other two, so there is regular communication. This is the picture of the mathematical analysis community in Barcelona that I got after discussing with Xavier Cabré about possibility of coming here. The UPC then felt like the university to which I would fit in best. Also beyond mathematics, Barcelona is a very interesting place. I had not really spent much time here before moving in, just a few days for conference trips, but so far I have been very happy.

Your master's degree at Aalto University suggests a strong interdisciplinary mix. How did that atmosphere impact on your professional education? Was it decisive in your research orientation? What was the topic of your master thesis?

The mission of my degree programme at Aalto University was to educate engineers with solid background in physics and mathematics to later move to real world applications in industry. This did not quite happen with me, and in retrospective I even feel I wasted a part of the opportunities that the interdisciplinary tendency around me could have brought. I ended up working on topics rather far from applications, the title of my master's thesis was *Local to global results for John-Nirenberg inequalities*, but I think it was very beneficial to study in an environment with good balance of pure and applied mathematics. I studied courses in numerics, physics and mechanics that I would not have taken elsewhere. But I can't say my research orientation were affected very much by that.

What was the topic of your PhD thesis? What are your recollections of that research period? How was the guidance by, and collaboration with your advisor, Juha Kinnunen?

The title of my PhD thesis was *Weights arising from parabolic partial differential equations*. A good part of mathematical analysis can be traced back to the study of harmonic functions, the solutions of the Laplace equation. One can go back to the basics and replace the stationary potential equation by an evolutionary diffusion equation and try to understand how such a change should be seen by inequalities in analysis that do not have anything to do with partial differential equations at the first sight, although they stem from the study of harmonic functions, in fact. My project was to apply this point of view to what is known as weighted norm inequalities.

The time of writing the PhD thesis was a very special period of my research career. There was a clear unique choice what to work on, as there was an advisor to tell me that, and except for the meetings with my advisor, I was mostly working alone in the beginning. Then the working routines changed at some point. Working extended periods on the same problem without too much external input was certainly a good learning experience, and it helped to build a solid understanding with no details omitted. The research as it started occurring later is more collaborative and also much more fast-paced.

Kinnunen was a very encouraging character for me as an advisor. He was actually not only my PhD advisor but also the advisor for my master's and bachelor's theses so I started working with him even before choosing mathematics as the major subject.

Can you tell us about your experiences as a visiting researcher for four months at UPV/EHU?

I was invited to Bilbao by Ioannis Parissis who had moved there after being a post-doc at Aalto University, and Carlos Pérez. At the time of staying in Bilbao I had already done the research for my thesis and was writing the final form of the thesis. I had therefore time to learn something new. The host group was focused on harmonic analysis whereas my home group at Aalto had been more of a PDE group. My thesis topic was something in between, so I found this change of environment extremely inspiring.

Let us move forward to your postdoctoral journey in Europe and the United States. What aspects of those stays would you like to highlight?

My first two short visits, Bonn and MSRI, were purely for research. During my later and longer stay at Bonn I was also involved in teaching. The research program in MSRI was of course an important experience and opportunity to get to know lots of colleagues and to get an idea about a wide range of topics that they were working on. It also showed clearly that one cannot stay on top of every currently evolving area in research that one understands to some extent but one has to focus a lot more. Researchwise, the most beneficial side of my postdoctoral positions has been the opportunity to widen one's mathematical perspective by getting to know new people and their research. Working with Pascal Auscher in MSRI or Christoph Thiele in Bonn has had a huge impact on how I see mathematics nowadays, probably even more than what I learned during my PhD thesis, not to mention the collaborations with younger colleagues, such as Simon Bortz and Moritz Egert whom I met in MSRI. It has also been interesting to see how differently things work in different countries. There are many ways to arrange research and teaching. Bonn was very different from Finland and the UPC must be another story again.

What vision do you have for your research during the RyC contract? What would you like to achieve? Do you have in mind collaborating with members of the mathematical community, or, more specifically, with participants of the Barcelona Analysis Seminar?

My current contract gives a planning security for a long time, which is an excellent thing to have. I have a few topics on the waiting list that I always wanted to look at more thoroughly when having more time: divergence equation on non-cylindrical domains, regularity of vertical maximal functions and convex sets as bilinear Fourier multipliers. But in reality, what I will end up doing also depends on the people around me. There are the collaborators far away with whom I am in a more or less regular correspondence, and I am also talking to people around here in Barcelona, for instance Albert Mas and Xavier Cabré at UPC. I chose to come here mostly because of the math community so I should try to take advantage of it.

In the mathematical ecosystem of the UPC, there is a wide room for teaching subjects in several studies offered by the FME and CFIS at various levels. Can you share how do you see your role in that respect for the next few years?

Well yes, this is a very nice question to answer. I heard, from several people, that year after year UPC manages to attract very strong students. On the other hand, there are not so many researchers in harmonic analysis involved in the teaching at the moment. There would be use for more people with my background. Going back to the first questions of this interview, choosing UPC instead of one of the other universities in Barcelona has a lot to do with this. I am currently teaching exercises for the advanced PDE in the master's program and supervising two master's theses. I will also be part of the teaching plans of our working group in the future. There will be more students with interest in analysis in the years to come, no doubt, and it will be good for them that I will be here. I don't have a good estimate on the number of students around here yet, but maybe there will even be a chance of introducing a course in harmonic analysis at some point in the future, who knows.

We hope that you will enjoy your contract and that it will turn out to be a most positive time for your academic career. We also wish you good luck!

Thank you!

Jaume Franch has recently finished his 8-year appointment as a dean of the Facultat de Matemàtiques i Estadística (**FME**) of the Universitat Politècnica de Catalunya (**UPC**). Before, he served as a vice-dean for almost 12 years under the deans Sebastià Xambó and Jordi Quer.

He obtained a "Llicenciatura en Matemàtiques" from the University of Barcelona in 1992. He got a position as full-time lecturer at **UPC** in 1993 and started his PhD in the Applied Mathematics program under the supervision of Enric Fossas, defending his thesis entitled *Flatness, tangent systems and flat outputs* in 1999. He became associate professor at **UPC** in 2002.

His research activity has always been mainly developed in the field of Control Theory, in particular in the study of the algebraic and geometric methods in nonlinear control and their applications to trajectory planning and control design. In collaboration with the research group led by Prof. Sunil K.

Agrawal first at the University of Delaware and later on at Columbia University, where he has been a visiting scholar several times, he has applied his theoretical results to robotics and mechanical systems in general.

NL. ***At the end of your second 4-year period serving as dean of the FME, what institutional accomplishments would you like to highlight?***

We have known, for a long time, that our studies are at an excellence level, and during these years we have been able to establish that fact: all our studies have been accredited with the excellence mark. We must take into account that only about 10% of the Spanish university studies get this level of assessment. Besides that, I would like to highlight the following facts:

- The number of applications to our master degrees have increased significantly, especially in the Master in Mathematics. At present, our two masters are the ones with the highest number of applications and registered students in their fields in Spain.
- We have increased substantially the budget of the **FME** by working in two directions: on one hand, increasing the rates of our studies and being able to get recognition for that, and, on the other hand, by augmenting the income through collaborations with private companies. With this increased budget, we have been able to improve our facilities, an achievement which is very important under circumstances that demand sustained attention to technology advances.
- Thanks to Mireia Ribera, our administrative secretary, we have improved our communication to the community and the society at large. An example of this is "Notícies FME", a weekly newsletter where we make announcements and provide all kinds of information related to the **FME**.
- When I started the first term as a dean of the **FME** the number of people working as a technical staff at the **FME** had dropped dramatically. One of my goals was to get more personnel and we also succeeded in this.
- We are participating in other studies that are also important for us: the master for preparing high school mathematics' teachers, whose lectures are now taught at the **FME**, and the Data Science and Engineering degree, shared with the schools ETSETB and FIB.
- Finally, we supported the creation of **IMTech** from the very beginning: The **FME** provided a small financial help to assist in its first steps, and it is where the **IMTech** has its headquarters. This means, in particular, that the **FME** purveys administrative assistance and facilities for **IMTech** activities.

Before your mandates as a dean, you served as vice-dean for the three preceding mandates. What is your balance sheet about those experiences?

Actuallly, I served as vice-dean for the four preceding mandates, two with dean Sebastià Xambó, and two with dean Jordi Quer. I was very young when Sebastià Xambó asked me to be in his team, and I have great memories from that time. We did a lot of work. Jordi Quer also trusted me and, specially in the last three years, he empowered me a lot. So, for me it was a natural step to stand for the dean position after that.

How do you see the future of the FME? Do you have any advice for the new dean's team?

My best advice is to not spoil anything that is working well. Besides that, I think it is important to have some influence in the decisions made by **UPC** and to increase the collaboration with companies. It is also worth mentioning the change that will imply the increment in the number of students in the undergraduate studies in mathematics in our faculty. It was something necessary to do due to the increasing number of students willing to study mathematics and to the very high demand of our graduates from companies. Thankfully, this process has been done with the agreement of everybody at the **FME**, which will make easy to deal with the challenges that will come up with a larger number of students

We would also like to know about your teaching experience. What subjects did you teach? For how long? What are your favorite topics?

I started teaching Real Analysis at the **FME** 25 years ago. This is my favourite subject to teach, but I like to teach any other subject related to Calculus and Analysis. Besides that I also enjoy teaching Control Theory, my research area. Lately, I have become the responsible for the 4th year compulsory subject Mathematical Models in Technology. I am trying that this subject evolves towards a kind of subject where companies and research groups present some projects and the students work in them during all the term.

Let's talk about research. What drove you into problems in control theory and its applications? How is this commitment related to you PhD thesis?

When I joined **UPC** I was asked to td the PhD in one of the research lines of my department. I attended a course in Graph Theory, a course in Criptography and a course in Control Theory. The latter was the one I liked the most, and I asked Enric Fossas to supervise my PhD in this topic. My PhD was quite theoretical, and later on I have been able to do some applications of my results, mainly in the field of Robotics.

Administration service usually holds back research to a good extend. How has it been in your case?

During my years as a vice-dean I was able to maintain my research, mainly through yearly visits to the group headed by Sunil K. Agrawal, then at University of Delaware. But once I became dean I stopped doing research. When I do research I need to be focused in a problem, and I am not able to do research only in the very few free hours that you have when you are a dean.

Could you explain in more detail the nature and scope of your collaboration with Sunil K. Agrawal and his research group?

We met for the very first time 25 years ago, in my first international conference when I was still doing the PhD. He was interested in my theoretical results since they could be applied to design controllers for mechanical systems. He started inviting me every year and we published a good number of papers together in my ten visits to his group. Later on, we gave a graduate course together, first at University of Delaware and later at Columbia University.

Returning to a more active research life may need some adjusting period. What are your plans in that regard for the years ahead?

I am very excited about returning to a more active research life. I have started a collaboration with some researchers at IRI (Joint Research Center of the Spanish National Research Council (CSIC) and the Technical University of Catalonia (UPC)). We have applied for a research project to the Spanish Ministry. I would also like to collaborate with Prof. Agrawal again.

Xavier Fernández-Real Girona pursued a CFIS double degree in Mathematics and Engineering Physics (his bachelor thesis in Mathematics, *Boundary regularity for the fractional heat equation*, was worked up in the academic year 2013/14 under the advise of Xavier Ros-Oton and Xavier Cabré Vilagut). Then he earned a Master's degree in Advanced Study in Mathematics (Part III of the Mathematical Tripos)indexMathematical Tripos from the University of Cambridge and a PhD in Mathematics from the ETH Zürich on *Regularity Theory for Thin Obstacle Problems* under the advise of Alessio Figalli. For his postdoc research, he joined the Maria Colombo group at EPFL. Since January 2023 he is a Bernoulli Instructor in this school, a position that will change next September to an Ambizione Fellow. His research interests focus on partial differential equations (PDEs) and calculus of variations, and more specifically on elliptic and parabolic PDEs, integro-differential operators, free boundary problems, and optimal transport and transport equations. Among the honors and awards he has received there are: Evariste Galois Prize 2016 from the Societat Catalana de Matemàtiques (SCM); Spanish National Award for Excellence in Academic Performance 2013-14, First Prize, 2018; ETH Medal for outstanding doctoral theses, 2021; Vicent Caselles Prize 2021 for best PhD Theses in Mathematics awarded by the Real Sociedad Matemática Española (RSME) and the BBVA Foundation; Dimitris N. Chorafas Prize 2021 for best doctoral students in the Hard Sciences; XXVI SeMA "Antonio Valle" Young Researcher Award 2023; Ferran Sunyer i Balaguer Prize 2023 for the book *Integro-Differential Elliptic Equations* (co-authored with Xavier Ros-Oton).

NL. ***To begin with, we would like to know when and how your interest for the "exact sciences" was ignited. Did it have to compete with other interests?***

My interest in the exact sciences was most likely ignited through the Cangur competitions that have been organized in Catalonia for many years. It was then consolidated through the mathematical Olympiads, which I was introduced to by Prof. Josep Grané Manlleu and Prof. José L. Díaz Barrero from UPC.

And yes, it has always had to compete with other interests, even to this day. I have always been interested in all sciences, and it was never easy for me to restrict myself to just one discipline. That's why I chose to study at CFIS, and it also influenced my choice of master, and even my choice of area within mathematics.

You took part in the Mathematical Olympiads and other similar contests, culminating with your winning a gold medal in the Spanish MO and hence taking part in 2010 in the IMO as a member of the Spanish delegation. Could you share your more salient memories of those experiences? Did they influence your choices when you entered the university?

The Mathematical Olympiads were probably the main reason why I chose Mathematics as my preferred degree when entering university. These competitions showed me what it really means to do math, and they also revealed how much there was to learn about it.

I have many fond memories of those days; the teachers at the Mathematical Olympiads were incredibly motivated, and we always felt fortunate and grateful to be there. Additionally, it was fascinating to meet people from all over the world who shared similar interests but had vastly different stories.

Your bachelor thesis shows that you had already a strong research drive as an undergraduate. What circumstances contributed to inspire you to follow that call?

I would say that I always knew I wanted to pursue basic research, but I was unsure of the specific area I would choose. My father is a researcher in medicine, and that undoubtedly influenced how I viewed researchers and their work. It was something that always fascinated me.

During my final year of my bachelor's degree, I was greatly inspired by Prof. Xavier Cabré and then-PhD student Xavier Ros Oton. They encouraged me to explore more advanced topics, and they were always patient and supportive. They helped me appreciate the beauty of PDEs and their connections with the world.

In 2015 you earned a master's degree from Cambridge University (Part III of the Mathematical Tripos), with distinction. How did this accomplishment unfold? What did it add to your research training?

I chose to pursue my master's degree at Cambridge University, specifically the Part III of the Mathematical Tripos program, for several reasons. One of the primary reasons was that it allowed me to postpone my decision on a specific research area. The program offered over 100 advanced courses in mathematics and physics, which provided me with the opportunity to study a diverse range of subjects such as Cosmology, Elliptic PDE, and Semigroups of Operators.

The program was intense, and there was a lot to learn in a short amount of time. However, it provided me with a solid foundation in the topics that would later be integral to my PhD thesis. I still consult some of my notes from my time at Cambridge today.

Your next major academic achievement was earning a PhD from ETH Zürich under the advise of Alessio Figalli. Could you please sketch the story of this outstanding experience?

Choosing where to pursue a PhD is always a difficult decision, and my experience was no different. At the time, Alessio Figalli was a professor at UT Austin, and I was applying to several universities in the US as a clueless master's student. I got very nice offers from some places, and I was full of doubts for some months. But Alessio (who at the moment hadn't yet won the Fields medal) seemed very approachable and very active, and this eased my decision to go to UT Austin (after one year, Figalli's group moved to ETH Zürich, where I finished my PhD). Looking back, it was one of the best academic decisions I've ever made, as Alessio proved to be an outstanding advisor who was always available and patient with my many questions.

Since 2016 you have published a remarkable number of works, with the last few still as arXiv manuscripts. Could you try to summarize the main questions you have probed, often with one or more collaborators?

I have worked on various topics, since the beginning of my PhD. I have studied the regularity of solutions to integro-differential equations, and my PhD thesis focused mostly on free boundary problems, and more precisely, on the thin obstacle problem. Roughly speaking, this problem attempts to understand the shape of a membrane that lives above a given fixed obstacle that is defined on a lower-dimensional set (a "thin obstacle"); and it has surprising connections in many different areas: from finance to ecology, biology, industry, physics, and to other areas of mathematics themselves. One of my favorite results, which I worked on with Joaquim Serra (currently a professor at ETH Zürich), was proving that minimizers of the area constrained to be above a thin obstacle are regular around contact points in all dimensions. This is in contrast to minimizers of the area without an obstacle, for which the Simons' cone is a counter-example to regularity from dimension 8 and higher.

Aside from my work on the thin obstacle problem, I have also been interested in transport equations and more recently, their relation to neural networks.

In your list of publications there are two recent books written in collaboration with Xavier Ros-Oton: Regularity Theory for Elliptic PDE (2022) and Integro-Differential Elliptic Equations (2023), which is the winner of the Ferran Sunyer i Balaguer Prize 2023. Could you comment on the genesis of these memoirs and broadly describe their goals and distinguishing features?

In both cases, the original seeds for the corresponding books were graduate courses that Xavier Ros-Oton gave some years ago both at UT Austin and at the University of Zurich. And in both cases, the books contain some of the things I learned from different places during my graduate studies; they are both books that I would have liked to have available when I started my PhD, and will be hopefully useful to other prospective PhD students.

The first book, *Regularity Theory for Elliptic PDE* is more "basic" in content, and in some settings, it could even be used in advanced master-level courses. This book contains a selection of fundamental results and techniques that we believe are essential in the understanding of Linear Elliptic PDE, but it also introduces, from an elementary viewpoint, some important nonlinear PDE problems.

The second book, *Integro-Differential Elliptic Equations*, is deeper in content, and with much more recent results, almost all of them obtained during the 21st century. The book is more advanced and covers, for the first time, an area that was until now contained in many different papers from the last 20 years. In this way, we try to give a first comprehensive understanding of the subject, we have simplified proofs and unified approaches, and in doing so we are even able to prove some new results that complement well the existing theory.

One aspect I particularly like about both books is that they are completely self-contained, which means that they can be used both as a reference and as a learning tool. We hope that they will be widely used by researchers and students interested in the topics covered in them.

With the Bernoulli instructorship and especially with the coming Ambizione Fellowship at EPFL it seems that you are going to have much freedom to undertake more enterprising research plans. Could you portray what is your vision about these plans and what aims you would like to achieve with them?

These fellowships are truly invaluable, as they offer a rare opportunity for intellectual freedom during a critical juncture in a researcher's career when such opportunities are hard to come by. Rather than constantly seeking out new postdoctoral positions and worrying about where to go next, I will have the stability of a fixed institution and my own research money for the next four years.

Thanks to the Ambizione Grant, I will have the opportunity to conduct research without the burden of heavy bureaucracy or extensive teaching obligations. Additionally, the grant provides substantial funding, allowing me to hire a PhD student and establish my own research group. Through this, I will be able to expand my collaborations with researchers from around the world. In my opinion, it is crucial to have research-focused positions at universities, and grants such as this one promote exactly that.

Looking at the panoply of your awards, prizes and distinctions, we would like to know your appraisal on their effectiveness to bolster research careers, and also to promote visibility of mathematics in the society.

Awards, prizes, and distinctions are undoubtedly valuable for researchers, especially those in the early stages of their careers. They can provide recognition and validation of one's work, and in some cases, they can even provide financial support to continue research. Moreover, they can also serve as a springboard for further career opportunities and collaborations.

I personally believe that major prizes and awards, such as the Fields Medal or the Abel Prize, can help highlight the importance and relevance of mathematics in various fields, as well as showcase the achievements of mathematicians to the wider public. They can also inspire younger generations to pursue mathematics and show that it can lead to meaningful and impactful contributions to society.

However, as with any form of recognition, it's important to remember that prizes can create an overemphasis on individual achievement and potentially overshadow the work of others. Therefore, it's crucial to maintain a balanced perspective and acknowledge the contributions of all researchers, regardless of whether they have received awards or not. Furthermore, I think it's important to ensure that resources are distributed fairly and that access to funding and opportunities are not tied solely to previous prizes or recognitions. Ultimately, research merits should be the sole consideration for such cases.

This year we have rejoiced with the awarding of the Abel Prize to Luis Ángel Caffarelli. How have you lived this memorable event?

It has now been some years that the researchers in the field have been wondering when (and not if) he would get the prize; so in a way, it has been to no one's surprise! Luis Caffarelli's work is one of the most influential contributions to mathematics in the 20th and 21st centuries, and having grown (mathematically) with his results and ideas I can only feel pride and admiration for him! His work in PDEs has greatly influenced many areas of mathematics and has had a profound impact on the development of the field.

This award is also a recognition of the importance of his field and its impact on mathematics and other disciplines. I believe that it will have a positive effect on all researchers in the area and that we are all very lucky to have someone like him working on the topics of our interest. (Of course, one could also argue that these topics are interesting to us thanks to his contributions!)

Congratulations for the FSB Prize, also to Xavier Ros-Oton. Does this prize have any special significance for you?

It is always special and meaningful to receive a prize, even more so when the prize is given by a Catalan foundation, named (and created) after a mathematician who worked in Analysis and was originally from the province of Girona (both traits that I share with him!). During my PhD studies, I looked up to this prize as a symbol of international recognition and prestige, and it is with great joy that I now find myself as a contributor to a book that has been awarded this honor!

Juanjo Rué is Associate Professor at the Department of Mathematics (**DMAT**), member of **IMTech**, member of the research group Geometric, Algebraic and Probabilistic Combinatorics (GAPCOMB) and researcher attached to Centre de Recerca Matemàtica (CRM). His research field is in the area of discrete mathematics, in a broad sense, including enumerative combinatorics, asymptotic enumeration, additive combinatorics and extremal combinatorics. In this context, he has been the Principal Researcher of two international projects: A Marie Curie Career Integration Grant, on *Enumeration of discrete structures: algebraic, analytic, probabilistic and algorithmic methods for enriched planar graphs and planar maps* (2014.3-2018.2) and a Bilateral project between Germany and France on *Analytic, probabilistic and geometric Methods for Random Constrained graphs* (2015.1-2016.12).

He received a degree in Mathematics from the **FME** (2005) and a degree in Telecommunications Engineering from the **ETSETB** (2007), a double degree program supervised by the then just created **CFIS**. Then he worked up a PhD thesis on *Enumeration and Limit Laws of Topological Graphs* under the advice of Prof. Marc Noy. After that, he was an ERC postdoctoral researcher at École Polytechnique (2009-2010) and a JAE-DOC postdoctoral researcher at ICMAT (2010-2013). Before joining **UPC**, he was a Professor–W1 in Discrete Mathematics group at the Freie Universität Berlin and member of the Berlin Mathematical School (2013-2016). On the administrative side, he has been Vice-dean of the **FME** (2019-2023) and since March 2023 he has been appointed its Academic Secretary.

J. Rué is also a steady outreach and dissemination writer, including several books. A faint gleam of this activities can already be perceived in the **IMTech** NLs published so far: NL01, *How to visit two million stars in the shortest time*, p. 14; NL02, a chronicle, co-authored with Guillem Perarnau, on *The European Conference on Combinatorics, Graph Theory and Applications* (Eurocomb'21), p. 18; a review of the paper *On a density conjecture about unit fractions*, by T. Bloom, in NL03, p. 24; and the chronicle James Maynard*: a Fields Medal for major advances in Analytic Number Theory* in NL04, p. 20. Recently he has been the winner of the Albert Dou Prize for Mathematical Dissemination awarded by the SCM.

NL. ***You have been the winner of the 2023 Albert Dou Prize awarded by the SCM. Would you please let us know the title of the essay you submitted, and describe its main features? What significance does this prize have for you?***

The essay is written in Catalan and its title is *De la teoria de grafs clàssica a l'anàlisi de les grans xarxes* (*From classical graph theory to the analysis of large networks*). My idea on writing this essay was to follow an historical path from a selection of very important and already classical results in graph theory (starting by the well known Euler's solution of the Königsberg bridge problem, but covering also Kuratowski's Theorem, Erdös probabilistic construction of graphs with large girth and large chromatic number, ...) until the recent developments in the context of graph limits. In particular I wanted to show that the first results in the area were a mixture of apparently disconnected results and tricks, which have evolved into an area in mathematics (and, of course, in theoretical computer science) with its own agenda and techniques. Hence, receiving this prize was a great satisfaction for me, as it also rewards my field of study.

The Albert Dou Prize is the last link in chain of mathematical dissemination writings, for example in the daily press. Could you summarize the main highlights of this outreach activity?

I am very happy to have received this prize. I believe that the figure of Albert Dou is not well-recognized and it deserves some words. He was a Jesuit (a member of the Society of Jesus), and as such, he received a very comprehensive education outside Spain (which is important to remark considering how was Spain after the Civil War). In short, he was a great promoter of mathematics in Spain when doing research in mathematics was a very challenging matter. Returning to your question, I like to collaborate from time to time with general media (such as El País) by explaining (as simply as I can) recent developments in the areas of mathematics I know a little. In these articles I have talked, for instance, about Ramsey Theory, prime numbers and (maybe the article I am happier) about the 2021 Abel prize awarded to Laslo Lovász and Avi Wigderson.

Less ephemeral than the daily press, you have published the books [1], [3], [4], [5] and [6] aimed at a rather general public, with the latter two covering applications of mathematics, and the book [2] that is more specialized. Could you explain what profiles of readers were you having in mind and describe what kind of comments from them would please you most?

In general terms almost all these books are addressed to people who have some minimal background in mathematics (secondary school) but who have not pursued technical studies. My philosophy is that they should be comfortable with some (small) degree of abstraction and, more importantly, to have a little predisposition to liking mathematics. However, for my book on transcendental numbers, written together with Javier Fresán, one needs to have some extra background (maybe 1-2 years of a bachelor in mathematics), as at the end we talk about modular forms, elliptic curves and other related number theoretical objects. Of course the idea with these books is to "hear the music" and not enter in the technical details.

The book [7] has a pedagogical character, but it is special in view of the problem solving approach to real analysis in a mathematics bachelor degree. In what context did it arise? What reception does it have on the part of the students?

This is a joint project with Santiago Boza from DMAT and Oscar Rivero, who was the living force of this project. The fact was that we were teaching this course at the **FME** just when pandemics started, and during the semester we prepared tons of ad hoc material for students. Oscar had the idea to use all this material to create a book, and the result happily ended with a nice publication. This is the first academic year the book is available, so we do not yet have feedback from the students, but I hope it will be useful to understand this course (which acts as a bridge between elementary calculus and a more abstract one).

It may be a good moment to ask you for your teaching record in terms of institutions, programs, and the specific subjects?

This is quite diverse. I have lectured bachelor, master and advanced courses in several countries and institutions, including of course Spain, but also France, Germany, Colombia and Peru. This goes from basic courses on linear algebra/calculus to advanced courses in extremal combinatorics or quantum computing. I like to change from time to time the courses I lecture. Typically, each year I take a course which I have not taught in the previous years.

Along your career you have also been committed to several administrative responsibilities. Can you specify what they have been in general, and at the FME in particular?

During the last years I have had some responsibilities at the **FME**. From march 2019 up to very recently (march 2023) I was Vicedean at **FME**, coordinating the **UPC** doctoral program in Applied mathematics as well as the master's degree in Mathematics. I had the chance to work with Jaume Franch [interviewed in this issue, page 4], who was a wonderful Dean and a great colleague, especially during the COVID confinement period (which was definitely new for everybody and we needed to use creativity to solve issues). Right now I am acting as Academic Secretary of the **FME** for the new managing team led by Jordi Guàrdia [interviewed in NL04, pages 6-8]. I thank him for the trust he put on me by making this proposal. Finally, apart from that, I have had several different administrative tasks in different institutions (for instance, inside the CRM and the BGSMath). Maybe the funnier one occurred some years ago, while in Paris, where I was the President of the Spanish researchers at the Cité Universitaire. Someone told me (I do not know if this is true, most likely not) that this job had an institutional status in France (10 or 12 levels below the ambassador), and this is why during that time I had meetings with, for instance, Ángel Gabilondo (who was at that time Minister of Education) to talk about the situation of Spanish expats researchers in France (2010 was an specially bad moment for researchers who were finishing their graduate studies).

Now we would like to turn to your mathematical pursuit. Can you share your memories about what sparked your mathematical vocation?

There is a funny story about that. As many people at a young age I liked most of all subjects during primary and secondary school. At 1st BUP (now this would be 3rd ESO) our Biology class had to make a visit to Delta de l'Ebre. The same day there was the Math Kangaroo contest [Cangur] and students had to decide what to do. I do not know why, but I chose to stay and participate at the Cangur. I got some good results and so I had to go from my hometown (Lleida) to the Institut d'Estudis Catalans to get the prize (which is something a little intimidating for someone coming from an small town). From this experience I saw that I liked mathematics, and so I wanted to learn more. I have to thank here the selfless work of many people specially around the **FME**, and among whom Josep Grané undoubtedly stands out.

What circumstances concurred in your decision about what to study at the university? Was it an easy choice? How do you value your university education?

I had the chance to move from Lleida to Barcelona to study at **UPC** thanks to the economic support of my family. I was trained at **UPC**, in what today is **CFIS**. Although **CFIS** was not yet created, I belonged to one of the first cohorts pursuing a double degree at **UPC**. I have to say that the level of studies I received at both **FME** and **ETSETB** was wonderful. At **FME**, which is the school I know best, I have to say that my impression is that the level we are offering has not gone down, and in fact that is has increased. In general, we can feel proud of the education level we are providing to our students.

We would also like to know about your research training, fundamentally about your doctoral and postdoctoral work.

I started my PhD under the guidance of Marc Noy here at **UPC** in the area of enumerative combinatorics, and more precisely on the study of random planar graphs (and related families). After defending my PhD in 2009 I started my first postdoc in Paris, under the guidance of Gilles Schaeffer, who is one of the top leading researchers in map enumeration and had at that time one of the first ERC projects. After that I moved to Madrid for 3 years to work with Javier Cilleruelo. During this 3 wonderful years at ICMAT I changed my main topic of research and started working on additive combinatorics. Finally, before moving to Barcelona in 2016, I was a Professor–W1 (Junior professor, which was essentially an 6-year assistant professorship in the Germany level but without tenure track at that time) at Freie Universität Berlin. The city was incredible and the German system was very interesting; I had lots of academic experiences during my stay in Berlin that definitely afforded me some perspective on how to manage research.

Can you summarize the main results of your research so far?

This is difficult to say, because I have worked in very different types of problems. Maybe the two results I am proudest of are a proof obtained with Javier Cilleruelo in 2008 (still during my PhD) solving a long-standing question in additive combinatorics (from 1992) of Vera Sós and András Sarközy. Also, in this "hungarian type" mathematics, jointly with Marc Noy and Vlady Ravelomanana from Paris we solved an even longer standing question posted by Paul Erdös and Alfréd Rényi in their seminal paper *On the evolution of random graphs*: we got the very precise expression for the probability of planarity of a random graph when taken inside the critical window. No such exact expression was known until our result appeared.

Looking ahead, how do you see your research in the next few years? What problems would you like to see solved?

The more I work, the more I see that I have to study more, and the more I note that there is a big sea still to learn. But in order to give one general idea, a very important trend nowadays in discrete mathematics is the study of limiting objects of discrete structures, which in many situations are continuous objects. This fact is an important leivmotiv in random discrete structures and I would like to explore a little more these type of results in the next years. In fact, the content of my Dou Prize essay goes in this direction!

Closing somehow a loop, we would like to know your experience in tutoring and advising students at all levels. Along with research and teaching, perhaps also some administration, it is generally regarded as one of the important roles a university professor has to play.

Definitely, and one of the most enjoyable activities in our job. Discussing with younger researchers, with more energy, is always a source of satisfaction. And of course, later these students become friends and colleagues. For instance, very recently, jointly with Oriol Serra and Miquel Ortega (who started his PhD at **UPC** this year) we have solved a conjecture on product-free sets on the free group. Sharing ideas with them and the very fresh contributions of Miquel have been very pleasant!

Finally, we would appreciate if you could speak about IMTech and put forward ideas and strategies that would enhance its positive future evolution.

IMTech has been a very important and transversal initiative at **UPC** level that I really appreciate, which it brings together the mathematical activity at **UPC**.

I think this is a very crucial point that must be remarked. I do not have much good advice and suggestions, but maybe **IMTech** could be the catalyst of the interplay between researchers and industry: this is specially challenging in the case of pure mathematicians.

Thank you so much for your time and insights, and congratulations for the Albert Dou prize.

Thanks for your time, and for your work preparing this material.

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Adrián Ponce Álvarez started with a Ramón y Cajal contract at the UPC in September 2022. His research is focused on the study of neural networks, at different scales and in different states. He uses a combination of data analysis and theory, by means of stochastic dynamical systems, information theory, statistical mechanics, machine learning, and network theory.

He completed his master's degree in Physics at University of Paris-Sud 11 in 2006 and did his PhD in Neurosciences in Aix-Marseille University, France. After finishing his PhD in 2010, he was a postdoctoral fellow at Gustavo Deco's lab, after which he moved to the Departament de Matemàtiques at UPC to start his current position.

NL. ***Welcome to the UPC, and congratulations for the Ramón y Cajal contract. Could you comment on what features of UPC contributed to your decision to come here with your RyC?***

Thank you, I am happy to be at the UPC and to be able to continue my research in the framework of the Ramón y Cajal. I highly value the commitment that the UPC has with scientific research and, before coming here, I already knew the theoretical neuroscience research groups that work in the Dynamical Systems Group of the Department of Mathematics at the UPC, specifically Gemma Huguet and Antoni Guillamon.

Your master's degree in Physics of Biological Systems at the University of Paris-Sud suggests a strong interdisciplinary mix. Was it decisive in your research orientation? What was the topic of your master thesis?

During my master's in Physics of Biological Systems I learned tools to study biological systems as interacting systems with emergent properties. That master focused mainly on dynamical systems and statistical physics. During those years, I devoured the book Mathematical Biology by James Murray. This background has allowed me to ask neuroscientific questions from a system perspective. My Master thesis was about resonance phenomena in a single-cell model, i.e., a leaky integrate-and-fire model with hyperpolarization-activated currents, supervised by Claude Meunier (Paris Descartes University).

What was the topic of your PhD thesis? What are your recollections of that research period?

During my PhD at Aix-Marseille University (funded by a personal PhD grant from the Mexican Government), I studied the neuronal activity during delayed motor preparation and decision-making, in Alexa Riehle's laboratory (CNRS). For this, I trained monkeys to perform cognitive tasks and recorded the neuronal activity in several cortical areas, as well as the animal's behavior. Due to this experience, I learned to design and conduct behavioral experiments, perform extracellular recordings in vivo, and analyze and interpret neuropsychological data. I used tools from information theory and machine learning to analyze the recorded data. Acquiring some experience on experimental neuroscience has been extremely useful to develop my research on theoretical neuroscience and to stablish collaborations with experimentalists.

Let us move forward to your postdoctoral position in Deco's lab. What aspects of those stays would you like to highlight?

My interest in the mechanisms underlying brain function led me to transit towards theoretical neuroscience. In 2011, I joined the group of Gustavo Deco (UPF), as a postdoctoral fellow, to study neural networks at the microcircuit and whole-brain levels. To study these models, I acquired expertise in stochastic dynamical systems, graph theory, and information theory. I used this knowledge to investigate how network statistics emerge and affect information processing. In a series of works, we studied the emergence of state-dependent neural interactions (among neurons or among brain regions) in different brain states, task conditions, diseases, and neuromodulatory processes, and their functional consequences in terms of information capacity and communication. Some of these models were included in The Virtual Brain, an open-source platform for whole-brain simulations constraint by biological connectivity data (*connectomes*) to test both scientific and clinical questions.

Along with the study of dynamical systems to understand neural systems, I studied these systems with a statistical mechanics approach, which has proven to be useful to link collective behavior in high-dimensional systems, model inference, and information. By inferring models from neuronal activity, we derived macroscopic properties that quantify the system's capabilities to process and transmit information. By studying these inferred models, together with simulations of neural networks, we have showed that cortical/brain states relate to phase transitions. Of particular interest for information processing, we found evidence of critical behavior in neural systems.

Gustavo Deco has supported me a lot, both scientifically and personally, allowing me to develop my own ideas.

You have collaborated with researchers from different disciplines, particularly biology and medicine. How has been the experience?

I truly admire experimental scientists. I'm always impressed by their intuitions about the mechanisms behind the biological phenomena they study. Nowadays experimental neuroscientists have a good understanding of mathematical models and, thus, in an authentic collaborative work, where time is invested, a fruitful dialogue between experiments and theory can be established. In recent years, I had the opportunity to participate in the design of experiments to answer theoretical questions.

Which are in your view the main open questions in theoretical neuroscience? How mathematics can contribute?

There are several open questions, but in my opinion, an interesting one is how neural systems self-organize during development and during interaction with the environment. We need analytic tools and models to understand how such a high-dimensional system, as the brain, self-organizes in interaction with a changing environment.

What vision do you have for your research during the RyC contract? What would you like to achieve?

In the following years, I would like to lead a research group to pursue my research on statistical mechanics and dynamical systems applied to study empirical and theoretical neural networks to answer fundamental neuroscience questions, in collaboration with national and international research institutions.

In the mathematical ecosystem of the UPC, there is a wide room for teaching subjects in several studies offered by the FME at various levels. Can you share how do you see your role in that respect for the next few years?

I'm really enjoying teaching within the Mathematical Models in Biology course of the Master's in Advanced Mathematics and Mathematical Engineering (MAMME). For me, mathematical biology has been always fascinating. I think it conveys a mix of esthetic experience, intellectual challenge, and social relevance. I would like to transmit this drive to the students.

You are strongly committed in favour of gender, race, and class equity in Academia. How do you see the current situation? How can we have more women and minorities involved in mathematics?

I consider it necessary to conduct research and teach from an inclusive-conscious perspective that seeks to include social minority groups in academia and to consider the mechanisms that could exclude these groups. I have co-organized meetings to discuss scientific papers on gender, race, sexual identity, and class equity in academia (Gender and Science Journal Club). Through this group, we have successfully convinced research centers to implement gender balanced policies and to confront unconscious biases. We have organized several PhD seminars on gender issues at the UPF-DTIC PhD program and we have been invited to round tables on gender-equity and equality plans at several research institutions in Barcelona, but also outside the academia, which is very important. Furthermore, I was member of the Center for Gender Studies (CEdGE) at the UPF since its foundation in 2015 until 2022. The CEdGE is a think tank that creates multidisciplinary knowledge and interdisciplinary research on gender by exploring the complex interactions between gender and class, identity, ethnicity, sexuality or functional diversity.

I think it is very important that discussions on discrimination within the academia also happens outside the academia and not only in the closed environment of the University.

We hope that you will enjoy your contract and that it will turn out to be a most positive time for your academic career. We also wish you good luck!

Thank you!

Research focus

Quantum character varieties

by Marta Mazzocco (School of Mathematics, Birmingham).
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In this note we will introduce the concept of character variety in the simple example of a torus with one disk removed. We will show how this is a surface in $\mathbb{C}^3$ defined by a cubic polynomial, called the *Markov cubic*. We will show the relation between the Markov cubic, singularity theory, Painlevé differential equations, and introduce a cluster algebra structure on it which is related to Markov numbers. We will discuss quantisation of the Markov cubic in the context of a wider class of surfaces and relate it to Sklyanin algebra.

Main motivation

"Symmetry, as wide or narrow as you may define its meaning, is one idea by which humans through the ages have tried to comprehend and create order, beauty and perfection" (Hermann Weyl). As mathematicians, we chase symmetry. The most profound and far reaching idea in physics is Emmy Noether theorem: the symmetries of a system imply the existence of conserved quantities along its evolution.

One of the most puzzling symmetries discovered nowadays is mirror symmetry (MS). In string theory, particles are replaced by strings and six extra small dimensions are needed to describe the universe. These are wrapped up in Calabi-Yau (CY) varieties that occur in pairs: CY in the same pair produce equivalent physical theories. Following Noether's idea, one way to comprehend MS is to study its fixed points, namely self mirrors. In this note I will discuss some examples of self mirrors which appear in several different contexts in mathematics.

Toy introduction to MS: Quantum mechanics analogy

Schrödinger picture	*Heisenberg picture*
State vectors evolve in time, operators mostly constant: $\lvert\psi(t)\rangle = U(t,t_0)\,\lvert\psi(t_0)\rangle$. Quantisation of the phase space: *geometric quantisation.*	State vectors are time independent and the observables evolve in time. This is the quantisation of the algebra of functions on the phase space: *deformation quantization.*

By the Stone–von Neumann theorem, these two pictures are equivalent (just a basis change in the Hilbert space).

Mirror symmetry

Similarly to the case of quantum mechanics, in MS we have two different sides: an A-side and a B-side.

Mirror pairs of CY varieties (compact Kähler manifolds with vanishing first Chern class and Ricci flat metric) are of very different nature but the symplectic geometry of the A-side is reflected in the algebraic geometry of its mirror:

A-side	***B-side***
Gromov-Witten (GW) invariants: two objects with the same complex structure have the same GW invariants. (Y, D) log symplectic CY *Strominger-Yau-Zaslow quantisation.*	Landau-Ginzburg models: two objects with the same symplectic structure are equivalent. Spec(Ring) *Deformation quantization* *(for example Etingof-Ginzburg).*

The physical theories produced by the two sides are equivalent.

The aim of this note is to present a class of examples (affine del Pezzo varieties) that are self mirrors (can be on both sides). These examples are relevant in several branches of mathematics:

- ❑ Number theory
- ❑ Singularity theory
- ❑ Orthogonal polynomials
- ❑ Moduli spaces
- ❑ Painlevé equations
- ❑ Cluster algebras

The Markov cubic

$$\mathcal{M} = \{(x_1, x_2, x_3) \in \mathbb{C}^3 | x_1^2 + x_2^2 + x_3^2 - 3x_1x_2x_3 = 0\}$$

$\mathcal{M} \cap \mathbb{Z}_+^3 = \{\text{Markov triples}\}$. Startig from $(1, 1, 1)$, all Markov triples are produced by permutations and Vieta jumping equivalent to the following three operations called "mutations":

$$\mu_1 : (x_1, x_2, x_3) \to \left(\tfrac{x_2^2+x_3^2}{x_1}, x_2, x_3\right)$$
$$\mu_2 : (x_1, x_2, x_3) \to \left(x_1, \tfrac{x_1^2+x_3^2}{x_2}, x_3\right)$$
$$\mu_3 : (x_1, x_2, x_3) \to \left(x_1, x_2, \tfrac{x_1^2+x_2^2}{x_3}\right)$$

Examples:

$$\begin{aligned}(1, 1, 1) &\xrightarrow{\mu_2} (1, 2, 1) \xrightarrow{\mu_1} (5, 2, 1) \xrightarrow{\mu_2} (5, 13, 1)\\ &\xrightarrow{\mu_3} (5, 13, 194) \xrightarrow{\mu_2} (5, 2897, 194)\\ &\xrightarrow{\mu_1} (1686049, 2897, 194)\\ &\xrightarrow{\mu_2} (1686049, 981277621, 194)\end{aligned}$$

Note that despite dividing by an integer at each step, the result is not a rational number, but an integer again. This is a consequence of the so-called Laurent phenomenon in cluster algebra.

Cluster algebras

This is a class of commutative rings introduced by Fomin-Zelevinsky in 2002. They are integral domains with some sets of size n called *clusters*. Given a cluster $(x_1, \dots, x_n)$, all other clusters are generated by mutations of the form

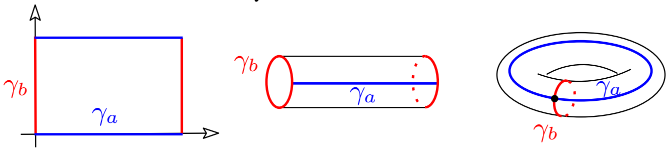
$$\mu_k : x_i \mapsto x_i \textbf{ for } i \neq k, \quad x_k \mapsto x_k' := \tfrac{m_1+m_2}{x_k},$$

where m_1 and m_2 are monomials in $(x_1, \dots, x_n)$ obeying some rules.

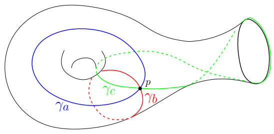
Theorem (Laurent phenomenon). Given any two clusters in a cluster algebra, $(x_1, \dots, x_n)$ and $(\tilde{x}_1, \dots, \tilde{x}_n)$, then $\tilde{x}_i$ is a Laurent polynomial of $(x_1, \dots, x_n)$, $i = 1, \dots, n$.

Example. The integers $\mathbb{Z}$ with the Markov triples form a cluster algebra, therefore any component of a Markov triple is a Laurent polynomial of $(1, 1, 1)$.

The Markov cubic is related to the character variety of a torus with one boundary.



Standard torus $\mathbb{T}^2$: $\pi_1(\mathbb{T}^2, p) = \langle \gamma_a, \gamma_b \rangle \simeq \mathbb{Z} \times \mathbb{Z}$, as $\gamma_a\gamma_b \sim \gamma_b\gamma_a$.



Torus with one boundary:
$\pi_1 = \langle [\gamma_a], [\gamma_b], [\gamma_c] \rangle$, $\gamma_a\gamma_b\gamma_c \sim \gamma_0$,
γ_0 loop around the removed disk.

Its character variety is defined as $\mathrm{Hom}(\pi_1, \mathrm{SL}_2(\mathbb{C}))/\mathrm{SL}_2(\mathbb{C})$. To write this as an affine surface, $\forall \gamma \in \pi_1$, $M_\gamma \in \mathrm{SL}_2(\mathbb{C})$, $M_aM_bM_c = M_0$ up to a global conjugation. Thus $\mathrm{tr}(M)$ are invariant and $x_i = \mathrm{tr}(M_i)$ for $i = a, b, c$ satisfy a cubic equation:

$$x_ax_bx_c - (x_a^2 + x_b^2 + x_c^2) = \text{constant}.$$

This is equivalent to the Markov cubic up to a rescaling of x_a, x_b, x_c.

Character variety of the 4-holed sphere

By a quadratic transformation, the Markov cubic is related to the character variety of a sphere with 4 punctures. More generarlly, for the case of 4 holes, this is:

$$\{(x_1, x_2, x_3) \in \mathbb{C}^3 | \\ x_1x_2x_3 - x_1^2 - x_2^2 - x_3^2 = \omega_1x_1 + \omega_2x_2 + \omega_3x_3 + \omega_4\},$$

where $\omega_1, \omega_2, \omega_3, \omega_4 \in \mathbb{C}$ are parameters.
–Monodromy manifold of the sixth Painlevé equation.
–Versal deformation of a du Val D_4 singularity at $(2, 2, 2)$.
–Most importantly, it is self mirror (see [1]).

I will show how to quantise a wide class of affine surfaces that contains this example as a sub-case.

Quantization of affine surfaces

We focus on a special class of affine surfaces of the form

$$\mathcal{M}_\varphi = \{(x_1, x_2, x_3) \in \mathbb{C}^3 | \varphi(x_1, x_2, x_3) = 0\},$$

where

$$\varphi(x_1, x_2, x_3) = x_1x_2x_3 + \phi_1(x_1) + \phi_2(x_2) + \phi_3(x_3),$$

with $\deg(\phi_i) \leqslant 6$. Examples:
– Affine del Pezzo surfaces: blowup of $9 - d$ points of $\mathbb{P}^2$.
– Deformations of elliptic singularities (e.g. affine cone surfaces with elliptic singularity) [2] and Kleinian sigularities [3] (weighted projective del Pezzo with a nodal singularity).

Poisson structure. Given any $\varphi \in \mathbb{C}[x_1, x_2, x_3]$, we get a Poisson bracket on $\mathbb{C}[x_1, x_2, x_3]$, i.e. a bilinear map

$$\{\cdot, \cdot\} : \mathbb{C}[x_1, x_2, x_3] \times \mathbb{C}[x_1, x_2, x_3] \to \mathbb{C}[x_1, x_2, x_3]$$

that is skew-symmetric and satisfies the Leibniz rule and the Jacobi identity. This is defined in terms of the potential φ as:

$$\{x_1, x_2\} = \tfrac{\partial\varphi}{\partial x_3}, \ \{x_2, x_3\} = \tfrac{\partial\varphi}{\partial x_1}, \ \{x_1, x_3\} = \tfrac{\partial\varphi}{\partial x_2}.$$

Note that the potential φ not only defines the Poisson relations but is also central $\{\varphi, \cdot\}$. Therefore the Poisson structure descends to the ring of functions on the surface

$Z(\varphi) = \{(x_1, x_2, x_3) \in \mathbb{C}^3 | \varphi(x_1, x_2, x_3) = 0\}$. In other words, $A_\varphi = (\mathbb{C}[x_1, x_2, x_3], \{\cdot, \cdot\})/\langle\varphi\rangle$ is a Poisson algebra, so that the affine del Pezzo surface given by the zero set of φ, $Z(\varphi)$, is Spec(A_φ).

For our chosen φ we have

$$\{x_1, x_2\} = x_1 x_2 + \frac{\partial \phi_3}{\partial x_3}, \text{ and cyclic.}$$

To quantize, we define a suitable Lie algebra isomorphism:

$(A_\varphi, \{\cdot,\cdot\})$	$\longrightarrow$	$(\mathcal{A}^\hbar_\Phi, [\cdot,\cdot])$
commutative Poisson symmetric algebra		non commutative flat deformation

Example: For

$$\varphi = x_1 x_2 x_3 - x_1^2 - x_2^2 - x_3^2 + \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 + \omega_4,$$

we obtain the Askey-Wilson algebra [4]

$$\begin{gathered} q^{-\frac{1}{2}} x_1 x_2 - q^{-\frac{1}{2}} x_2 x_1 = (q^{-1} - q) x_3 - \Omega_3 x_3, \\ q^{-\frac{1}{2}} x_2 x_3 - q^{-\frac{1}{2}} x_3 x_2 = (q^{-1} - q) x_1 - \Omega_1 x_1, \\ q^{-\frac{1}{2}} x_3 x_1 - q^{-\frac{1}{2}} x_1 x_3 = (q^{-1} - q) x_2 - \Omega_2 x_2, \\ [\Omega_i, \cdot] = 0,\ i = 1, \ldots, 4. \end{gathered}$$

Semiclassical limit: $\frac{[x_i, x_j]}{1-q} \underset{q\to 1}{\longrightarrow} \{x_i, x_j\}$ gives the character variety of a sphere with four boundaries.

The Painlevé–Sklyanin algebra

Definition. For any choice of the scalars

$$\alpha_i,\ \beta_i,\ a_i,\ b_i\ \in \mathbb{C},\ i = 1, 2, 3,$$

such that α_i are not roots of unity, the *generalised Sklyanin-Painlevé algebra* is the non-commutative algebra with generators X_1, X_2, X_3 defined by the relations:

$$\begin{aligned} X_2 X_3 - \alpha_1 X_3 X_2 - \beta_1 X_1^2 + a_1 X_1 + b_1 = 0, \\ X_3 X_1 - \alpha_2 X_1 X_3 - \beta_2 X_2^2 + a_2 X_2 + b_2 = 0, \\ X_1 X_2 - \alpha_3 X_2 X_1 - \beta_3 X_3^2 + a_3 X_3 + b_3 = 0. \end{aligned}$$

We fully characterise for which cases the generalised Sklyanin-Painlevé algebra is a Calabi Yau algebra has nice properties [5]:

Theorem. For specific choices of the parameters as follows:

1. $\alpha_1 = \alpha_2 = \alpha_3 \neq 0$ and $(\alpha_1^3, \beta_1\beta_2\beta_3) \neq (-1, 1)$,
2. $(\alpha_1, \alpha_2, \alpha_3) \neq (0, 0, 0)$ and either $\beta_1 = \beta_2 = \alpha_1 - \alpha_2 = 0$ or $\beta_3 = \beta_1 = \alpha_3 - \alpha_1 = 0$ or $\beta_2 = \beta_3 = \alpha_2 - \alpha_3 = 0$,
3. $\beta_1 = \beta_2 = \beta_3 = 0$ and $(\alpha_1, \alpha_2, \alpha_3) \neq (0, 0, 0)$,

the generalised Sklyanin-Painlevé algebra is Calabi-Yau, has a Hilbert series with polynomial growth and is Koszul.

Note that for $a_i = b_i = 0$, $i = 1, 2, 3$, the Sklyanin–Painlevé algebra restricts to the so called Artin–Schelter–Tate–Sklyanin algebra with three generators. For $\beta_1 = \beta_2 = \beta_3 = 0$ and $(\alpha_1, \alpha_2, \alpha_3) \neq (0, 0, 0)$ this algebra produces the monodromy manifolds of the Painlevé differential equation in the semi-classical limit.

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Reducing the dynamics of a large interacting system,

by Marina Vegué (DMAT)

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The natural world is full of large systems of interacting units. A human brain, for example, contains around 86 billion ($86 \cdot 10^9$) neurons whose activities evolve in time as a function of the activities of the neurons they are connected to. An ecosystem can also be regarded as an interacting network in which the units are the different species and their abundances are variables that depend on the abundances of the species they interact with. A social network is another environment through which information, opinion or diseases can spread or disappear depending on the structure of its interactions.

Systems as the ones defined by these examples are usually dubbed "complex", a broad term that is used to emphasize the fact that their behavior is often difficult to study and predict. A natural question to ask is whether a generic complex system composed of N interacting units or nodes (e.g., neurons, species, individuals) can be reduced to a simpler system composed of $n \ll N$ units. In other words: if one can construct a smaller system that is easier to treat and understand while preserving some of the characteristic properties of the original system. Imagine an ecosystem in which the species' abundances are relatively stable in time. These abundances may change smoothly as some perturbation is introduced, for example a change in the average annual temperature, which alters the capacity of some species to survive or reproduce. If the perturbation is too strong, however, this equilibrium could disappear and the system might abruptly drift to a state characterized by a partial or total species extinction. It is thus important to know what is the critical temperature at which such a catastrophe occurs, and we would like a reduced version of the system to have an analogous transition at roughly the same temperature. Being able to construct such a reduced system is not only useful for practical reasons (i.e., simulating it and predicting its behavior using less resources) but also from a purely theoretical point of view, because in doing so we can learn about the inner workings of the original system and the reduced dimension n is a measure of the "effective complexity" that characterizes it.

In 2016, Gao, Barzel and Barabási [1] proposed a way to reduce a system on N nodes whose activity evolves in time according to

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} w_{ij}\, g(x_i, x_j), \tag{1}$$

where $x_i = x_i(t)$ is the activity of node i at time t and $w_{ij} > 0$ stands for the weight of the positive interaction from node j to node i. Functions f and g define the nodes' self-dynamics and the dynamical coupling between pairs of nodes, respectively.

They proposed to reduce the system to a system of dimension $n = 1$ whose only *effective* (or *macroscopic*) activity $\mathcal{X}_{\text{eff}}$ is a linear combination of the activities in the original system according to

$$\mathcal{X}_{\text{eff}} = \sum_{i=1}^{N} a_i\, x_i, \qquad a_i = \frac{w_i^{\text{out}}}{\sum\limits_{j=1}^{N} w_j^{\text{out}}} = \frac{\sum\limits_{j=1}^{N} w_{ji}}{\sum\limits_{j,k=1}^{N} w_{kj}}. \tag{2}$$

The quantity w_i^{out} is the (weighted) out-degree of node i. Eq. (2) thus states that the effective activity is a weighted average of the microscopic activities according to a positive vector $\boldsymbol{a} = (a_i)_{i=1}^N$ with sum 1 whose entries are proportional to the nodes' out-degrees. The authors showed that $\mathcal{X}_{\text{eff}}$ approximately follows an ODE which has the same structure as the original dynamics:

$$\dot{\mathcal{X}}_{\text{eff}} \approx f(\mathcal{X}_{\text{eff}}) + \mathcal{W}_{\text{eff}}\, g(\mathcal{X}_{\text{eff}}, \mathcal{X}_{\text{eff}}), \tag{3}$$

the effective interaction weight $\mathcal{W}_{\text{eff}}$ being the weighted average, according to the same vector $\boldsymbol{a}$, of the in-degrees:

$$\mathcal{W}_{\text{eff}} = \sum_{i=1}^{N} a_i\, w_i^{\text{in}} = \sum_{i,j=1}^{N} a_i\, w_{ij}. \tag{4}$$

They argued that such a one-dimensional reduction can capture the tipping points of several complex systems as different parameters are perturbed.

Despite some systems may be effectively reduced this way, it is likely that other systems will not be properly represented by a system of dimension just one. So can we generalize this result in some way? Can we construct a set of $n \ll N$ linear effective variables —called *observables* from now on— and a reduced matrix of interactions $\boldsymbol{\mathcal{W}} = (\mathcal{W}_{\nu\rho})_{\nu,\rho=1}^n$ whose dynamics, at least approximately, has the same form as the original one? In what follows I will outline a possible way to that end (exposed in detail in Ref. [2]), assuming that the original system's evolution is given by Eq. (1). Fig. 1 provides a visual sketch of the process.

Imagine that we start by partitioning the units in our network according to their connectivity properties so that nodes within a group connect similarly to nodes in other groups. A neuronal network, for instance, might contain functional modules composed of neurons that are densely connected among them but sparsely connected to neurons in other modules. In an ecological network, species that exploit similar niches (and thus interact with other species in a similar way) could be grouped together. If we do not have any information about the presence of modules in the network, we could use a community-detection algorithm on the matrix of interactions $\boldsymbol{W} = (w_{ij})_{i,j=1}^N$ and try to identify them.

Our reduced system will have as many variables as the number of groups in the partition, n. We denote by $G_1, \cdots, G_n$ these groups. We require the ν-th observable to be a weighted average of the activities within group G_ν, that is, denoting by $m_\nu = |G_\nu|$ the size of G_ν, we want to find a non-negative vector $\boldsymbol{a}_\nu = (a_{\nu i})_{i=1}^{m_\nu} \in \mathbb{R}^{m_\nu}$ with sum 1 and construct from it the observable associated to G_ν,

$$\mathcal{X}_\nu = \sum_{i=1}^{m_\nu} a_{\nu i}\, \widehat{x}_{\nu i}, \tag{5}$$

where $\widehat{\boldsymbol{x}}_\nu = (\widehat{x}_{\nu i})_{i=1}^{m_\nu} = (x_j)_{j\in G_\nu}$ is the vector of activities of nodes in group G_ν. We call the vectors $\boldsymbol{a}_1, \cdots, \boldsymbol{a}_n$ the *reduction vectors*.

The next step is to write down the exact ODE for the evolution of the ν-th observable, which is a function of $f(\widehat{x}_{\nu i})$ and $g(\widehat{x}_{\nu i}, \widehat{x}_{\rho j})$ for $i \in \{1, \cdots, m_\nu\}, \rho \in \{1, \cdots, n\}$ and $j \in \{1, \cdots, m_\rho\}$, and, generically, cannot be expressed as a function of the observables only. To approximately close the observables' dynamics, we hypothesize that, if the node partition is defined properly, nodes within the same group will have similar activities. Thus, these activities will be close to the corresponding observable and it makes sense to use first-order Taylor approximations of $f(\widehat{x}_{\nu i})$ and $g(\widehat{x}_{\nu i}, \widehat{x}_{\rho j})$ around $\mathcal{X}_\nu$ and $(\mathcal{X}_\nu, \mathcal{X}_\rho)$, respectively. In doing so, we obtain an approximate system of ODEs that will not be closed unless the reduction vectors fulfill the following conditions:

$$\boldsymbol{K}_{\nu\rho}\, \boldsymbol{a}_\nu = \mu_{\nu\rho}\, \boldsymbol{a}_\nu \tag{6a}$$

$$\boldsymbol{W}_{\nu\rho}^T\, \boldsymbol{a}_\nu = \lambda_{\nu\rho}\, \boldsymbol{a}_\rho, \tag{6b}$$

where $\nu, \rho \in \{1, \cdots, n\}$, $\boldsymbol{K}_{\nu\rho} = \text{diag}(w_1^{\text{in},\rho}, \cdots, w_{m_\nu}^{\text{in},\rho})$ is the $m_\nu \times m_\nu$ diagonal matrix whose diagonal is the vector of in-degrees of nodes in G_ν taking into account connections coming from G_ρ only, and $\boldsymbol{W}_{\nu\rho}$ is the $m_\nu \times m_\rho$ matrix of interactions from nodes in G_ρ to nodes in G_ν. The scalars $\mu_{\nu\rho}$ and $\lambda_{\nu\rho}$ are additional parameters to be found. Eqs. (6a),(6b) are called the *compatibility equations*. They impose constraints on the reduction vectors.

Once the compatibility equations are fulfilled, the approximate reduced dynamics has the form

$$\dot{\mathcal{X}}_\nu \approx f(\mathcal{X}_\nu) + \sum_{\rho=1}^{n} \mathcal{W}_{\nu\rho}\, g(\mathcal{X}_\nu, \mathcal{X}_\rho), \quad \mathcal{W}_{\nu\rho} = \sum_{i=1}^{m_\nu} a_{\nu i}\, w_i^{\text{in},\rho}. \tag{7}$$

The matrix $\boldsymbol{\mathcal{W}} = (\mathcal{W}_{\nu\rho})_{\nu,\rho=1}^n$ is thus the *effective interaction matrix* in the reduced system. Notice that both the approximate reduced dynamics and $\boldsymbol{\mathcal{W}}$ have the same form as Eqs. (3) and (4) proposed by Gao et al., for an arbitrary n. The difference with their approach is the construction of the observable vectors, as we will show now.

In our approach, we must solve the compatibility equations to find the reduction vectors. If the nodes in G_ν have similar connectivity properties, their in-degrees from G_ρ will be close to one another and $\boldsymbol{K}_{\nu\rho}$ will be close to a multiple of the identity matrix so Eq. (6a) can be assumed to be automatically fulfilled. We thus concentrate in solving Eq. (6b) for all ν, ρ, that is,

$$\boldsymbol{W}_{\nu\rho}^T\, \boldsymbol{a}_\nu = \lambda_{\nu\rho}\, \boldsymbol{a}_\rho, \qquad \nu, \rho \in \{1, \cdots, n\}. \tag{8}$$

For $n = 1$, this is a single equation and it states that the vector $\boldsymbol{a}_1$ must be an eigenvector of the transposed connectivity matrix $\boldsymbol{W}^T$. If this matrix is positive, since we ask the reduction vector to be non-negative, then it has to be the dominant, or Perron-Frobenius, eigenvector of $\boldsymbol{W}^T$, normalized to have sum 1. Despite in some particular situations the normalized dominant eigenvector and the vector of normalized out-degrees proposed by Gao et al. coincide, in general they are not the same.

For $n > 1$, Eqs. (8) are coupled, but we can decouple them using, again, the Perron-Frobenius theorem when $\boldsymbol{W}$ is a positive matrix. In this case, Eqs. (8) are equivalent to the following decoupled equations:

$$\boldsymbol{W'}_{\nu\rho} \boldsymbol{a}_\nu = \lambda'_{\nu\rho} \boldsymbol{a}_\nu \qquad \nu, \rho \in \{1, \cdots, n\},$$

$$\boldsymbol{W'}_{\nu\rho},\, \lambda'_{\nu\rho} := \begin{cases} \boldsymbol{W}_{\nu\nu}^T, & \lambda_{\nu\nu} \quad \text{if } \nu = \rho, \\ \boldsymbol{W}_{\rho\nu}^T \boldsymbol{W}_{\nu\rho}^T, & \lambda_{\rho\nu}\lambda_{\nu\rho} \quad \text{if } \nu \neq \rho. \end{cases} \tag{9}$$

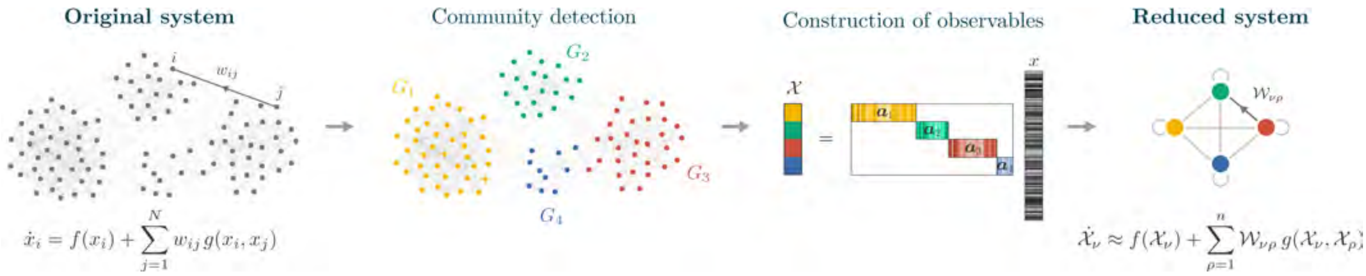

Figure 1: Schematics of the dimension-reduction strategy.

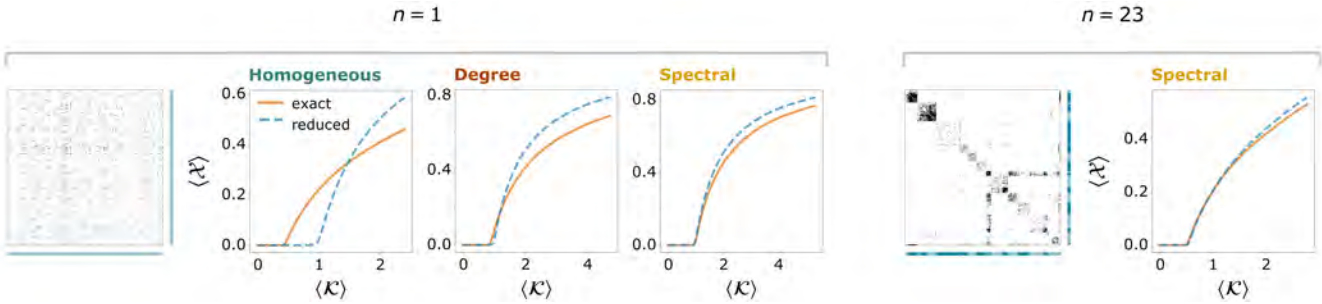

Figure 2: Dimension reduction results for a susceptible-infected-susceptible (SIS) infectious dynamics given by Eq. (1) with $f(x) = -x$ and $g(x, y) = (1 - x)y$ on a network based on Facebook contacts with $N = 362$ nodes [2]. We plot the average observable at equilibrium, $\langle \mathcal{X} \rangle$, as a function of the average weighted in-degree of the reduced system, $\langle \mathcal{K} \rangle$, (both of them weighted by the relative sizes of the different groups) for the exact and the reduced dynamics. The parameter $\langle \mathcal{K} \rangle$ increases as a consequence of multiplying all the interaction strengths by a global factor. For $n = 1$ (i.e., when considering the whole network as a unique group), we compare three reductions: a homogeneous reduction in which all the nodes contribute equally to the observable (left), the degree-based reduction defined by Gao et al. [1] (middle), and our spectral reduction (right). The last diagram shows the spectral reduction results when the nodes have been partitioned into $n = 23$ groups.

For a fixed ν, Eqs. (9) state that $\boldsymbol{a}_\nu$ has to be, simultaneously, the (normalized) dominant eigenvector of a collection of n positive matrices. Of course, such a vector will not exist in general because these matrices will not share their dominant eigenspace. To approximately solve the problem, we propose to set $\lambda'_{\nu\rho}$ to the dominant eigenvalue of $\boldsymbol{W}'_{\nu\rho}$ (this is what it would be if there were an exact solution) and then find the vector $\boldsymbol{a}_\nu$ that minimizes the quadratic error associated to Eqs. (9). We call this dimension-reduction strategy the *spectral reduction*.

Fig. 2 shows an application of the spectral reduction to a susceptible-infected-susceptible (SIS) infectious dynamics on a friendship network obtained from Facebook contacts, where nonexisting interactions were assumed to have an arbitrary small value [2]. In this example every unit's activity represents the probability of the node being infected. We plot the value of the average observable at equilibrium as we increase, along the x-axis, a global factor that multiplies all the interactions. There is a point from which the disease cannot be eradicated: the average observable becomes different from zero. This critical point is well predicted by the $n = 1$ spectral reduction, and it is better captured than when the reduction vector is either homogeneous or proportional to the vector of out-degrees. Moreover, the spectral diagram becomes more accurate as n increases.

What is the interpretation of the spectral reduction vectors for $n > 1$? From the decoupled compatibility equations [Eqs. (9)], we know that $\boldsymbol{a}_\nu$ has to be the dominant eigenvector of a collection of matrices. The first matrix is $\boldsymbol{W}^T_{\nu\nu}$, whose entries are the weights of the inverted interactions from nodes in G_ν to nodes in G_ν. The other matrices are $\boldsymbol{W}^T_{\rho\nu}\boldsymbol{W}^T_{\nu\rho}$ for every $\rho \neq \nu$, whose entries represent the weight sum of all the inverted 2-step paths from nodes in G_ν to nodes in G_ν going through nodes in G_ρ. The i-th component of the (normalized) dominant eigenvector of a non-negative matrix is the so-called "eigenvector centralitiy" of node i in the corresponding network: a measure of the relative importance of the node in the network (a variant of which is used by Google to rank web pages upon a search, for example).

In summary, our findings suggest that in order to reduce the dynamics by means of grouping nodes and defining linear observables within the groups, the contribution of each node to its corresponding observable should be the relative eigenvector centrality of the node within that group, with this centrality taking into account both direct (1-step) and indirect (2-step) interactions via nodes in other groups.

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PhD highlights

Alberto Larrauri defended his PhD thesis *First Order Logic of Random Sparse Structures* on March 03, 2023.

The thesis was produced within the **UPC** doctoral program on Applied Mathematics and his advisor was Marc Noy.

Starting May 2023, he will be a postdoctoral researcher at the University of Oxford in the group of Standa Živný.

Thesis summary

In the case of graphs, first order (FO) logic speaks about adjacencies using quantification over vertices and Bolean connectives. A natural question in this area is: given a sequence of random graphs $\mathcal{G}_n$, does the limit probability that $\mathcal{G}_n$ satisfy P exist for every FO property P? When the answer is yes, we say that $\mathcal{G}_n$ satisfies a FO *convergence law*. As a follow-up to the last question, we can also ask what are the possible values of those limit probabilities. We may also be interested in studying the FO properties P that hold in $\mathcal{G}_n$ with high probability (w.h.p.). This thesis makes contributions in all these directions, focusing on *sparse* (as opposite to dense) random structures.

In the first part, published in [1], we look at a binomial model of random relational structures. We prove that a FO convergence law holds in this model when each relation is expected to grow linearly with the number of elements in the structure. Moreover, we show that the limit probability of each FO statement is given by an analytic expression in terms of the density of each relation. We extend this result to more complex models where symmetry and anti-reflexivity constraints are imposed on the random structure. Finally, we give an application our convergence results to the study of random SAT, suggested by Albert Atserias. A line of research here tries to establish the existence of efficient algorithms that accurately decide satisfiability of random CNF formulas with high probability. Our contribution here states, roughly, that if P is a FO statement that implies unsatisfiability of CNF formulas, then w.h.p. P does not hold in random CNF formulas with linear number of clauses.

In the second part we consider several random models where a FO convergence law is known to hold, and we study the set L of limiting probabilities of FO statements. More concretely, we consider the linear ranges of binomial random graphs and binomial random uniform hypergraphs [2], as well as of random graphs with given degree sequences. Our main result here is that L is dense in the whole interval [0,1] exactly when the random model in question contains some cycle with asymptotic probability exceeding $1/2$. Otherwise the closure of L is a finite union of closed intervals with some 'gaps' in between.

The final chapter is devoted to the study of probabilistic versions of FO preservation theorems. These are classical results that relate semantic and syntactic classes of FO sentences. For instance, Lyndon's Theorem states that any monotone sentence is equivalent to some positive sentence, and Łoś-Tarski Theorem states that any sentence closed under extensions is equivalent to some existential sentence. Crucially, these results are stated for the class of all structures (both finite and infinite), and fail when restricted to finite structures. We define probabilistic versions of those two preservation theorems where logical equivalence is replaced by almost sure equivalence, and ask whether those new results hold in several random graphs, obtaining multiple positive results. We consider the binomial random graph at the connectivity threshold, in the linear range, and in the sublinear range. Additionally, we also look at uniform random graphs from addable minor-closed classes.

Selected Publication: [2].

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Franco Coltraro defended his PhD thesis *Robotic manipulation of cloth: mechanical modeling and perception*, supervised by Professors Jaume Amorós and Maria Alberich-Carramiñana, on March 30th, 2023 within the UPC doctoral program in Applied Mathematics. Currently, he is a postdoctoral researcher at the Institut de Robòtica i Informàtica Industrial (IRI), CSIC-UPC in Barcelona.

Thesis summary

We introduce a new cloth model for the dynamics of textiles as inextensible surfaces. This assumption challenges most models in literature where elasticity is allowed, sometimes by necessity (Textile Engineering) or in the pursuit of spectacularity (Computer Graphics). Inextensibility is modeled as follows: we assume that our cloth S is a surface (with boundary) moving through space whose metric (first fundamental form) is preserved. In order to implement these conditions (which are in fact PDE's) on a computer, we assume that S has been triangulated (or quadrangulated) and then apply a novel and non-trivial Finite Element discretization to the inextensibility constraints [1]. If we denote by $\varphi(t)$ the position of the nodes of the polyhedron, this gives raise to a smooth (actually quadratic) constraint function $\mathbf{C}(\varphi) = 0$ which must be preserved at all times. Making use of Signorini's contact model, the dynamic equations of motion then would be:

$$\begin{cases} \mathbf{M}\ddot{\varphi} = \mathbf{F}(\varphi, \dot{\varphi}) - \nabla\mathbf{C}(\varphi)^{\intercal}\boldsymbol{\lambda} + \nabla\mathbf{H}(\varphi)^{\intercal}\gamma, \\ \mathbf{C}(\varphi) = 0, \\ \mathbf{H}(\varphi) \geqslant 0, \quad \gamma \geqslant 0, \quad \gamma^{\intercal} \cdot \mathbf{H}(\varphi) = 0, \end{cases} \tag{10}$$

where we have grouped in the force term $\mathbf{F}$ damping, gravity, stiffness, aerodynamics, friction, etc. On the other hand, $\mathbf{H}(\varphi) \geqslant 0$ contains the implicit equation of a given obstacle (e.g. a table) and in addition self-collision constraints. Since the collision and inextensibility forces turn out to be very stiff, the system must be integrated implicitly. This is done by solving a sequence of quadratic problems with linear constraints (basically by linearizing all non-linearities in (10)). In order to solve these quadratic programs efficiently, we develop a novel active-set numerical algorithm which takes into account which constraints were active from one iteration to the next [2]. To our knowledge, our method is the first that results in a non-decoupled resolution of contacts, friction and inextensibility for cloth simulation in a single pass.

The inextensibility assumption is shown to be realistic by comparing simulations to experimental data: we record in a laboratory setting —with depth cameras and motion capture systems— the motions of seven types of textiles (including e.g. cotton, denim and polyester) of various sizes and at different speeds and end up with more than 80 recordings. The scenarios considered are all dynamic and involve rapid shaking and twisting of the textiles, collisions with frictional objects and even strong hits with a long stick. Then we compare the recorded textiles with the simulations given by our inextensible model and find that on average the mean error is of the order of 1 cm even for the largest sizes (DIN A2) and the most challenging scenarios [1, 2].

Furthermore, we also tackle other relevant problems to robotic cloth manipulation such as cloth perception and classification of its states. We present a reconstruction algorithm based on Morse theory that proceeds directly from a point-cloud to obtain a cellular decomposition of a surface with or without boundary: the results are a piecewise parametrization of the cloth surface as a union of Morse cells. From the cellular decomposition, the topology of the surface can be then deduced immediately [3]. Finally, we study the configuration space of a piece of cloth: since the original state of a piece of cloth is flat, the set of possible states under the inextensible assumption is the set of developable surfaces isometric to a fixed one. We prove that a generic simple, closed, piecewise regular curve in space can be the boundary of only finitely many developable surfaces with nonvanishing mean curvature [4]. Inspired by this result we introduce the dGLI cloth coordinates, a low-dimensional representation of the state of a piece of cloth based on a directional derivative of the Gauss Linking Integral. These coordinates —computed from the position of the cloth's boundary— allow us to distinguish key qualitative changes in folding sequences [5].

Selected Publication: [1].

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Anastasia Matveeva defended her PhD thesis on *Poisson Structures on Moduli Spaces and Group Actions*, supervised by Professor Eva Miranda Galceran, on October 3, 2022, within the UPC doctoral program in Applied Mathematics. Currently, she leads a team of analysts at a Californian startup developing tools for mobile performance optimization.

Thesis summary

Symplectic and Poisson geometry fields arise at the intersection of geometry and physics. Motivated by understanding the dynamics of mechanical systems, they consider the phase space of such a system as a manifold with a prescribed geometric structure. Understanding the geometric properties of these manifolds brings insights into mechanical systems' behavior. Symplectic structures cover a large part of the examples coming from classical mechanics and provide very applied techniques. Poisson manifolds, more general, can be viewed through the prism of the symplectic foliation. One of the good examples where symplectic methods shine at their best is the problem of finding periodic orbits (if they exist). Another splendid application comes from the simple idea that any symmetry of a system reduces the number of its degrees of freedom, simplifying the system itself. In physical language, this would be formulated as conservation laws and first integrals. In geometric language, this concept can be encoded as a reduction theorem. The celebrated Marsden-Weinstein reduction reveals an exciting phenomenon that for a group of dimension k, the reduction can be doubled: the system can be simplified by $2k$ degrees of freedom [1].

Marsden-Weinstein quotients are naturally connected to certain moduli spaces. In their seminal article, Michael Atiyah and Raoul Bott unveiled the symplectic structure on the space of flat connections. The Riemann-Hilbert problem explores the correspondence between the moduli space of flat connections of Fuchsian systems (i.e., differential systems with simple poles) on a sphere and the monodromy data's moduli space (i.e., representations of the fundamental group of a punctured sphere). There are few cases where the solution can be constructed explicitly. For Riemann spheres, positive results of a classical Riemann-Hilbert problem are usually existence theorems. In that case, Riemann-Hilbert correspondence turns out to be a Poisson morphism.

In recent years, there has been an increasing interest in b-symplectic (together with more general b^m- and E-symplectic) geometry. The corresponding manifolds can be viewed as stepping out of the symplectic category toward Poisson, allowing certain types of singularities in the 2-form, which is no longer symplectic. This approach enables a careful transfer of symplectic techniques to larger classes of Poisson structures while tracking which properties break or change. Many aspects of b, b^m and E-symplectic geometry are studied in numerous works of Eva Miranda and her collaborators [2–4].

This thesis studies the analog of Marsden-Weinstein reduction in the context of singular symplectic and singular quasi-Hamiltonian structures taking as a motivating example a singular version of the Atiyah-Bott structure on the moduli space of flat connections.

In the second part of this thesis, we turn to Poisson structures on moduli spaces of flat connections and monodromy data related to the Riemann-Hilbert correspondence. It turned out recently, in the work of Irina Bobrova and Marta Mazzocco, that another interesting example of non-autonomous b-symplectic structures appears in the context of Painlevé transcendents [5]. Sigma-coordinates for Okamoto Hamiltonian of the second Painlevé equation lead to a natural b-symplectic structure [6]. For other Painlevé equations $P_{III} - P_{VI}$, the Poisson structure takes a more complex form. We consider Poisson structure on moduli spaces of flat connections and monodromy data related by the Riemann-Hilbert correspondence for P_{VI} and P_V. For P_{VI} and the other Fuchsian equations, we explicitly construct such a structure on the corresponding character variety leading us to a conjecture for P_V, which can be seen as a counterpart of the same structure on flat connections and coincides with obtained in [7].

Highlighted publication: [8]

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Patricio Almirón Cuadros defended his PhD thesis *Analytic invariants of isolated hypersurface singularities and combinatorial invariants of numerical semigroups* (pdf), supervised by Professors Maria Alberich Carramiñana and Alejandro Melle Hernández, on 11 July 2022 within the UCM doctoral program in Mathematical Research. Currently, he is a postdoctoral researcher at Instituto de Matemáticas Universidad de Granada (IMAG), through a postdoctoral contract in the Maria de Maeztu Programme for Centres of Excellence.

Thesis summary

The identities between the inner invariants of a mathematical object provide patterns and restrictions that allow us to better understand its inherent properties. The main purpose of this thesis has been to offer a new understanding of the identities that appear in the interplay between analytic and topological invariants of hypersurface singularities, as well as those arising between different combinatorial invariants of numerical semigroups and their associated additive structures.

The first part of the thesis deals with analytic and topological invariants of an isolated hypersurface singularity. Two prototypical examples are the Milnor number, μ, which is a topological invariant, and the Tjurina number, τ, which is an analytic invariant. Both are closely related through the study of small perturbations of the singularity. From this point of view, it is natural to try to understand up to what extent the topology, for example encoded in μ, of the germ of a singularity constrains its analytical properties, for example τ.

My main contributions in the first part of the thesis are the following: first we provide a closed formula for the minimal Tjurina number in a fixed topological class of an irreducible plane curve in terms of topological invariants of the branch [1]. Secondly, I address a question of Dimca and Greuel about the quotient of the Milnor and Tjurina numbers of an isolated plane curve singularity [1, 3, 4] and extend it to higher dimensions. The strategy of understanding Dimca and Greuel's question for higher dimensional singularities was the clue to provide a complete answer to it [3]. Moreover, I show the connection of the extended question with an old standing conjecture about surface singularities posed by Durfee.

To finish the first part, I study another set of invariants, which in this case are of Hodge theoretical nature, called the spectrum. For them, we establish K. Saito's continuous limit distribution for the spectrum of Newton non-degenerate isolated hypersurface singularities [9]. Moreover, we link Saito's distribution problem with our generalization of Dimca and Greuel's question. As a consequence, this provides a new way of understanding the important role of Durfee's conjecture in the context of isolated hypersurface singularities.

The second part deals with numerical semigroups and their combinatorics. First, we address Wilf's conjecture on numerical semigroups, which asks for a lower bound of its conductor in terms of the genus and the embedding dimension of the numerical semigroup. In this direction, we present two necessary conditions for a numerical semigroup to have negative Eliahou number, which is a number whose positivity implies Wilf's conjecture [8]. One of our contributions to Wilf's conjecture is to propose its extension to modules over a numerical semigroup, which provides a new insight in some related problems to Wilf's conjecture [7]. We provide a formula for the conductor of a semimodule over a numerical semigroup with two generators [5]. As a consequence we prove the generalization of Wilf's conjecture in this particular case and reveal some interesting symmetries in the set of gaps of a numerical semigroup with two generators [6].

Finally, we also study the value set of modules over the local ring of an irreducible plane curve singularity with one Puiseux pair providing a partial generalization of a Theorem of Bresinsky and Teissier about the value semigroup of an irreducible plane curve singularity. As a consequence, we deduce some new features about the value set of Kähler differentials of an irreducible plane curve singularity with one Puiseux pair [2]. As a sort of conclusion, this final chapter shows how the understanding of the combinatorics of a numerical semigroups can help in order to discover new properties of some analytical and topological invariants of curve singularities.

Highlighted publication: [3]

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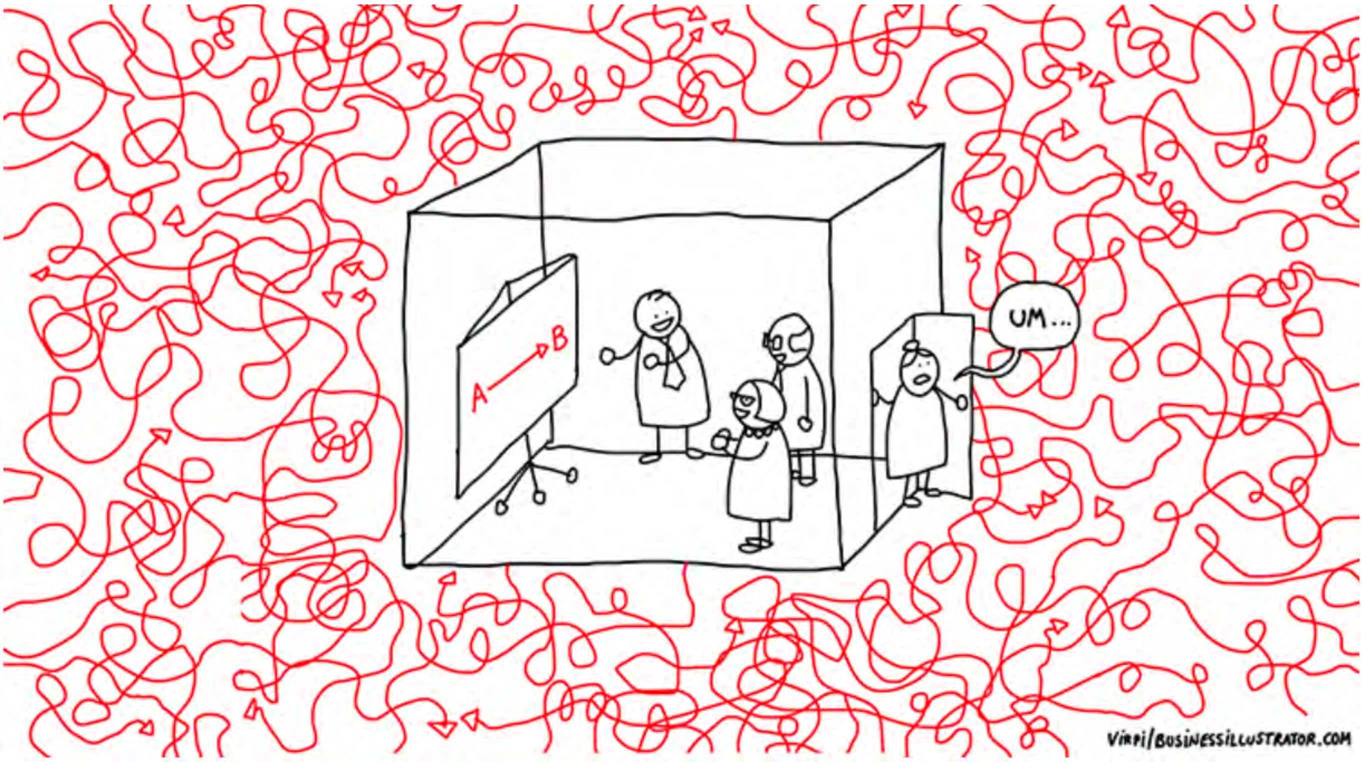
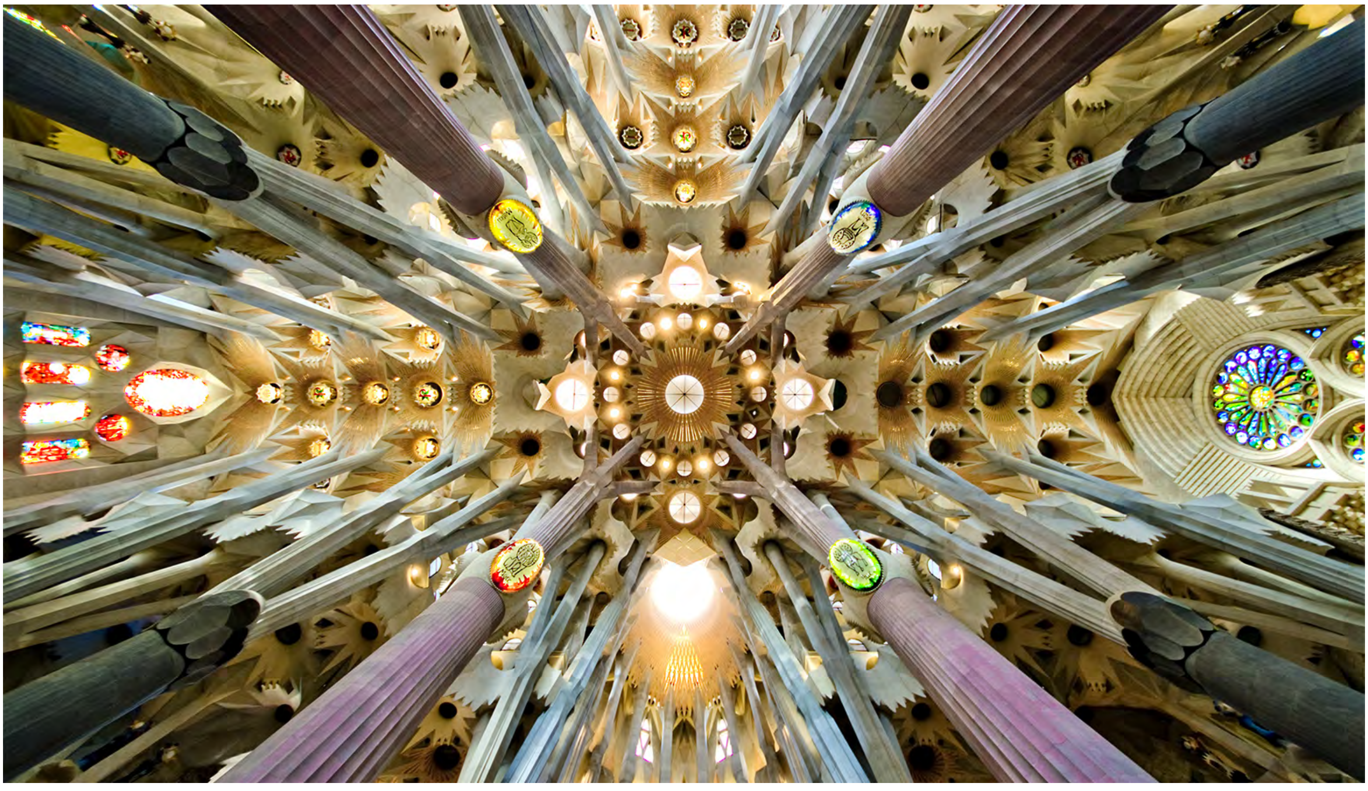


Theory and practice, or ivory tower and world complexity: Courtesy of businessillustrator.com: "Complexity and policy making don't mix very well."



Gaudí's hyperbolic vault of the Sagrada Familia nave (Barcelona).

Outreach

The heat equation, by Luis A. Caffarelli
(Deparment of Mathematics, UT at Austin).


Facultat de Matemàtiques i Estadística
Universitat Politècnica de Catalunya
Lección inaugural del curso 2003-2004
22 de setiembre de 2003

La ecuación del calor

Professor Luis Caffarelli
Departament of Mathematics
University of Texas at Austin

On September 22, 2003, Luis A. Caffarelli delivered the inaugural lecture of the FME 2003-04 term. The image corresponds to the heading of the corresponding chapter in the booklet distributed by the FME on that occasion. Now, after twenty years, he has been awarded the 2023 Abel Prize and this NL rejoices in this great honor by offering a translation into English from the Spanish original of that lecture.

Fourier

The heat equation was proposed by Fourier in 1807—in his memoir on the propagation of heat in solid bodies.

In it, he also proposed the germ of what would become the theory of Fourier series.

So controversial was the latter that it took fifteen years, until 1822, for the Academy of Sciences to decide to publish it.

Mathematical models

The heat equation is a mathematical model (perhaps the simplest) that tries to describe the evolution of temperature in a solid body.

Let us consider, to simplify the presentation, an isolated metallic bar of length one ($0 \leqslant x \leqslant 1$), initially at zero temperature, which after a certain time, t_0, we have heated to a temperature $T(x, t_0)$ keeping its ends, $x = 0$ and $x = 1$, at zero temperature.

From that instant, t_0, we let the temperature $T(x, t_0)$ evolve freely and we are interested in a mathematical model that allows us to predict the temperature $T(x, t)$ for all x in the interval $[0, 1]$, for any future time (that is, for all $t > t_0$), from our knowledge of $T(x, t_0) = T_0(x)$ and from the fact that for $x = 0$ or $x = 1$ the temperature remains equal to zero.

Naturally there is no "one model". There are infinitely many, depending on the precision and the range of values in which we intend it to be valid (high or low temperatures will change the behavior of the material, impurities could be relevant, etc.).

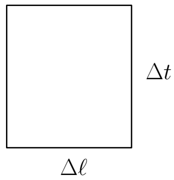
The model proposed by Fourier can be summarized as follows:

1. The (caloric) energy required to bring a piece of the bar of length $\Delta\ell$ from zero temperature to temperature T is proportional to $\Delta\ell \times T$ (i.e., the energy density, $e = kT$, is proportional to the temperature, with k a characteristic constant of the material).

2. Energy flows from areas of higher temperature to those of lower temperature. More precisely, the energy flux density is
$$f(x) = -\theta D_x T$$
(or $\boldsymbol{f}(x) = -\theta \nabla T$ in various dimensions), where θ is again a characteristic constant of the material.

3. The energy is conserved. If we take a piece of the bar, $\Delta\ell$, the energy contained in $\Delta\ell$ at the instant t_2 is equal to the energy that was in $\Delta\ell$ at the instant t_1 plus the "energy flux" that penetrated the extremes x_1, x_2 in the time interval from t_1 to t_2. Mathematically:
$$\int_{\Delta\ell} (e(x, t_2) - e(x, t_1))\, dx = \int_{t_1}^{t_2} (-f(x_2, t) + f(x_1, t))\, dt.$$

If we draw the rectangle $\Delta\ell \times \Delta t$,



the first integral occurs at the top and bottom edges, while the second occurs at the sides. In order to compare them, we need to be able to write them in a common domain, namely the rectangle. We do this by taking derivatives:
$$\int_{\Delta\ell \times \Delta t} D_t e(x, t) dx dt = \int_{\Delta\ell \times \Delta t} -D_x f(x, t) dx dt.$$

Since this relationship must be verified for any rectangle, no matter how small, the integrands must necessarily be equal: $D_t e + D_x f = 0$.

Remembering the expressions for e and f as a function of T, we obtain the equation
$$k D_t T = \theta D_{xx} T.$$

In present day terms, the relationships 1) and 2) are called *constitutive laws*, and they establish specific relationships between the state variables, e, f, T and their derivatives, which depend on the characteristics of the state. materials etc. Relationship 3), on the other hand, is of a different nature, it is a conservation law, and establishes that certain quantities (mass, energy, etc.) are conserved through a process. That does not mean that they are point-wise constant. In a gas, for example, mass flows from one part to another. What a conservation law does is postulate the existence of a conserved variable, e, and a flow, $\boldsymbol{f}$, that satisfy
$$D_t e + D_x f = 0.$$
(or $D_t e + \operatorname{div} \boldsymbol{f} = 0$).

In short, writing a mathematical model consists of *choosing those state variables that are relevant to the phenomenon we want to describe*, finding (usually experimentally) *their constitutive laws*, and how they are conserved.

Existence and uniqueness

Perhaps the most important variation that these ideas have undergone today is in taking into account random effects.

Regardless of how good a mathematical model is at representing reality, it must have a minimum of internal consistency.

If the relationships we specify are excessive, they will generally be contradictory and our problem may not have a solution.

If they are too few, we may have many different solutions, when in reality we expect to have a single solution.

So Fourier's next step was to try to find a solution to the problem. Given the initial temperature, $T_0(x)$, and the condition

$$T(1,t) = T(0,t) = 0$$

for all $t > t_0$, it is a matter of proving that there is a unique function $T(x,t)$ that satisfies the equation $T_t = T_{xx}$ (we set $k = \theta = 1$).

Let's first try to find some solutions for particular T_0 of the form

$$T(x,t) = T_0(x)g(t).$$

This requires that

$$g'(t)T_0(x) = g(t)T_0''(x)$$

or that

$$\frac{g'(t)}{g(t)} = \frac{T_0''(x)}{T_0(x)} = \lambda \text{ constant}$$

(the only possible way for two functions of distinct variables to be equal is for them to be constants, since we can set t and vary x over all possible values).

Since $T_0(0) = T_0(1) = 0$, the only possible pairs are

$$T_0(x) = \sin(n\pi x)$$
$$g(t) = e^{-(n\pi)^2 t}.$$

But the problem we were considering is linear. Therefore, any combination of solutions

$$T(x,t) = \sum c_n \sin(n\pi x) e^{-(n\pi)^2 t}$$

is a new solution, with initial data

$$T_0(x) = \sum c_n \sin(n\pi x).$$

Fourier then proves that any function T_0 (say continuously differentiable, with $T_0(0) = T_0(1) = 0$) can be expressed that way, and gives a formula for the coefficients.

This is how Fourier analysis was born, so revolutionary that it took fifteen years for mathematicians of the time to accept that a series of highly oscillating functions such as $\sin(n\pi x)$ could represent, for example, an arc of a parabola or a polygonal.

Harmonic analysis

Fourier analysis has come to be called harmonic analysis. We can say that it consists of describing a function not by its special characteristics (where it is large, where it is small), but by the influence that each frequency ($\sin \lambda x$) has on its composition. As such, it occupies a fundamental place in everything related to wave theory, transmission of all kinds of signals, ultrasound image reconstruction, spectral analysis, etc.

In the last years a way of decomposing functions into "elementary chunks" that describe the oscillatory properties of the function simultaneously in physical space (the variable x) and frequency space (the variable n or λ) has acquired great prominence.

These elementary chunks are called *wavelets* and have revolutionized image compression, data transmission, etc.

The Gaussian kernel and random walks

There is actually a more convincing way of representing the solution $T(x,t)$, one that more clearly exhibits the qualitative properties of heat propagation. It consists of initially putting "point masses".

Suppose now that the bar is infinite, it is at zero temperature and we manage to place a "point mass" of one heat unit at the origin and at the instant t_0. In other words, we were able to concentrate a quantity $c = 1$ of heat energy at the origin so quickly that it is instantaneous for our time scale.

How does the temperature evolve next?

A little self-similarity analysis: if $T(x,t)$ is a solution of the equation, so is $aT(bx, b^2t)$, which allows us to calculate that in this case

$$T(x,t) = \frac{1}{(\pi t)^{1/2}} e^{-x^2/4t} = G(x,t).$$

This is the Gaussian kernel ("the bell"), or error dispersion formula.

If we translate the point mass to x_0, the new formula is

$$T\left(x,t\right) = G\left(x - x_0, t\right),$$

since the equation is translation invariant .

If we superimpose point masses of intensities c_i on the points x_i,

$$T\left(x,t\right) = \sum c_i G\left(x - x_i, t\right)$$

and finally, for an energy density $e = T_0\left(x\right)$,

$$T\left(x,t\right) = \int G\left(x_0, t\right) T\left(x_0\right) dx_0.$$

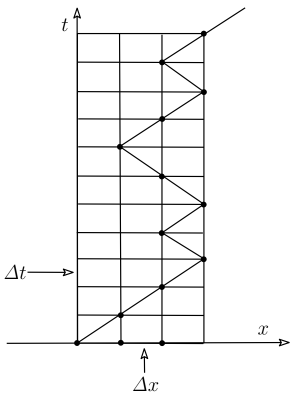
This representation immediately tells us, among other things, that:

a) If the original temperature is positive, it remains positive.

b) The effect of any change in temperature is felt instantly throughout the bar.

c) The temperature T_0 can be highly discontinuous and an instant later it becomes regular.

But what is the relationship between the heat equation and error propagation?



Suppose that at the instant t_0 we are standing at the origin. We flip a coin; if heads, we take one step, Δx, to the right. If tails, to the left. Every interval Δt, we repeat the operation.

What is the probability $u\left(x,t\right)$ that at time t we find ourselves in position x?

It seems complex to calculate, but we can see that at the instant $t - \Delta t$ we were either at $x + \Delta x$ or at $x - \Delta x$ and that from there we moved with probability ½ to (x,t), that is,

$$u\left(x,t\right) = \frac{1}{2}\left(u\left(x + \Delta x, t - \Delta t\right) + u\left(x - \Delta x, t - \Delta t\right)\right)$$

or

$$\begin{aligned} u\left(x,t\right) - u\left(x, t - \Delta t\right) = \tfrac{1}{2}\big(&u\left(x + \Delta x, t - \Delta t\right) \\ &+ u\left(x - \Delta x, t - \Delta t\right) \\ &- 2u\left(x, t - \Delta t\right)\big). \end{aligned}$$

Everything now depends on the balance between Δt and $(\Delta x)^2$. If we choose $\frac{(\Delta x)^2}{\Delta t} = 1$, we can divide both sides by Δt and we get:

$$\frac{\Delta u}{\Delta t} = \frac{\Delta^2 u}{(\Delta x)^2},$$

which is a discrete form of the heat equation.

That is, in the limit $\Delta t \to 0$, u converges to the solution of the heat equation. But at the initial instant, we are standing at the origin with probability 1, that is,

$$u(x,t) = G(x,t).$$

This is a version of the central limit theorem which says that if we independently repeat n times the same zero expectation experiment X_i, then the probability distribution of

$$X = \frac{\sum X_i}{\sqrt{n}}$$

converges to a Gaussian.

Nonlinear diffusions

That is why a heat-type equation is often called a diffusion equation. Diffusion equations appear in various fields. For example, in population dynamics the energy density e is replaced by the population density σ, and one of the many reasons why a population migrates is to go to areas of lower density, that is, the population flow has the form

$$f = -\nabla\sigma + \cdots \text{ (other reasons)}$$

and therefore the corresponding equations will be of the form

$$D_t\sigma = \Delta\sigma + \cdots$$

Or, in an epidemic, the probability of infection at a place x, at an instant of time t, depends monotonically on the probabilities of adjacent points a few hours earlier. This gives rise, infinitesimally, to an equation of the form

$$D_t e = F(D_x^2 e, \nabla e)$$

where e is the expectation of infection at x, t.

In a viscous fluid, particles adjacent to a given one try to "drag" or "slow down" it if it is slower or quicker, respectively, then the others.

The point I want to emphasize is that, in all these phenomena, the "diffusion" or "viscosity" term induces a process of "flattening" or "averaging" of the state variables that characterizes diffusive or viscous problems.

The influence of the theory of "parabolic equations" is today immense, in fluid equations (Navier–Stokes, flow in porous media, phase change equations), in optimal control theory and game theory (totally non-linear equations), modeling of population dynamics, epidemiology, mathematics of finance, etc.

Research and Development at Basetis: The strength of blending Mathematics and Artificial Intelligence,

by Andreu Masdeu and by José Luis Muñoz

Received June 1, 2023

Basetis is a Barcelona-based IT consulting firm comprising over 350 employees with a strong focus on social change.* For instance, we have adopted the Teal philosophy, which stands on three pillars: self-management, wholeness, and evolutionary purpose. As a result, decision-making and leadership responsibilities are distributed among **Basetis** personnel rather than following a rigid hierarchical structure.** Our business encompasses a variety of ICT services, including software and mobile application development, graphic design, cloud infrastructure management, as well as data analytics and artificial intelligence (AI) services.

In these latter domains, Mathematics constitutes a pivotal component in the successful execution of our projects. For this matter, **Basetis** employs numerous mathematicians and physicists. In fact, most of the very first members of **Basetis** were alumni of the **FME**, creating a close connection between the company and the faculty. As a result, many **FME** students come to **Basetis** every year for internships, some of them subsequently joining the company on a permanent basis, constituting a significant portion of the company.

Mathematicians at **Basetis** possess a deep intellectual curiosity which led the company to strategically establish its own AI Team five years ago. This decision capitalized on the intersection between Mathematics and AI, leveraging the strengths of the firm's STEM profiles to develop cutting-edge AI solutions for our clients. **Basetis** already provided data analytics solutions since its foundation, so the incorporation of AI was a natural next step as its relevance increased within the IT industry.

From a mathematical perspective, AI projects usually come with intriguing challenges. There are a wide variety of clients, with different needs and data, hence our spectrum of services is rather broad. In all cases, Mathematics is the backbone of our work, as it provides the theoretical foundation for the algorithms and models that power every aspect of AI technology. These projects can be divided into three distinct categories:

Fraud detection in bank transactions

1. **Advanced and Predictive Analytics**
Developing accurate predictive models that can forecast future or real-time outcomes based on historical data is one of the most important challenges faced by the AI Team. Examples of such applications include auto-

mated fraud detection in bank transactions, energy consumption forecasting, and stock and demand predictions, among others. The successful execution of these projects requires a profound understanding of statistical principles and techniques. Initially, historical datasets must be subjected to rigorous statistical analysis and interpretation to identify the relevant variables and create new synthetic variables from the data. Also, a comprehensive grasp of the mathematics and complexity underlying the typical models used in machine learning is vital for effective model selection.

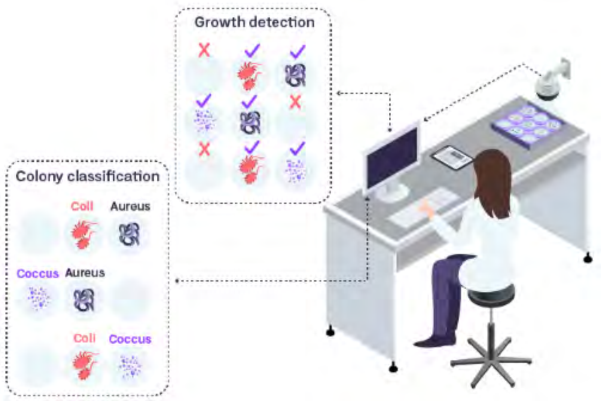
All models inherently involve a procedure for error minimization at their core. However, the specifics of this procedure may vary depending on the model structure, leading to different model behaviors. For instance, decision trees, gradient boosting machines, and logistic regression are examples of machine learning models with a different structure but which can solve the same task. Once the models are fitted to the data using the selected variables, it is crucial to carry out a thorough analysis of their performance. This analysis enables the identification of potential risks and biases associated with the model, enabling appropriate steps to mitigate such issues.

2. **Optimization**
Another key focus of the AI Team is mathematical optimization. Depending on the problem, we can differentiate between continuous optimization and discrete optimization. While continuous optimization problems can be tackled efficiently with machine learning methods, discrete optimization relies on fundamentally different techniques such as graph-based algorithms, mixed-integer programming, Monte Carlo methods, and hybrids of those.

Assigning drivers to services

For instance, the team used graph theory to solve the problem of assigning drivers to services, using one graph to parameterize optimal service concatenation through a minimum path cover problem, and another one to determine the optimal matching between drivers and routes. In such a way, the number of drivers required and the waiting times were minimized and the service efficiency was improved. On a different project, graphs were used to represent a set of academic problems that a student had to face and the dependencies between them. The optimal education itinerary for individual students could be computed using Dijkstra's algorithm and Monte Carlo-based optimization methods.

3. **AI for perception tasks**
In addition to these challenges, we also work on developing natural language processing (NLP) systems, computer vision (CV) algorithms, and other AI applications that require a strong mathematical foundation. Using state-of-the-art techniques, our team has deployed a wide variety of successful AI solutions.



Detecting the growth of bacterial colonies

The AI team has built a chatbot able to function as a drive-through window assistant for a fast-food restaurant chain, using NLP technology to make it capable of understanding the intention of the client's words when listening, and guiding the conversation in order to deliver the order efficiently. On the CV front, the team has developed a system to automatically identify, classify, and quantify olives from images taken of the conveyor belt for real-time quality monitoring. Similarly, an algorithm for detecting the growth of bacterial colonies on a Petri dish was developed, being able to identify typical species and quantify their amount, which can be very helpful for early diagnosis. Moreover, our team has also worked on audio-perception solutions, for instance developing a device able to anticipate malfunctions on industrial machinery by processing sensor data and sound records of the machines.

Overall, the work of the AI team at **Basetis** is highly technical and requires a strong background in mathematics. By leveraging our expertise in math and AI, we are able to provide innovative solutions that help our clients to achieve their business goals.

Press headlines

- **El País**/Business Management:
  Companies witout a boss work (and very well).
- Indicador d'Economia/TIC:
  End hierarchies to empower talent

* Marc Castells and Víctor Roquet founded Base Technology and Information Service (**Basetis**) on 6 November 2009, a company in the ICT industry initially focused on providing professional services.

** ***Basetis*** *is made up of a team of people who are passionate about information and communications technologies. We share an entrepreneurial spirit and base our way of doing things on trust.*

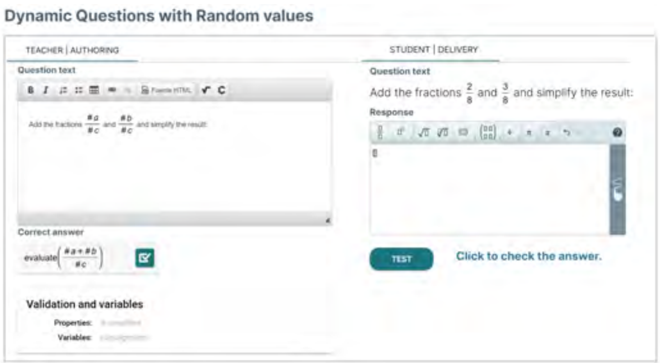
Enhancing STEM Learning: online solutions for Education,

by Ramon Eixarch (**Wiris** CEO).

Received June 2, 2023

The democratization of digital technology has been underway for some time now. In today's world, it would be hard to understand human life without the influence of technology. Moreover, the pandemic has led to the rapid adoption of digital tools to enable various activities to continue remotely, such as virtual meetings, hybrid education, and remote work. Most of these trends continue even after the pandemic has receded. However, with the growing reliance on technology in daily life, not all disciplines were prepared for this context, and some still need to be adapted. STEM[*] communication and assessment, in particular, have been challenging areas to adapt to the digital environment. This is where **Wiris** has made significant strides, with the aim of making people's STEM work more meaningful.

Wiris is a math and science software company that aims to simplify the work of STEM professionals by creating tools that facilitate writing, communicating, and assessing math using digital technology. Our products aim to enable users to have the best experience in STEM, and we accomplish this by integrating our products into all environments to make it as seamless as possible. Currently, **Wiris** offers two primary products: MathType and WirisQuizzes. MathType is a software program that enables the creation of mathematical notation to be inserted into both desktop and web applications. One of its most valuable features is handwriting recognition. On the other hand, WirisQuizzes is an authoring and assessment tool that allows educators to create and deliver math assessments online. Virtual learning environments often pay little attention to tools for mathematical content creation [1], and WirisQuizzes aims to address this need.



WirisQuizzes interface: Example of a dynamic question with random variables.

Wiris' beginnings date back over 25 years ago when a group of mathematics students at the Faculty of Mathematics and Statistics (FME) of the Technical University of Catalonia (UPC), myself included, worked out research projects that coalesced into enough know-how to launch a company. A few years later, as the company evolved, its primary goal was to improve and develop digital tools specialized in STEM subjects. **Wiris** released its first product, **Wiris CAS**, in 2002, an online-based computer algebra system that quickly became popular in mathematical environments. Over the years, **Wiris** collaborated with academic institutions and publishing houses, and by 2007, the **Wiris** product family started to flourish, including the first versions of our web-based formula editor (known by then as Wiris editor) and our mathematical assessment tool (WirisQuizzes). In the upcoming years, we continued evolving our products, migrating them to Javascript, and integrating them into popular LMS and HTML editors. The company merged forces with Design Science in 2017, and Wiris editor was rebranded as MathType, becoming a leading solution for mathematical equations. WirisQuizzes also evolved and became compatible with popular LMS systems, and added features such as random learning units [2].

The 2020 pandemic had a significant impact on **Wiris**. Even prior to it, the use of technology in the classroom was a growing trend and a significant concern in educational centers. The lockdown forced the acceleration of this trend as educational institutions globally had to adapt their teaching methods to virtual environments. Most education institutions shifted their learning formats to online or hybrid models, resulting in an unprecedented revolution in education. Both **Wiris**' products played a critical role in this digital transformation regarding STEM subjects. MathType experienced more than 400% growth during the lockdown months, and many educators opted for WirisQuizzes to assess their STEM subjects. During Covid, **Wiris** offered MathType for Google Workspace for free to provide free service to millions of math and science teachers and students who required assistance during a stressful situation. Given the success, we rapidly adapted and integrated with Google Slides by the end of 2020, achieving the perfect solution for educational institutions.

The usage of both products has continued to increase even after the pandemic, demonstrating the sustained demand for digital solutions in STEM education. To put this into perspective, we have experienced incredible growth in MathType users from half a million to double-digit million users, and the figure continues to grow at a fast rate.

Our progress does not end here. As technology continues to advance, **Wiris** recognizes the increasing opportunities to improve STEM education and make it more accessible to people of all ages and backgrounds. The company aims to remain at the forefront of the EdTech landscape, which is evolving at a breakneck pace with the integration of AI into everyday life. We will continue to look ahead and work towards our mission by leveraging all that technology has to offer.

As we expand technologically, Wiris is also rapidly growing in terms of team members. From 2018 onwards, the company has experienced accelerated growth, and today we are about 100 employees with offices in Barcelona and Long Beach, California. We stand out for promoting and harnessing the potential of talent by providing opportunities for individuals and helping them develop their skills to reach their full potential. We believe this is the key to our success and what maintains us at the forefront of the EdTech landscape.

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Chronicles

A personal glimpse on Professor Luis Caffarelli on the occasion of the Abel Prize 2023

by Juan Luis Vázquez (RAC, UAM, UCM)

Received on March 30, 2023.

(This text is an English version, slightly edited, of the article just published in Spanish as [1]).

A week ago, the Norwegian Academy of Science and Letters awarded the Abel Prize 2023 to the Argentine-American mathematician Luis Ángel Caffarelli in an announcement made in Oslo. Luis is well known in the international mathematical community for his fundamental contributions to the theory of regularity for nonlinear PDEs (Partial Differential Equations), which include free boundary problems, the equations of viscous fluids, the Monge-Ampère equation, optimal transport equations and many other topics. After last week's brief scientific presentation [2] today we would like to point out some more personal aspects of the first Spanish-speaking Abel prize winner and also highlight his intense relationship of many years with Spain.

Luis Caffarelli was born in 1948 in Buenos Aires, Argentina, and has lived in the US since 1973. His early American years were spent in Minnesota, a distant region of the Midwest that brings back such fond memories to so many Spanish scientists of my generation. Luis rose to fame at the end of that decade for his surprising work on the regularity of the so-called free boundaries, known to the public today largely thanks to his work. In fact, he surprised the whole world with the article [3], a work whose novelty and brilliance was the basis of his future fame. Indeed, his initial studies on the Obstacle Problem, [4], are already a classic in pure and applied mathematics. Another name to remember in this topic is the Stefan Problem, which models, among other applications, the evolution of the ice-water system with its separation interface.

A second hit came in 1982 with the article [5], produced during his 2-year stay at the mythical Courant Institute, where he collaborated with Robert Kohn and Louis Nirenberg. The latter, also an Abel Prize winner (2015), was his supporter in those years and was his friend for life. Years go by and this beautiful result, the CKN theorem, continues to stand out as the last great contribution made in the study of the regularity of the solutions of the Navier-Stokes equations for viscous fluids, which is one of the problems of the Millennium of the Clay Foundation.

The eighties were a prodigious time for Luis and a cascade of articles with various collaborators marked the breadth and depth of his mathematical ability and established him as the best worldwide representative of the legacy of the great Ennio de Giorgi on how to study the regularity inherent in the solutions of the problems of the so-called Calculus of Variarions, which is today a main branch of mathematics. Luis added to the program of the great Ennio the free boundary problems that had bewildered the best experts during the 60s and 70s due to their intricate combination of analytical and geometric difficulties.

Over the years, Luis has been and is one of the world's leading experts in the field of nonlinear Partial Differential Equations. The PDEs are a discipline established within the body of Mathematics in the 18th-19th centuries, one of the many brilliant daughters of Calculus, and today it is experiencing one of its golden moments due to the enormous influence of its results and techniques in the most diverse directions, ranging from the fundamental equations of Physics to the mathematical models of various engineering and other sciences of growing social interest.

The mathematics of the 20th century have been excellent in the study of non-linear processes, which represent a stage of difficulty higher than the study of linear processes. Although Nature has had the good taste of resorting to linear processes for many of its basic models (such as the propagation of waves or heat, and also the basic equations of quantum theory), today it is well known that many of the fundamental processes of Science are non-linear, and understanding that added difficulty is the glory and the cross of the current mathematical profession. Briefly stated, nonlinearity is an infinite source of complexity.

From 1986 to 1996 Luis was a permanent member of the Institute for Advanced Study of Princeton. He was later a professor at the Courant Institute of Mathematical Sciences in New York, before joining the University of Texas at Austin in 1997 as the Sid Richardson Chair in Mathematics. Early recognized as a scientific referential in the US, he has been a member of the US National Academy of Sciences since 1991. But he was no less recognized in Spain where he was bestowed a Doctorate Honoris Causa by the Autonomous University of Madrid in 1992.

A series of relevant prizes show the impact of his scientific contributions. In this century, for instance, we count the Rolf Schock Prize, from the Royal Swedish Academy of Sciences (2005); the Leroy P. Steele Prize "for Lifetime Achievement" from the American Mathematical Society (AMS, 2009); the very prestigious Wolf Prize in Mathematics (2012); the Solomon Lefschetz Medal, from the Mathematical Congress of the Americas (2013); a Leroy P. Steele Prize again, this time "for Seminal Contribution to Research", AMS (2014, shared with Robert Kohn and Louis Nirenberg); as well as the Shaw Prize in Mathematics, (2018). Since 2015 he has been a foreign member of the Spanish Royal Academy of Sciences, where I had the honor of introducing him.

Luis Caffarelli has had an enormous influence on the development of partial differential equations in several countries, mainly the US, Argentina, Spain, Italy and Greece (to make the list short). His collaboration with Spanish authors dates from the 1980-90s. After meeting him at a conference on free boundaries in Italy in the summer of 1981, I enjoyed long stays in Minnesota, where Luis Caffarelli was a professor at the time, and contact with him was strengthened by his regular visits to Spain. The collaborations carried out include a good number of Spanish co-authors, in particular Xavier Cabré (with whom he wrote a famous book, [6]; see Reviews in this issue), Antonio Córdoba, Ireneo Peral (1946-2021), Rafael de la Llave, Fernando Soria, María del Mar González and the author of this glimpse, among others. These collaborations with Spanish authors deal with regularity problems of highly nonlinear equations and phase change problems and free boundaries, issues in which his leadership is uncontested. I am the co-author of 8 articles with Luis and my best-known book [7] is dedicated to him because of the many pages that refer to his ideas.

Luis Caffarelli and Juan Luis Vázquez at work (UAM, 2017)

Luis has participated in numerous courses and schools in Spain, particularly in the Summer Courses of the UIMP in the incomparable framework of the Palacio de la Magdalena de Santander, courses that Luis inspired and that UAM co-sponsored, a university that he visited with some frequency from 1986 to 2017. I am witness to this since I participated in all these courses, which had the strong support of the rector Ernest Lluch (1937-2000), eminent patron of science at the UIMP. The strong international orientation responded to Luis's ideas but was not easy to implement with the existing local rules. The series of courses of the 80s and 90s continued under the rector Salvador Ordóñez between 2010 and 2015. And we must not forget Luis' strong ties with Barcelona and Granada, for example. In recent years Luis has been involved with us in the study of anomalous diffusion problems with non-local operators, another source of friendships, travels, and mathematical anguish and pleasure. His most notable paper on this subject is [8], a true best seller in the area.

Luis has written more than 320 mathematical articles with more than 130 collaborators, from the most reputed authors to bright young people looking for a future in mathematics. He has advised more than 30 PhD students. The depth of his ideas, the breadth of his subject matter, coupled with his generosity and easy manner, have cemented his fame on all continents, and by all continents we mean people we know on all of them. *Luis alone has been a non-linear university for the world.*

In addition, he has the well-deserved reputation of being a great cook in the Argentine-Italian tradition. Congratulations, Maestro.

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Mark Braverman's Work on Information Complexity

Albert Atserias (CS/UPC & IMTech)

Received on July 1st, 2023.

The IMU Abacus Medal 2022 was awarded to Mark Braverman for his "path-breaking research developping the theory of information complexity" [1]. The Abacus Medal recognizes outstanding contributions in Mathematical Aspects of Information Sciences to a mathematician under 40 and is the continuation of what used to be called the Rolf Nevanlinna Prize until 2018. Braverman joins this way an impressive list of awardees featuring for example who would later be the 2021 Abel Prize co-winner Avi Wigderson [2].

The citation for Braverman's award qualifies information complexity as the "interactive analogue of Shannon's information theory". To recall, Shannon's theory is an elegant and immensely useful framework (a.k.a. language) to reason about the amount of information in a message as transmitted from a sender, Alice, to a receiver, Bob. The key concept in Shannon's theory is the *binary entropy* $H(X)$ of a discrete random variable X, defined by the formula

$$H(X) := \sum_{a \in \mathrm{Supp}(X)} \Pr[X = a] \log_2(1/\Pr[X = a]).$$

The *unit* of entropy is the *bit*, or *binary digit*. The importance of this definition stems from two theorems as proved by Shannon himself. The first one, called the Source Coding Theorem, is the most relevant for us. This states that $H(X) \leqslant C(X) < H(X) + 1$, where $C(X)$ is the average codeword length of an optimal binary code associated to the support of the random variable X. Equivalently,

$$\lim_{n\to\infty} \frac{1}{n} C(X^n) = H(X), \tag{1}$$

where X^n denotes n independent copies of the random variable X. Thus, the binary entropy is an analytic quantity associated to a random variable that exactly captures the combinatorial notion of optimal binary code (for the noiseless channel). In Shannon's information theory, entropy is such a fundamental concept that it appears as a building block for several other definitions, such as in the definition of *mutual information* between two random variables X and Y; i.e.,

$$I(X;Y) := H(X) - H(X|Y) = H(Y) - H(Y|X).$$

But what could the "interactive analogue" of Shannon's theory possibly be? To explain this we should imagine a scenario in which both Alice and Bob hold a realization of their own random variables X and Y, respectively. Both are allowed to send messages, back and forth, and interactively, following a protocol. Their goal is set more generally as that of performing a task $T(X,Y)$ and to do so by exchanging the fewest possible number of bits of communication. For example, if X and Y are binary strings of length n, say $X_1 \cdots X_n \in \{0,1\}^n$ and $Y_1 \cdots Y_n \in \{0,1\}^n$, then their task could be to determine if the bit-strings have no bit in common, i.e., if there is no index $i \in \{1,\ldots,n\}$ such that $X_i = Y_i = 1$. This is called the *set-disjointness problem* in *communication complexity*. A related problem is *equality*, where Alice and Bob are required to determine whether their input n-bit strings X and Y are equal. It should be stressed that in communication complexity the players are allowed to exchange several messages, interactively, until they agree on the result, and that only communication counts in that both Alice and Bob are considered all-powerful in performing any action they need on their available information at any given time.

Analyzing the optimal communication complexity of a task poses formidable questions. To appreciate the difficulties, the reader is encouraged to think a little what should optimal protocols be, first for equality, and then for set-disjointness (expect no definitive answers). Not even the start is easy as it boils down to Shannon's theory. Indeed, an obvious upper bound is given by the protocol that instructs Alice to send her input X to Bob, and Bob to complete the task entirely on his own. Vice-versa, they could agree to have Bob send his input Y to Alice, and have Alice complete the task. It follows then, from the Source Coding Theorem, that any task can be accomplished with no more than $\min\{H(X), H(Y)\}$ bits of communication on average with respect to the joint distribution μ of (X, Y). Thus, if T_ϵ denotes the task of performing T with success probability at least $1-\epsilon$, then the (distributional) *communication complexity* $DCC(T_\epsilon, \mu)$ of T_ϵ with respect to μ is at most $\min\{H(X), H(Y)\} + O(\epsilon)$. Naturally, the communication complexity of the optimal protocol could be much smaller when interaction is genuinely taken into account beyond the possibility of both sending a single message

Following the tradition of computational complexity, the actual definitions are typically made for the worst-case scenario. The (worst-case) *communication complexity* $CC(T)$ of the task T is defined as the minimum amount of communication of an interactive protocol that solves task T for any two inputs X and Y. The (worst-case) *randomized communication complexity* $RCC(T_\epsilon)$ is defined as the minimum amount of communication of an interactive protocol that solves task T with probability at least $1-\epsilon$ for any two inputs X and Y when Alice and Bob are allowed to access a common source of random bits Z. A basic fact in communication complexity, known as Yao's Minimax Principle, is that

$$RCC(T_\epsilon) = \max_\mu DCC(T_\epsilon, \mu),$$

which follows from standard linear programming duality. This relates worst-case randomized communication complexity with distributional communication complexity. A landmark early result of communication complexity is the fact that $RCC(\mathrm{DISJ}_{n,1/2}) = \Omega(n)$, where $\mathrm{DISJ}_{n,\epsilon}$ denotes the set-disjointness task for n-bit strings with success probability at least $1-\epsilon$. In contrast, there are several ways to show that $RCC(\mathrm{EQ}_{n,1/2}) = O(\log n)$, where $\mathrm{EQ}_{n,\epsilon}$ denotes the equality task for n-bit strings. For more on the definitions and results of communication complexity we refer the reader to Yao's original article [3] and the excellent textbook by Kushilevitz and Nisan [4].

The concept of *information complexity* mentioned in Mark Braverman's citation is arguably the right analogue of entropy for the analysis of interactive communication protocols. In brief, it is defined as follows. For a fixed protocol π for solving task T with respect to distribution μ on (X, Y), the information complexity of protocol π with respect to μ is defined as

$$IC(\pi, \mu) := I(Y; \Pi \mid X) + I(X; \Pi \mid Y),$$

where Π is the realization of π on inputs (X, Y). The (distributional) *information complexity* of task T with respect to μ is then the infimum of $IC(\pi, \mu)$ over all protocols π that solve T (inf instead of min is needed here!). The theorem that justifies referring to IC as the right analogue of entropy is the one that states that *information complexity equals amortized communication* [6]. In symbols:

$$\lim_{n\to\infty} \frac{1}{n} DCC(T_\epsilon^n, \mu^n) = IC(T_\epsilon, \mu). \tag{2}$$

Note the similarity of this equation with Equation (1).

The theorem underlying Equation (2), and its method of proof, has found many important applications. An impressive one is the determination of the *optimal constant* in the randomized communication complexity of set-disjointness [5]:

$$RCC(\mathrm{DISJ}_{n,0^+}) = c_{\mathrm{DISJ}} \cdot n + o(n)$$

where $c_{\mathrm{DISJ}} = 0.4827\ldots$ The importance of this result stems from the fact that it proves information complexity as the right tool to analyze randomized communication complexity for such foundational problems as set-disjointness, which lies at the heart of many other communication complexity tasks. Indeed, (the complement of) set-disjointness is known to capture (i.e., is complete for) the class NP_{cc} of communication problems that can be solved by a non-deterministic interative communication protocol with polylogarithmic $(\log n)^{O(1)}$ many bits of communication on n-bit inputs. Thus, set-disjointness plays a role in communication complexity that parallels the role that the satisfiability problem SAT plays in classical computational complexity. Furthermore, the connection does not end at the level of analogy since it is well known that further progress on certain line of problems in communication complexity known as direct-product problems, to which set-disjointness belongs, would lead to progress on open problems in computational complexity in the ballpark of the famous P vs. NP [7].

Other applications of information complexity span such diverse areas as the analysis of distributed algorithms, lower bounds for streaming algorithms, design of protocols for interactive computation over a noisy channel, and even quantum information theory. Mark Braverman's foundational contributions to developping information complexity and its applications to all these areas promises to be a fruitful source of inspiration for the next generation of mathematicians and theoretical computer scientists.

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Algebraic and topological interplay of algebraic varieties

A Conference in Honor of the 60th Birthday of Enrique Artal and the 55th Birthday of Alejandro Melle (A60+M55) held June 12-16, 2023, in the Palacio de Congresos of Jaca, Spain.

by Josep Àlvarez Montaner (DMAT & **IMTech**)

Received on July 12th, 2023.

Enrique Artal (right) and Alejandro Melle (left)
Photo: Manuel González Villa

This conference in honor of Enrique Artal Bartolo and Alejandro Melle Hernández, on the occasion of their 60th and 55th birthdays (see Poster), convened 130 participants from 23 different countries, with a remarkable age spread ranging from graduate students currently working on their PhD theses to Professors that earned their PhD as far back as the '60s.

The Opening Ceremony was presided by José Antonio Mayoral, the Rector of the University of Zaragoza; Juan Manuel Ramón Ipas, Mayor of Jaca; Pedro Miana Sanz, Director of the IUMA Institute; María Pe Pereira, Vice-Dean of the School of Mathematics of the University Complutense de Madrid; and José Ignacio Cogolludo Agustín, Chair of the Organizing Committee. In his opening words, J. I. Cogolludo thanked the University of Zaragoza for their extensive support; the city of Jaca, represented by its Mayor, for lending the Congress Hall (Palacio de Congresos) as the conference venue; the Institute IUMA for their funding and unconditional support; and the School of Mathematics of the University Complutense de Madrid for their contribution.

The profile of the honorees was reviewed in a Special Session by three speakers: Anatoly Libgober, Alexander Suciu, and Ignacio Luengo Velasco. They highlighted the quality of their research and the many international connections and contributions. In the words of Eva Elduque, a young researcher with a Ramón y Cajal contract at the Mathematics Department of the UAM, "The speakers shared multiple anecdotes about the two honorees, showcasing how much the community appreciates them (both mathematically and at a personal level), as well as their work as mentors to younger mathematicians".

One valuable feature of the program was the inclusion of three "In-a-nutshell" courses. They were much appreciated by all participants. The first two consisted of two ninety-minute lectures and the third, three ninety-minute lectures. The speakers and titles of these courses were the following:

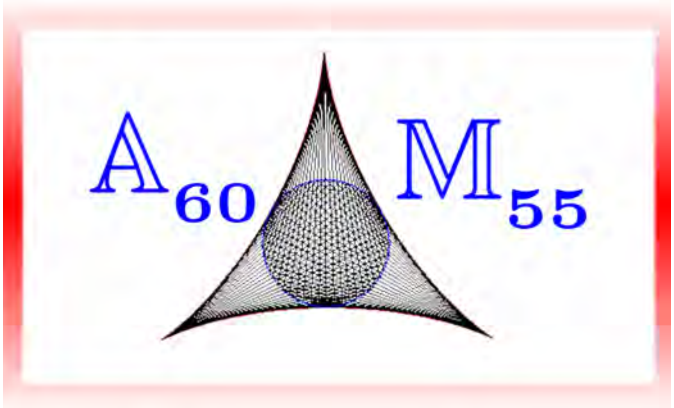
- Patrick Popescu-Pampu: *Combinatorics of Plane Curve Singularities* (Monday and Tuesday). Pdf slides.
- Laurentiu-George Maxim: *Fundamental groups of complex quasiprojective manifolds and their Alexander type invariants* (Monday and Wednesday). Pdf slides.
- Masahiko Yoshinaga: *Hyperplane Arrangements* (Wednesday, Thursday and Friday). Pdf slides.

In the page Plenary Speakers you can find the full list of names with the corresponding titles and abstracts, and via the Program page you can access the pdf slides of most of the lectures. To note that there was a participation of members of **IMTech**, including two plenary speakers.

Maria Alberich Carramiñana's lecture was on *The minimal Tjurina number in an equisingularity class of planar branches* (Pdf slides). It is well-known that the Tjurina number of a plane branch varies when deforming this plane branch preserving its Milnor number. This shows how subtle are analytic invariants of singularities compared to its topological counterparts. In this talk, she gave a closed formula for the minimal Tjurina number in such deformation which leads to a solution of a question posed by Alexandru Dimca and Gert-Martin Greuel on the comparison of the Milnor and Tjurina numbers.

Sebastià Xambó-Descamps presented *The discrete charms of Kähler Geometry* (a view of June Huh's Kähler package), Pdf slides, where he gave a gentle overview of the breakthrough results of June Huh and his coworkers emphasizing the algebraic geometry techniques they used in the resolution of deep conjectures in combinatorics. This talk connected nicely with Masahiko Yoshinaga's course.

The rest of the talks featured interesting aspects of singularity theory that give a clear picture of the latest results in the area. In this regard, we highlight Javier Fernández de Bobadilla's lecture on *Equimultiplicity of families with constant Milnor number* (Pdf abstract), where he presented a proof of Zariski's multiplicity conjecture that has been open for more than fifty years.



Five video recordings are available, one for each day of the conference. They can be accessed via these links: Monday 12, Tuesday 13, Wednesday 14, Thursday 15, and Friday 16.

The conference also programmed four Poster sessions that displayed sixteen posters in all. These sessions spawned lively discussions between the authors and the participants visiting them. Here is a link to the list of Authors and abstracts. In it we can find the poster presented by the UPC/CFIS student Roger Gómez López on *Stratification of the roots of the*

Bernstein-Sato polynomial for deformations of plane branches (Pdf abstract). The roots of the Bernstein-Sato polynomial behave quite mysteriously in a deformation of a plane branch. In this work, an effective method is presented, based on a careful study of the residues of the poles of the Archimedean zeta function, which provides a stratification of the space of parameters of the deformation with a fixed set of roots at each stratum. The set of roots of the generic stratum were conjectured by Tamaki Yano and proved by Guillem Blanco (see his Research focus on *Yano's conjecture* in NL03, page 10).

Reviews

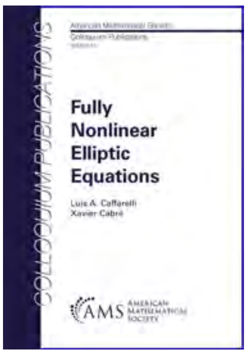
Analysis in Euclidean Spaces (World Scientific 2023)
by Joaquim Bruna.
Reviewed by Sebastià Xambó.

This treatise, published in the series *Essential Textbooks in Mathematics*, is based on the notes written by the author in his long teaching experience. It is organized in 21 chapters, each with an introduction summarizing its contents, and an additional chapter with 122 miscellaneous exercises. The wealth of materials it presents, with emphasis on concepts and rigorous proofs, with many examples and exercises inserted throughout, are tied together by very careful reasoning all along, and are handy for a variety of purposes. It can be used as a textbook for mathematics and physics undergraduate students on subjects such as differentiation theory in several real variables, measure and integration in several real variables, ordinary differential equations, linear partial differential equations, vector analysis, and curves and surfaces. Graduate students may use this book for an introduction to geometric measure theory and integral geometry, as well as advanced topics in vector analysis. It is also very suitable for self-study, including revision of topics by readers wishing to refresh them.

One valuable feature of this text is that there are a good number of original presentations of classic results. Among them, the treatments of the Riemann integral (Chapter 11) and the Lebesgue integral (Chapter 12); the multidimensional version of the fundamental theorem of calculus (Section 12.8); and refined versions of the Helmholtz decomposition of vector fields (Chapter 20, particularly §20.4). No doubt this fresh view has also much interest in teaching. For example, the author's version of the fundamental theorem of calculus allows to give "the correct proof" of the change of variable theorem in an integral (Theorem 13.3), as well as the basic theorems of vector calculus, with minimal regularity assumptions (Chapters 16, 17, 18). Another example is Poincaré's lemma (Theorem 18.5). Actually, there are jewels that are rare to find elsewhere, as the phrasing in modern language of Darboux's proof of Liouville's theorem on the rigidity of conformal applications in dimension > 2 (§10.4), and the results of the French classics on systems of triply orthogonal surfaces and Lamé surfaces (Chapter 10).
Although the text is basically a book on differentiation and integration in several variables, there are many links and pointers to many other areas. For instance: analysis of one complex variable (§9.5 and §17.3); ordinary differential equations (§8.2); partial differential equations (§4.7); classical theory of curves and surfaces (Chapter 16); geometric analysis (Chapter 7); integral geometry (§15.2). In this way the author fosters a living image of the core subject matter within mathematical analysis that contrasts with isolated treatments found in the literature.
Above all, this volume provides a road to learn how to think mathematically in real analysis, and its applications in various fronts, with no more prerequisites than a basic course in linear algebra and a standard first-year calculus course in differentiation and integration.

Fully Nonlinear Elliptic Equations
by Luis Caffarelli and Xavier Cabré, [18].
Reproducing the review by John Urbas in the Book Reviews of the Bulletin of the AMS, with an annex by the NL editors collecting a few follow up notes.

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 34, Number 2, April 1997, Pages 187–191
S 0273-0979(97)00704-0



One of the major advances in the theory of partial differential equations during the last twenty years has been the development of techniques for studying fully nonlinear second-order elliptic equations. These are equations of the general form

$$F[u] = F(D^2u, Du, u, x) = 0 \quad (3)$$

where F is nonlinear with respect to D^2u as well as possibly with respect to u and Du. Ellipticity means that F is a monotone function of the second derivative variables, in the sense that for any $(M, p, z, x) \in \mathbf{S}^{n\times n} \times \mathbf{R}^n \times \mathbf{R} \times \mathbf{R}^n$, where $\mathbf{S}^{n\times n}$ denotes the space of $n \times n$ real symmetric matrices, we have

$$F(M + N, p, z, x) > F(M, p, z, x) \quad (4)$$

for any positive definite $N \in \mathbf{S}^{n\times n}$. In addition, F is said to be uniformly elliptic if for any positive $N \in \mathbf{S}^{n\times n}$ we have

$$\lambda\|N\| \leqslant F(M+N, p, z, x) - F(M, p, z, x) \leqslant \Lambda\|N\| \quad (5)$$

for all $(M, p, z, x) \in \mathbf{S}^{n\times n} \times \mathbf{R}^n \times \mathbf{R} \times \mathbf{R}^n$, where λ and Λ are some positive constants, called the ellipticity constants of F. Examples of such equations are the Bellman equation

$$F[u] = \inf_{\alpha\in\mathcal{A}}\{L_\alpha u - f_\alpha(x)\} = 0 \quad (6)$$

and Isaacs' equation

$$F[u] = \sup_{\alpha\in\mathcal{A}} \inf_{\beta\in\mathcal{B}}\{L_{\alpha\beta} u - f_{\alpha\beta}(x)\} = 0 \quad (7)$$

where each L_α, $L_{\alpha\beta}$ is a linear elliptic operator of the form

$$L = a_{ij}(x)D_{ij} + b_j(x)D_i + c(x). \quad (8)$$

These are uniformly elliptic if each L_α, $L_{\alpha\beta}$ is uniformly elliptic with ellipticity constants independent of α and β. Bellman and Isaacs' equations arose originally in stochastic control theory and stochastic games theory, but it turns out that many equations arising elsewhere can also be written in these forms.

Two main strategies have been developed for solving fully nonlinear elliptic equations. One approach is to prove the existence of classical solutions of, say, the Dirichlet problem in a smooth bounded domain $\Omega \subset \mathbf{R}^n$ directly using the continuity method. For this, one needs to prove a priori estimates for solutions in the space $C^{2,\alpha}(\overline{\Omega})$ for some $1 < \alpha < 1$; i.e., one needs to bound u and its derivatives up to second order as well as the α Hölder seminorm of the second derivatives. This approach has led to the existence of classical solutions for a wide variety of fully nonlinear elliptic equations subject to various boundary conditions, but without doubt the central results are the interior second derivative Hölder estimates of Evans [6,7] and Krylov [12] and the corresponding global estimate of Krylov [13]. These results in turn depend on the Harnack inequality for linear elliptic equations in nondivergence form due to Krylov and Safonov [15,16].

The second approach is to prove the existence of some kind of generalized solutions and then to establish their uniqueness and regularity. The concept of generalized solution which has evolved in the work of Evans [4,5], Crandall and Lions [3], and Lions [17] is that of viscosity solution, although for specific classes of equations there are also other notions. A continuous function u defined on a domain in $\mathbf{R}^n$ is said to be a *viscosity subsolution* (respectively, *viscosity supersolution*) of the elliptic equation

$$F(D^2u, x) = 0 \tag{9}$$

if for any C^2 function ϕ on Ω and any local maximum (respectively, minimum) $x_0 \in \Omega$ of $u - \phi$ we have $F(D^2\phi(x_0), x_0) \geqslant 0$ (respectively, $\leqslant 0$). A *viscosity solution* is a continuous function that is both a viscosity subsolution and supersolution. The point is that the test function ϕ should satisfy the inequalities which would hold by virtue of the maximum principle and the ellipticity of the equation if u were a C^2 solution. The notion of viscosity solution turns out to be very useful, because it is stable under locally uniform convergence of both u and F and because existence and uniqueness results for such solutions can be proved under very general conditions, and in particular for equations for which the existence of classical solutions is not known (and perhaps not true), such as Isaacs' equation in dimensions greater than two.

A major breakthrough in the theory of viscosity solutions was made by Jensen [11], who proved a comparison principle which implied the uniqueness of viscosity solutions of the Dirichlet problem for (7), at least for F independent of x. Later refinements allowed this assumption to be relaxed. Using these results, Ishii [9,10] observed that the existence of viscosity solutions of the Dirichlet problem followed from the Perron method.

Finally we come to the main topic of this book, which is the regularity theory for viscosity solutions of uniformly elliptic equations of the general form

$$F(D^2u, x) = f(x). \tag{10}$$

It is based on a series of lectures given at New York University in 1993. The central results are the $W^{2,p}$, $C^{2,\alpha}$, and $C^{1,\alpha}$ estimates of the first author [1,2]. To describe these, it is probably best to follow the book and to recall the corresponding estimates for linear equations. Let u be a bounded solution of the uniformly elliptic equation

$$Lu = a_{ij}(x)D_{ij}u = f(x) \tag{11}$$

in the unit ball $B_1 \subset \mathbf{R}^n$. Then the following are true:

(i) (Cordes-Nirenberg type estimates) Let $0 < \alpha < 1$ and suppose that

$$\|a_{ij} - \delta_{ij}\|_{L^\infty(B_1)} \leqslant \delta(\alpha)$$

for sufficiently small $\delta(\alpha) > 0$. Then $u \in C^{1,\alpha}(\overline{B}_{1/2})$ and

$$\|u\|_{C^{1,\alpha}(\overline{B}_{1/2})} \leqslant C\big(\|u\|_{L^\infty(B_1)} + \|f\|_{L^\infty(B_1)}\big)$$

(ii) (Schauder estimates) If a_{ij} and f belong to $C^\alpha(\overline{B}_1)$, then $u \in C^{2,\alpha}(\overline{B}_{1/2})$ and

$$\|u\|_{C^{2,\alpha}(\overline{B}_{1/2})} \leqslant C\big(\|u\|_{L^\infty(B_1)} + \|f\|_{C^\alpha(\overline{B}_1)}\big)$$

(iii) (Calderón-Zygmund estimates) If a_{ij} are continuous in B_1 and $f \in L^p(B_1)$ for some $1 < p < \infty$, then $u \in W^{2,p}(B_{1/2})$ and

$$\|u\|_{W^{2,p}(B_{1/2})} \leqslant C\big(\|u\|_{L^\infty(B_1)} + \|f\|_{L^p(B_1)}\big).$$

These estimates are obtained by first deriving the estimate for solutions of $\Delta u = f$ and then using a perturbation technique. The idea in the nonlinear case is similar: if one has suitable existence results and interior estimates for solutions of the "constant coefficient" equation

$$F(D^2u, x_0) = f(x_0), \tag{12}$$

then under certain assumptions on F and f one can derive $C^{1,\alpha}$, $C^{2,\alpha}$, or $W^{2,p}$ estimates for viscosity solutions of (8) by a perturbation argument. The basic assumption on F required to carry out this procedure is that the quantity

$$\beta(x, x_0) = \sup_{M \in \mathbf{S}^{n\times n}} \frac{|F(M,x) - F(M,x_0|}{\|M\|}$$

is sufficiently small if $|x - x_0|$ is small. The precise nature of this smallness condition and the estimates required for solutions of (9) depend on which estimate one is considering, but in each case the smallness of $\beta(x, x_0)$ is measured in the L^n norm rather than the L^∞ norm. The techniques thus give improved versions of the classical linear estimates stated above.

The book begins with some preliminary material concerning tangent paraboloids and second-order differentiability. In Chapter 2 viscosity solutions are introduced, and the class $\mathcal{S}(\Lambda, \lambda, f)$ of "all viscosity solutions of all elliptic equations of the form (8) with ellipticity constants Λ and λ" is defined using Pucci's extremal operators. This is important in what follows because it allows the authors to avoid the traditional Bernstein method of differentiating the equation to obtain linear differential inequalities for the derivatives of the solution. Clearly, this procedure is not possible if F and u are not sufficiently smooth.

Chapters 3 and 4 deal with two crucial tools from the linear theory. These are the Alexandroff-Bakelman-Pucci estimate and maximum principle and the Harnack inequality of Krylov and Safonov. These are proved for viscosity solutions rather than classical solutions, so the proofs are a little more complicated in certain parts than those presented elsewhere, for example in [8] or [14]. An important consequence of the Harnack inequality is the C^α interior regularity of solutions of (8).

The following two chapters deal with the existence and uniqueness results and estimates for solutions of uniformly elliptic equations of the form

$$F(D^2u) = 0 \tag{13}$$

which are necessary to carry out the perturbation procedure mentioned above. A proof of Jensen's comparison principle for viscosity solutions of (11) is given, but the existence of solutions of the Dirichlet problem is not proved. It is only remarked that the Perron method can be used for this once one has a comparison principle. A $C^{1,\alpha}$ interior estimate for solutions of (11) is proved, and in Chapter 6 a version of the Evans-Krylov $C^{2,\alpha}$ interior estimate for C^2 (in fact, even $C^{1,1}$) solutions of concave equations of the form (11) is presented. In addition, a new proof of the $C^{1,1}$ interior regularity of viscosity solutions of such equations is given.

In the following two chapters these estimates are combined with delicate perturbation arguments to obtain the $W^{2,p}$, $C^{2,\alpha}$, and $C^{1,\alpha}$ estimates mentioned above. This is the most technical part of the book and cannot be described in detail here. The key idea, however, is to consider paraboloids of the form $P(x) = u(x_0) + l(x - x_0) \pm \frac{1}{2}M|x|^2$, where l is a linear function, and to show that the measure of the complement in $B_{1/2}$ of the set of points x_0 at which there is such a paraboloid touching the graph of u from above (or from below) must decay sufficiently quickly as M gets large. This gives control of the distribution function of second-order difference quotients of u, leading to $W^{2,p}$ estimates. The $C^{2,\alpha}$ (respectively, $C^{1,\alpha}$) estimates are proved by showing that the existence and regularity results for solutions of (10) imply that the solutions of (8) are well approximated by quadratic polynomials (respectively, affine functions).

In the final chapter the authors present an alternative proof of the $C^{1,1}$ interior estimate for smooth solutions of concave equations of the form (11). In addition, they describe the proof of the classical solvability of the Dirichlet problem for such equations using the continuity method. Most of the necessary estimates are proved in detail, but the proof of Krylov's boundary gradient Hölder estimate is omitted.

The book is well written, with the arguments clearly presented. There are helpful remarks throughout the book, and at several points the authors give the main ideas of the more technical proofs before proceeding to the details. No previous knowledge of viscosity solutions is assumed, but readers who are not familiar with the existence and uniqueness theory of viscosity solutions will probably want to consult other sources for this, as these aspects are not covered in detail. The book will certainly be of interest to researchers and graduate students in the field of nonlinear elliptic equations.

2023 Annex by the NL editors

The review [19] summarizes the contents of the book and ends with this appraisal: "This book provides a self-contained and detailed presentation of the regularity theory for viscosity solutions of fully nonlinear elliptic equations as developed in the last decade. It can be highly recommended to researchers as well as to graduate students who are interested in this area."

In [20], which also provides a detailed review, it is asserted that: "The book [...] is clearly written and gives an excellent impression of progress with the regularity theory for equations of types (1) [$F(D^2u, x) = f(x)$] and (2) [$F(D^2u) = 0$]; it highlights in a most useful way some significant recent advances (in which the authors have been very active) which play a dominant part in the theory. [...] All in all, the book marks an important stage in the theory of nonlinear elliptic problems. Its timely appearance will surely stimulate fresh attacks on the many difficult and interesting questions which remain."

The book has been a bestseller within the AMS bookstore at different periods. As of this writing (March 31st, 2023), it has 955 citations in MathSciNet and 1.726 in Google Scholar.

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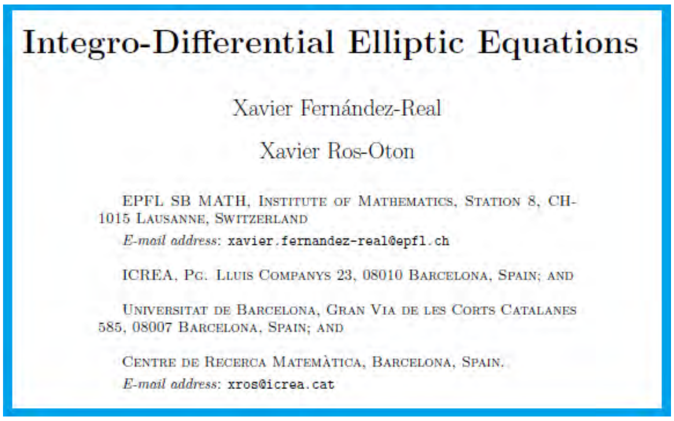
Integro-Differential Elliptic Equations

by Xavier Fernández-Real Girona and Xavier Ros-Oton. Reviewed by Sebastià Xambó.



Integro-Differential Elliptic Equations

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This memoir, henceforth named IDEE, is the winner of the Ferran Sunyer i Balaguer Prize 2023. This prestigious prize is awarded yearly by the Ferran Sunyer i Balaguer Foundation since 1993 (except the years 1995, 2015, 2019) and the winning memoirs are published in the Birkhäuser book series Progress in Mathematics. About the authors, see the interviews in this issue with Xavier Ros-Oton (page 2), and with ▷ Xavier Fernández-Real Girona (page 6). For a biography of FSiB, see [1].

Ferran Sunyer i Balaguer (1912-1967)
Credit: FSB Foundation

The members of the Scientific Committee appointed to decide this edition of the FSB Prize were Antoine Chambert-Loir (winner of the 2017 edition [2]), Ruth Kellerhals, Marta Sanz-Solé, Kristian Seip, and Yuri Tschinkel. They had to consider nine candidates and their report on IDEE says:

"This book presents a deep and solid study about the existence and regularity theory for nonlocal elliptic equations. This type of equations somehow extends the PDE notion to a more general setting, in which the underlying operators are integral (instead of purely differential) operators. The study of nonlocal equations is motivated by several applications within Mathematics (in Probability, Geometry, or Fluid Mechanics) and in other sciences (Physics, Biology, or Finance). This is a quite young subject, central in the PDE theory, in which the authors have contributed with breakthroughs, and are among the world leaders."

The authors' back-cover summary of the book is also very informative about its aims and nature:

"This book aims to provide a self-contained introduction to the regularity theory for integro-differential elliptic equations, mostly developed in the 21st century. Such a class of equations often arises in analysis, probability theory, mathematical physics, and in several contexts in applied sciences. The authors give a detailed presentation of all the necessary techniques, primarily focusing on the main ideas rather than proving all results in their greatest generality. The book starts from the very basics, studying the square root of the Laplacian and weak solutions to linear equations. Then, the authors develop the theory of viscosity solutions to nonlinear equations and prove the main known results in this context. Finally, they study obstacle problems for integro-differential operators and establish the regularity of solutions and free boundaries. Almost all the covered material appears in book form for the first time, and several proofs are different (and shorter) than those in the original papers. Moreover, several open problems are listed throughout the book."

IDEE appears to be germane of [3], by the same authors. The sequential treatment is similar in both: linear equations first, then nonlinear equations, and finally the application to obstacle problems. The main difference lies in the kind of operators used: while in [3] they are "classical" local operators, in IDEE they are 'integro-differential" operators, a more involved breed whose study has taken off in recent times. In any case, they are remarkably self-contained and comprehensive, and are thus the sort of materials that should be useful to researchers, and even more to PhD students wishing to enter the realm of integro-differential operators. These books, and especially IDEE, also supply bibliographic references for results that are scattered in papers published in the last two decades, often with trimmed down proofs and with a good many new results. The technical aspects are dealt with thoroughly and efficiently, with a clear and engaging writing style.

To end, let us describe briefly IDEE's contents. The focus of the first chapter is the square root of the Laplacian, which is used to present a fairly basic introduction to nonlocal operators. The second chapter is about general linear integro-differential operators. After a motivating discussion about their origin and their probabilistic interpretation, the theory of interior and boundary regularity is presented. To note that the authors use, and prove, the optimal ellipticity hypotheses, a move not covered before in the literature. In the third chapter, "fully nonlinear" equations are studied and the complete general theory of viscosity solutions for "fully nonlinear nonlocal" operators is developed. Finally, in the last chapter the "obstacle problem" is presented as an example of "free boundary problems" with integro-differential operators.

Acknowledgments. The reviewer is thankful to the authors for sharing the prized memoir and other materials, and also for their readiness to provide helpful feedback on various aspects of their work. Thanks also to the FSB Foundation for the permission to reproduce the verdict on IDEE.

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The Gaussian Double-Bubble and Multi-Bubble Conjectures

by Emanuel Milman and Joe Neeman, [1].
Reviewed by Joaquín Pérez.

For the Euclidean space $\mathbb{R}^n$, the multiple-bubble problem consists of finding the smallest area that encloses and separates $q \in \mathbb{N}$ regions of prefixed positive volumes. Therefore, the classical isoperimetric problem is obtained for $q = 1$ and the double-bubble problem is the case $q = 2$. The double-bubble problem was solved in $\mathbb{R}^3$ by M. Hutchings, F. Morgan, M. Ritoré and A. Ros, [2], and for general dimension n by B. Reichardt, [3]. The solution to this problem is the standard spherical double-bubble given by three spherical pieces that intersect each other forming a dihedral angle of $2\pi/3$ along their intersection, a configuration that is easily produced when blowing soap bubbles. For the q-bubble case, $3 \leqslant q \leqslant n+1$, the optimal way to enclose and separate q regions of prescribed volume in $\mathbb{R}^n$ is conjectured to be the standard spherical q-bubble, i.e., a general version of the standard double bubble above. This question is still open.

The case of the Gaussian space is a relevant one related to several questions in PDEs, Probability, Banach spaces and Geometry. The Gaussian space $(\mathbb{R}^n, \gamma^n)$ is the Euclidean space endowed with the probability measure

$$\gamma^n = (2\pi)^{-n/2} \exp\left(-|x|^2/2\right)\, dx.$$

For $q = 2$, the isoperimetric problem in the Gaussian space and its solution was obtained in the 1970s by V.N. Sudakov and B.S. Tsirelson, [4], and independently by C. Borell, [5]. They proved that a hyperplane dividing the space in the two prescribed Gaussian measures is a minimizer for this problem. Later, E.A. Carlen and C. Kerce, [6], proved that halfspaces are in fact the unique minimizing clusters minimizers for the Gaussian isoperimetric inequality.

As in the Euclidean case, for $3 \leqslant q \leqslant n+1$ there is a natural Gaussian multi-bubble conjecture, which states that the least perimeter way to decompose the Gaussian space $(\mathbb{R}^n, \gamma^n)$ into q cells is the symmetric simplicial cluster given by the Voronoi cells determined by q equidistant points. In this paper the authors resolve the conjecture:

Gaussian Multi-Bubble Theorem: *For all $2 \leqslant q \leqslant n+1$, (symmetric) simplicial q-clusters are the unique minimizers of the total Gaussian perimeter in $(\mathbb{R}^n, \gamma^n)$ among all q-clusters of prescribed Gaussian measures.*

To understand the above statement, we need some concepts:

1. A *q-cluster* $\Omega = (\Omega_1, \ldots, \Omega_q)$ is a q-tuple of pairwise disjoint Borel subsets $\Omega_i \subset \mathbb{R}^n$ (called *cells*), of finite Gaussian perimeter and such that $\gamma(\mathbb{R}^n \setminus \cup_{i=1}^q \Omega_i) = 0$ (cells are not necessarily connected).

2. The total Gaussian perimeter of a q-cluster Ω is

$$P_\gamma(\Omega) := \frac{1}{2}\sum_{i=1}^{q} P_\gamma(\Omega_i),$$

where the Gaussian perimeter of a Borel set $U \subset \mathbb{R}^n$ is

$$P_\gamma(U) = \sup\left\{ \int_U (\mathrm{div} X - \langle \nabla W, X\rangle) d\gamma \mid X \in C_c^\infty(\mathbb{R}^n, T\mathbb{R}^n),\ \|X\| \leqslant 1 \right\}.$$

3. The Gaussian measure of a q-cluster Ω is the element $\gamma(\Omega)$ in the $(q-1)$-dimensional simplex $\Delta^{(q-1)} = \{v \in \mathbb{R}^q \mid v_i \geq 0, \sum_{i=1}^q v_i = 1\}$ given by

$$\gamma(\Omega) = (\gamma(\Omega_1), \ldots, \gamma(\Omega_q)) \in \Delta^{(q-1)}.$$

4. A *simplicial q-cluster* is the set of Voronoi cells of q equidistant points $x_1, \ldots, x_q \in \mathbb{R}^n$. These Voronoi cells are

$$\Omega_i = \mathrm{int}\left\{ x \in \mathbb{R}^n \mid \min_{j=1,\ldots,q} \|x - x_j\| = \|x - x_i\| \right\}, \quad i = 1, \ldots, q.$$

Therefore, the cells of a simplicial 2-cluster are precisely half-spaces in $\mathbb{R}^n$, and and the single-bubble Gaussian conjecture for $q = 2$ holds by the classical Gaussian isoperimetric inequality. For $q = 3$ we have the double-bubble Gaussian case (with prescribed Gaussian volume pair $v \in \mathrm{int}\,\Delta^{(2)}$), whose minimizer is given by three half-hyperplanes meeting along an $(n-2)$-dimensional subspace with dihedral angles of $2\pi/3$.

The proof of this groundbreaking result is based on the application of the maximum principle for a certain fully non-linear second-order elliptic PDE that involves the *Gaussian isoperimetric profile* $I^{(q-1)} \colon \Delta^{(q-1)} \to \mathbb{R}$ given by

$$I^{(q-1)}(v) = \inf\left\{P_\gamma(\Omega) \mid \Omega \text{ is a } q\text{-cluster with } \gamma(\Omega) = v\right\}.$$

Remarks. The authors of the paper under review posted [7] in May 2022. This work deals with bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$ (Euclidean case), is not cited in [1], and seemingly has not been published yet. It is reviewed in AMR by F. Morgan.

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Events

INGRID DAUBECHIES wins the 2023 Wolf Prize in Mathematics

Excerpts from [1]:

- The 2023 Wolf Prize in Mathematics is awarded to INGRID DAUBECHIES, Duke University, USA, *for work in wavelet theory and applied harmonic analysis.*

- Her research has revolutionized the way images and signals are processed numerically, providing standard and flexible algorithms for data compression. This has led to a wide range of innovations in various technologies, including medical imaging, wireless communication, and even digital cinema.

- The Wavelet theory, as presented by the work of Professor DAUBECHIES, has become a crucial tool in many areas of signal and image processing. For example, it has been used to enhance and reconstruct images from the early days of the Hubble Telescope, and to detect forged documents and fingerprints. In addition, wavelets are a vital component of wireless communication and are used to compress sound sequences into MP3 files.

- Beyond her scientific contributions, Professor DAUBECHIES also advocates for equal opportunities in science and math education, particularly in developing countries. As President of the International Mathematical Union, she worked to promote this cause. She is aware of the barriers women face in these fields and works to mentor young women scientists and increase representation and opportunities for them.

For more information on her life, work, and academic accomplishments, see the web pages [2–5]. The citation of [5] (2013):

- This year's BBVA Foundation Frontiers of Knowledge Award in Basic Science goes to two mathematicians: Professor INGRID DAUBECHIES for her work on wavelets, and Professor DAVID MUMFORD for his contributions to algebraic geometry and to the mathematics of computer vision. These works in pure mathematics have strongly influenced several fields of application, ranging from data compression to pattern recognition.

- Professor DAUBECHIES is a leader in theoretical signal processing, with pioneering contributions to the theory and application of wavelets and filter banks. Her work resulted in a new approach to data compression, with a strong impact on a multitude of technologies, including efficient audio and video transmission and medical imaging.

- Professor MUMFORD introduced the modern approach to algebraic geometry into a classical area through his work on geometric invariant theory. He also applied tools of variational calculus to the theory of vision and developed statistical models for imaging and pattern recognition. His work has had a lasting impact in both pure and applied mathematics.

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[2] Scholars@Duke, 2023. Ingrid Daubechies.

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IMTech member ALBERT ATSERIAS awarded ICREA Academia distinction

The ICREA Academia program was launched in 2008 with the aim of promoting and rewarding the research excellence of professors at public universities in Catalunya, who are in an active and expansive phase of their research career. Recognized researchers receive a research grant of 40,000 euros a year for a period of five years to promote their research. In the 2022 call, a total of six UPC researchers have been recognized with the ICREA Academia Prize.

Albert Atserias is full professor at the UPC Department of Computer Science, and a member of IMTech and Centre de Recerca Matemàtica. He was the principal investigator of the ERC-CoG project (2015–2020) AUTAR: A Unified Theory of Algorithmic Relaxations, funded by the research funding agency of the European Commission (ERC). His groundbreaking result with Moritz Müller on the computational complexity of proof search, published in the J. of the ACM in 2020, was outlined in a Research focus published in NL01 (pages 6-7).

His research focuses on logic and the theory of computation. His expertise is on computational and algorithmic complexity, more particularly, on descriptive complexity and proof complexity. During the next five years his efforts will pursue the research line on algorithmic complexity that aims at establishing variants of the most important conjectures in the area, such as the famous $P \neq NP$ or $P = BPP$, in non standard models of computation. The goal of the research is to establish these conjectures in non standard models which are however realistic enough so that it is still possible to deduce some of its most relevant consequences for the foundations of algorithmic complexity, for instance the existence of generators of guaranteed pseudorandom bits. Computational complexity studies the limitations of algorithms: what they can do and, especially, what computers can't do. The conjecture predicts that for solving some algorithmic tasks we will need more resources than we can ever have. The conjecture P $\neq$ NP is one of the seven millennium problems established by the Clay Foundation.

Jordi Guàrdia, new Dean of the FME

Photos: Conchi Martínez

Professor Jordi Guàrdia Rúbies took office on March 28 as Dean of the Faculty of Mathematics and Statistics (FME) of the **UPC**. He is associate professor of the **UPC** Departament of Mathematics and succeeds Jaume Franch, who has been Dean of the **FME** for the last eight years (see the interview with him in this issue, and the inteview with Jordi Guàrdia in NL04, pages 6-8).

The ceremony was held at the **FME** and was chaired by the rector of the UPC, Daniel Crespo, accompanied by Jaume Franch, Jordi Guàrdia, and the general secretary of the UPC, Ana B. Cortinas.

Jordi Guàrdia has a Doctorate in Mathematics from the UB (1998) and he teaches at the **FME** and at the EPSEVG. He has also taught at the UOC (during the period 1998-2009) and at the UB (1990-2001). His research has focused on number theory, mainly in arithmetic geometry and computational algebraic number theory. He is presently working in the interaction between local number theory and singularities of curves through valuation theory. He is an active member of the Seminari de Teoria de Nombres de Barcelona and has collaborated in the organization of several international conferences and workshops, being one of the founders of the "Jornadas de Teoría de Números", which this year has reached the ninth edition.

He has served as teaching deputy of the DMAT for eleven years, and has promoted various teaching initiatives, such as the Jornada Docent of the DMAT (Department's Teacher's Day—DTD), or the coordination of teaching materials for the different engineering schools of the UPC.

In 2022 his project *Watches, dresses and roller coasters: designing with mathematics* (*Relotges, vestits i muntanyes russes: dissenyant amb matemàtiques*), developed in the context of the subject *Mathematics for Design*" (MADI for short) was distinguished with the 25th UPC Prize for Teaching Initiatives (Press release) and with the Vicens Vives Award for Teaching Quality.

The Dean's Team: 1 Jordi Guàdia, Dean; 2 Juanjo Rué Perna, Academic Secretary; 7 María Paz Linares Herreros, Vice-dean for Equality, Inclusion and Equity; 3 Sebastià Martín Molleví, Vice-dean for Promotion and External Relations; 6 Jordi Saludes Closa, Vice-dean for Teaching Innovation and New Technologies. 9 Jesús Fernández Sánchez, deputy director for the mathematics degrees; 10 Lourdes Rodero de Lamo, deputy director for the statistics degrees; 4 Gemma Flaquer, Academic Resources delegate; 8 Mireia Ribera Mitjans, Promotion and Internal Coordination delegate; 5 Jaume Fusté, Head of the FME Management.

Eva Miranda's Hardy tours

On page 6 of the Issue 505 of the LMS Newsletter (March 2023), we read the following announcement: "The London Mathematical Society is pleased to announce that the LMS Hardy Lecturer 2023 is Professor Eva Miranda (**UPC** and CRM–Barcelona).

The Hardy Lectureship was founded in 1967 in memory of G. H. Hardy in recognition of outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is *a lecture tour of the UK by a mathematician with a high reputation in research*. Professor Miranda will undertake three lecture tours of the UK between May and September 2023, which will include the Hardy Lecture at the Society Meeting on Friday 30 June in London. Further details about the Hardy Lecture Tour 2023 are available on the website Hardy Lectureship."

G.H. Hardy, LMS President 1926-1928 and 1939-1941

E. Miranda (UPC Barcelona) Hardy Lecturer 2023

The latter web page provides details about dates, institutions, lecture titles and abstracts of the nine scheduled lectures. See also the LMS Poster on next page. The title of the LMS Hardy Lecture 2023 was *From Alan Turing to Fluid computers: Explored and unexplored paths*. For a timely synopsis of some of the ideas in Miranda's lecture, the reader may consult the Research focus written by Robert Cardona, Eva Miranda and Daniel Peralta-Salas on pages 9-11 of NL01.

LMS Hardy Lecture Tour 2023

G.H. Hardy, LMS President
1926–1928 and 1939–1941
Photo Courtesy of Master and Fellows of
Trinity College Cambridge

Eva Miranda (UPC-Barcelona)
Hardy Lecturer 2023

The Hardy Lectureship was founded in 1967 in memory of G.H. Hardy in recognition of outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is a lecture tour of the UK by a mathematician with a high reputation in research. The 2023 LMS Hardy Fellow is Professor Eva Miranda (UPC-Barcelona).

Professor Miranda will visit the UK in May, June, July and September 2023 and she will give talks at:

Cambridge 30 May;
Counting periodic orbits
Organiser: Maciej Dunajski

Royal Institute, London 1 June;
From Alan Turing to contact geometry: towards a "Fluid computer"
Organiser: Saksham Sharma

Birmingham 26 June;
Desingularizing singular symplectic structures
Organiser: Marta Mazzocco

Warwick 28 June;
Euler flows as universal models for dynamical systems
Organiser: José Rodrigo

Mary Ward House, London 30 June;
From Alan Turing to fluid computers: Explored and unexplored paths
Organiser: London Mathematical Society

Oxford 4 July;
Singular Hamiltonian and Reeb Dynamics: First steps
Organisers: Andrew Dancer and Vivat Nanda

Loughborough 6 July;
Action-angle coordinates and toric actions on singular symplectic manifolds
Organisers: Sasha Veselov and Alexey Bolsinov

Edinburgh 19 September;
From Symplectic to Poisson manifolds and back
Organiser: José Figueroa-O'Farrill

Glasgow 21 September;
Quantizing via Polytope counting: Old and new
Organiser: Ian Strachan

For further information on attending each lecture, please visit the LMS website here:
lms.ac.uk/events/lectures/hardy-lectureship#Hardy%20Current
For general enquiries about the Hardy Lectures, please contact (lmsmeetings@lms.ac.uk).

Hardy lectureship. General Meeting of the LMS, 30 June

The LMS Hardy Lecture 2023 was preceded by Sir Roger Penrose's lecture with the title *Non-computability in Physics?*

Top row: Title slides of lectures at Birmingham (June 26) and Loughborough (July 6). ***Second row:*** Lecturing at Birmingham (left) and Warwick (right). ***Third row left:*** LMS Hardy Lecture: Eva Miranda was introduced by Ulrike Tillman, the President of the LMS. ***Third row right:*** A view of the LMS Hardy Lecture room. ***Bottom left:*** Eva Miranda delivering the LMS Hardy Lecture. ***Bottom right:*** Eva Miranda, Sir Roger Penrose, Alina Vdovina, Arthur Jaffe; (back) Jens Marklof, Stephen Hugget. These pictures are courtesy of Eva Miranda.

On Saturday, May 13, 2023, the front page of the newspaper Ara featured the headline Eva Miranda, the most awarded mathematician, pointing to the two pages of the Science section dedicated to an interview with her. The interview of Prof. Eva Miranda (**IMTech**, full professor at UPC, and ICREA Academia) was conducted by the journalist Toni Pou. Her mathematical achievements, which are and have been attracting much international media attention, are regularly reported in this NL: Hardy tour (this issue, pages 1, 35-37); Distinctions: NL01, pages 1, 2, 21 and NL03, page 1 and page 20; Research focus: NL01, pages 9-11 (with Robert Cardona and Daniel Peralta-Salas).

In the interview, various topics about Mathematics of great interest to the general public were discussed, and Professor Miranda provided her personal view of them. Here is a summary gleaned by this NL. From an early age, Eva Miranda found solace and fascination in the world of mathematics. It provided her with a platform to solve problems within defined parameters and explore the depths of her own imagination. Describing mathematics as a means to solve equations and a discipline that allows us to model and address problems using formulas, she believes it encompasses a wide range of applications.

Prof. Miranda acknowledges the often-intimidating reputation that mathematics holds and attributes it to the way the subject is taught. She draws a charming parallel to writing a book, explaining that to truly appreciate mathematics, one must learn its language and engage with its internal rules and structures. She emphasizes that mathematics, much like literature, possesses a creative aspect. While there are rules to follow, mathematicians venture into uncharted territories to solve problems, a process she finds endlessly captivating. She admits that teaching mathematics is a formidable challenge, as conveying its creative dimension can be difficult, particularly when students have varying levels of interest, and she concedes that efforts have been made to highlight the creative aspects of mathematics and improve teaching methods to make it more engaging.

Prof. Miranda often tries to persuade those who have struggled with mathematics that it is a fascinating subject by emphasizing its presence in everyday life. From the coordination of traffic lights to the functioning of computers, mathematics permeates our surroundings. She is convinced that, by recognizing mathematics as an integral part of daily life, the fear associated with the subject can be overcome, transforming it into an exciting and empowering experience.

Prof. Miranda discusses the social recognition of mathematicians in Catalonia and its disparity when compared to that of countries like France, where mathematicians enjoy significantly higher prestige. She cites examples of accomplished mathematicians who receive little attention in the media, which connotes a low social recognition of the field.

The aesthetic and elegance found in mathematical procedures are qualities often mentioned by mathematicians. Prof. Miranda draws a parallel between mathematics and art, highlighting that mathematicians, like poets or painters, work with patterns made of ideas rather than words or colours. The beauty lies in capturing the essence of things and distilling complex concepts into simple and elegant explanations. Prof. Miranda textually asserts: "I lay claim to this aesthetic part of mathematics and science because, in a way, it humanizes us and makes us closer to art and creativity."

Mathematics has both theoretical and practical applications. While society increasingly demands immediate practical applications for everything, Prof. Miranda emphasizes that mathematics may not always provide immediate utility, but often yields applications in the long run. As an illustration, she considers the example of computers, which emerged from Alan Turing's abstract definition of computation during his doctoral thesis, initially without any apparent practical use.

Prof. Miranda's research explores various areas, often seeking interdisciplinary connections. She mentions a recent project related to fluid dynamics equations, motivated by a question of the renowned mathematician Terence Tao related to one of the Millennium Prize problems, which led to insights in geometry and has applications in studying ocean currents and meteorology.

In addressing the ethical implications of mathematics, Prof. Miranda highlights the significance of algorithms and artificial intelligence (AI). She recognizes that algorithms shape the information we receive and that AI can influence people's opinions, emphasizing the need for ethical guidelines in their development and usage. Prof. Miranda suggests involving ethics committees and taking a responsible approach to ensure these technologies benefit society as a whole.

Overall, Eva Miranda's journey in mathematics is characterized by her passion for abstract structures, her drive to uncover creative dimensions within the discipline, and her commitment to making mathematics accessible and recognized in society.

Sir Roger Penrose's quotations

- My own way of thinking is to ponder long and I hope deeply on problems and for a long time which I keep away for years and years and I never really let them go.
- Sometimes it's the detours which turn out to be the fruitful ideas.
- Some people take the view that the universe is simply there, and it runs along—it's a bit as though it just sort of computes, and we happen by accident to find ourselves in this thing. I don't think that's a very fruitful or helpful way of looking at the universe.
- The image of Stephen Hawking—who has died aged 76—in his motorised wheelchair, with head contorted slightly to one side and hands crossed over to work the controls, caught the public imagination as a true symbol of the triumph of mind over matter.

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Alphabetical Index NL05

A. Dou Prize, 9, 10
A. Montaner, J., 1, 28
A. Valle Prize, 2, 6
A60+M55, 28
Aalto University, 3, 4
Abacus Medal, 26
Abel Prize, 1, 7, 8, 25, 26
Agrawal, S. K., 5
AI, 22, 24
Aix-Marseille Univ., 10
Alberich-Carramiñana, M., 16, 18, 28, 39
Albert Dou Prize, 8
Allard, A., 1
Almirón, P., 1, 18
Ambizione Fellow, 6
Amorós, J., 16
AMR, 33
AMS, 1, 25
Ara, 38
Arias, I., 39
Artal, E., 1, 28
Atiyah, M., 17
Atserias, A., 1, 26, 34
Auscher, P., 4

Basetis, 1, 22
BBVA Foundation, 6, 34
Berlin Math. School, 8
BGSMath, 9
binary entropy, 26
Birkhäuser, 32
Birmingham Univ., 37
Blanco, G., 29
Bloom, T., 8
Bobrova, I., 18
Bolsinov, A., 36
Bortz, S., 4
Bott, R., 17
Boza, S., 8
Braverman, M., 26
Bruna, J., 1, 29
Bull. AMS, 29

Cabré, X., 1–4, 6, 25, 29
Caffarelli, L., 1, 2, 7, 20, 25, 29
Calabi-Yau, 11
Cangur, 9
Cardona, R., 35, 38
CAS, 24
Caselles, V., 2, 6
Castells, M., 23
CFIS, 6, 8, 9
Chambert-Loir, A., 32
character varieties, 11
Cilleruelo, J., 9
Cité Universitaire, 9
Clay Foundation, 25
Cluster algebras, 12
CNRS, 10
Cogolludo, J. I., 28
Colombo, M., 6
Coltraro, F., 1, 16
Columbia Univ., 5
connectome, 10
Córdoba, A., 25
Cortinas, A. B., 35
Courant Institute, 25
COVID, 9
Covid, 24
Crespo, D., 35
CRM, 8, 9
CS/UPC, 26
CSIC, 5, 16
CV, 23

D. N. Chorafas Prize, 6
Dancer, A., 36
Daubechies, I., 1, 34
de Giorgi, E., 25
de la Llave, R., 25
Deco, G., 10, 11
Desrosiers, P., 1
Díaz Barrero, J. L., 6
Dimca, A., 28
DMAT, 8, 10, 28, 35
doctorate HC, 25
Dunajski, M., 36

E. Galois Prize, 6
École Polytechnique, 8
Egert, M., 4
Eixarch, R., 1, 24
El País, 8, 23
Elduque, E., 28
elliptic equations, 29
EMS, 2
EPFL, 6
EPSEVG, 35
EPV/EHU, 3
ERC, 8
ERC Starting Grant, 2
Erdös, P., 9
ETH Zürich, 6, 7
ETSEIB, 5
ETSETB, 8, 9
Eurocomb'21, 8

F. de Bobadilla, J., 28
FBBVA, 2
Fernández, J., 35
Fernández-Real, X., 1, 2, 6, 32
FIB, 5
Fields Medal, 2, 7, 8
Figalli, A., 2, 6
Figueroa-O'Farrill, J., 36
Flaquer, G., 35
FME, 1, 2, 5, 8, 9, 20, 22, 24, 35
Fossas, E., 4
Franch, J., 1, 4, 9, 35
Freie Univ. Berlin, 8, 9
Fresán, J., 8
FSB Prize, 6, 7, 32
FSiB Prize, 1
Fusté, J., 35

G. Deco's lab, 10
Gabilondo, A., 9
GAPCOMB, 8
Gaudí, A., 19
Gaussian kernel, 21
Giacomini, M., 39
Gómez López, R., 28
González, M. del M., 25
Grané, J., 2, 6, 9
Greuel, G.-M., 28
Guàrdia, J., 1, 9, 35
Guillamon, A., 10

Hardy lectures, 35
Hardy, G., 35
harmonic anal., 21, 34
Hawking, S., 38
heat equation, 1, 20
Hubble Telescope, 34
Hugget, S., 37
Huguet, G., 10, 39
Huh, J., 28

IAS, 25
ICM-2018, 2
ICMAT, 8, 9
ICREA, 2
ICREA Academia, 1, 34
ICT, 22, 23
IDEE, 32
IEC, 9
IMAG, 18
IMTech, 8, 10, 26, 28
IMU, 26, 34
IMUVA, 28
inform. complexity, 27
IRI, 5, 16

Jaffe, A., 37

Kähler package, 28
Kellerhals, R., 32
Kinnunen, J., 3
Kohn, R., 25

L. P. Steele Prize, 25
Larrauri, A., 1, 16
Laurentiu, M., 28
Lefschetz Medal, 25
Libgober, A., 28
Linares, M. P., 35
Lluch, E., 26
LMS, 35, 36
Loughborough U., 37
Lovász, L., 8
Luengo Velasco, I., 28

MADI, 35
MAMME, 11
Maria de Maeztu, 18
Marklof, J., 37
Markov cubic, 12
Martín, S., 35
Martínez, C., 35
Mas, A., 4
Masdeu, A., 1, 22
math. models, 20
MathType, 24
Matveeva, A., 1, 17
Maynard, J., 8
Mayoral, J. A., 28
Mazzocco, M., 1, 11, 18, 36
Melle, A., 1, 18, 28
Meunier, C., 10
Miana Sanz, P., 28
millenium problems, 25, 38
Milman, E., 1, 33
Milnor number, 28
Miranda, E., 1, 17, 35, 38
Mirror symmetry, 11
Montaner, J. A., 1, 28
Morgan, F., 33
MSRI, 3, 4
Mumford, D., 34
mutual information, 26
Muñoz, J. J., 39
Muñoz, J. L., 1, 22

Nanda, V., 36
NAS, 25
Neeman, J., 1, 33
Nirenberg, L., 25
NLP, 23
Noy, M., 8, 9, 16, 39

obstacle problem, 25
Ordóñez, S., 26
Ortega, M., 9

P vs. NP, 27
Parissis, I., 4
PDE, 2, 6, 25
Pe Pereira, M., 28
Penrose, Sir R., 37, 38
Peral, I., 25
Peralta Salas, D., 35, 38
Perarnau, G., 8
Pérez, C., 4
Pérez, J., 1, 33
Ponce Álvarez, A., 1, 10
Popescu-Pampu, P., 28
Pou, T., 38
Princess of Girona, 2

Quer, J., 4, 5

R. Schock Prize, 25
RAC, 2, 25
Ramón Ipas, J. M., 28
Ramón y Cajal, 3, 10, 28
random walks, 21
Ravelomanana, V., 9
Rényi, A., 9
Ribera, M., 5, 35
Riehle, A., 10
Rivero, O., 8
Rodero de Lamo, L., 35
Rodrigo, J., 36
Roquet, V., 23
Ros-Oton, X., 1, 2, 6, 7, 32
RSME, 2, 6
Rubio de Francia, 2
Rué, J. J., 1, 8, 35

Saari, O., 1, 3
Saludes, J., 35
Sanz-Solé, M., 32
Sarközy, A., 9
SAT, 27
Schaeffer, G., 9
SCM, 2, 6, 8
Seip, K., 32
SeMA, 2
Serra, J., 2, 7
Serra, O., 9
Sharma, S., 36
Shaw Prize, 25
Soria, F., 25
Stampacchia GM, 2
Stefan problem, 25
STEM, 22, 24
Strachan, I., 36
Suciu, A., 28
Sós, V., 9

Tao, T., 38
teal philosophy, 22
The Virtual Brain, 10
Thibeault, V., 1
Thiele, C., 3
Thile, C., 4
Tillman, U., 37
Tjurina number, 28
Tschinkel, Y., 32
Turing, A., 38

U. of Zürich, 2
UAB, 3
UAM, 25, 26, 28
UB, 3, 35
UCM, 18, 25
UIMP, 26
Univ. of Bonn, 3
Univ. of Cambridge, 6
Univ. of Delaware, 5
Univ. of Oxford, 16
Univ. Paris-Sud, 10
Univ. Zaragoza, 28
UOC, 35
UPC, 2–5, 9, 10, 16, 24, 35
UPC Prize, 35
UPF, 10
Urbas, J., 1, 29
UT Austin, 2, 6, 20, 25

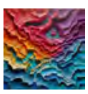
V. Caselles Prize, 2, 6
Vázquez, J. L., 1, 25
Vegué, M., 1, 13
Veselov, S., 36
Vicens Vives Award, 35
Vvodina, A., 37

Warwick Univ., 37
wavelet theory, 34
Wigderson, A., 8, 26
Wiris, 1, 24
WirisQuizzes, 24
Wolf Prize, 1, 25, 34

Xambó, S., 4, 5, 28, 29, 32, 39

Yano's conjecture, 29
Yano, T., 29
Yoshinaga, M., 28

Živný, S, 16



Background image of Eva Miranda's title slide of her 3rd LMS Hardy Tour lecture: *Desingularizing singular symplectic structures* (Birmingham, 26 June 2023). Generated with her Midjourney account.